

187(1): The Complete Equations of Torsional Cosmology

These must include the Einstein identity:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa\mu\nu}{}_\mu, \quad - (1)$$

the equation of metric compatibility:

$$D_\rho g_{\mu\nu} = 0 \quad - (2)$$

and the antisymmetry of the metric connection:

$$\Gamma^{\kappa}_{\mu\nu} = -\Gamma^{\kappa}_{\nu\mu} \quad - (3)$$

From eq. (3), we obtain the result:

$$R^{\kappa\mu\nu}{}_\mu = 0 \quad - (4)$$

Therefore the two equations are:

$$\boxed{\begin{aligned} D_\mu T^{\kappa\mu\nu} &= 0, & - (5) \\ D_\rho g_{\mu\nu} &= 0 & - (6) \end{aligned}}$$

These must be solved simultaneously.

Written out in full, eqs. (5) and (6) are:

$$D_\mu T^{\kappa\mu\nu} + \Gamma^{\kappa}_{\mu\lambda} T^{\lambda\mu\nu} - \Gamma^{\lambda}_{\mu\nu} T^{\kappa\lambda\mu} - \Gamma^{\lambda}_{\mu\nu} T^{\kappa\mu\lambda} = 0 \quad - (7)$$

$$D_\rho g_{\mu\nu} - \Gamma^{\lambda}_{\rho\mu} g_{\lambda\nu} - \Gamma^{\lambda}_{\rho\nu} g_{\mu\lambda} = 0 \quad - (8)$$

Eq. (7) is the rule for the covariant derivative of a rank three tensor, and eq. (8) is the covariant

2) derivative of a rank two tensor. The symmetries are:

$$g_{\mu\nu} = g_{\nu\mu} \quad (9)$$

$$T^{\mu}_{\nu\sigma} = -T^{\sigma}_{\nu\mu} \quad (10)$$

$$\Gamma^{\mu}_{\nu\sigma} = -\Gamma^{\sigma}_{\nu\mu} \quad (11)$$

For a spherically symmetric spacetime: (12)

$$g_{\mu\nu} = \begin{bmatrix} \exp(2\alpha(r,t)) & 0 & 0 & 0 \\ 0 & -\exp(2\beta(r,t)) & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2 \phi \end{bmatrix}$$

In general, nothing is known about metrics because of the many errors of twentieth century cosmology. Therefore the set of equations (7) to (12) must be solved by computer algebra. For the solar system it is known that the following results of AFT 186 give a good description:

$$\Gamma^0_{10} = \frac{r_0}{2r(1-r_0)}, \quad \Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{23} = \tan \phi \quad (13)$$

However this is not a fully self consistent description because of the following. From eqs. (7) and (13):

$$D_1 T^0_{10} = \partial_1 T^0_{10} + \Gamma^0_{1\lambda} T^\lambda_{10} - \Gamma^\lambda_{11} T^0_{\lambda 0} - \Gamma^\lambda_{10} T^0_{1\lambda} = 0 \quad (14)$$

3)

i.e.

$$\partial_1 T^0_{10} = \Gamma^{\lambda}_{10} T^0_{1\lambda} - \Gamma^0_{1\lambda} T^{\lambda}_{10} \quad - (15)$$

$$= 2(\Gamma^{\lambda}_{10} \Gamma^0_{1\lambda} - \Gamma^0_{1\lambda} \Gamma^{\lambda}_{10})$$

$$= 0$$

However:

$$\partial_1 T^0_{10} = \frac{\partial}{\partial r} \left(\frac{r_0}{r(r-r_0)} \right) \quad - (16)$$

$$= \frac{r_0(r_0 - 2r)}{r^2(r-r_0)^2}$$

So eqs. (15) and (16) are self-consistent only in the limit:

$$\frac{r_0}{r^2} \rightarrow 0 \quad - (17)$$

where

$$r_0 = \frac{2MG}{c^2} \quad - (18)$$

for the solar system this happens to be an excellent approximation, but it is not exactly correct. The solar system is described to an excellent approximation by one element of the solar:

$$T^0_{10} = \frac{r_0}{r(r-r_0)} \sim \frac{2MG}{c^2 r^2} \quad - (19)$$

and no curvature because of eq. (4).