

1) Generally Covariant Lorentz Force Law
Standard Model

The law is part of the transformation:

$$F'^{\mu\nu} = \frac{\partial x'^{\mu}}{\partial x^{\rho}} \frac{\partial x'^{\nu}}{\partial x^{\sigma}} F^{\rho\sigma} \quad - (1)$$

of the field tensor. In matrix notation eq. (1) is:

$$F' = AFA^{-1} \quad - (2)$$

In the standard model the Lorentz transformation is used from frame K to frame K' moving at \underline{v} with respect to K . In SI units this gives:

$$\underline{E}' = \gamma \left(\underline{E} + \underline{v} \times \underline{B} \right) - \frac{\gamma^2}{\gamma+1} \frac{\underline{v}}{c} \left(\frac{\underline{v}}{c} \cdot \underline{E} \right) \quad - (3)$$

$$\underline{B}' = \gamma \left(\underline{B} - \frac{\underline{v}}{c^2} \times \underline{E} \right) - \frac{\gamma^2}{\gamma+1} \frac{\underline{v}}{c} \left(\frac{\underline{v}}{c} \cdot \underline{B} \right) \quad - (4)$$

The Lorentz force law is part of eq. (3), i.e. for $v^2 \ll c^2$:

$$\underline{F} = e\gamma \left(\underline{E} + \underline{v} \times \underline{B} \right) = e\underline{E}' \quad - (5)$$

Note that there are some internal inconsistencies

2) in the standard model of electrodynamics, notably, it is a theory of special relativity, one frame K' can only translate w.r.t respect to another frame K at a velocity \underline{v} . Secondly the field is different from the frame, and thirdly the spacetime is confined to flat or Minkowski spacetime, i.e. the metric is:

$$\eta_{\mu\nu} = \eta^{\mu\nu} = \text{diag}(1, -1, -1, -1). \quad (6)$$

This means that electrodynamics cannot be unified w.r.t gravitation in the standard model.

ECE Field Theory

The Lorentz force law is made generally covariant and unified w.r.t gravitation. This means that a frame K' can move w.r.t respect to a frame K in any way. The electromagnetic part of the unified field is the Cartan torsion of spacetime, w.r.t the general metric $g_{\mu\nu}$. This is no longer equal to the Minkowski metric $\eta^{\mu\nu}$.

Wu refers to paper 43: "General Covariance and Coordinate Transformation in Classical and Quantum Electrodynamics", the general tensor

3) We follow eqns (16) ff of this paper. The generally covariant Lorentz force law is postulated transformation:

$$T^{a'}_{\mu'\nu'} = \Lambda^{a'}_a \frac{dx^\mu}{dx^{\mu'}} \frac{dx^\nu}{dx^{\nu'}} T^a_{\mu\nu} \quad (7)$$

of the Cartan torsion tensor:

$$T^a_{\mu\nu} = \partial_\mu v_\nu^a - \partial_\nu v_\mu^a + \omega_{\mu b}^a v_\nu^b - \omega_{\nu b}^a v_\mu^b \quad (8)$$

The electromagnetic field is:

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \quad (9)$$

so the most general Lorentz force (cf. eq. (5)) is proportional to:

$$F^{a'}_{\mu'\nu'} = A^{(0)} T^{a'}_{\mu'\nu'} \quad (10)$$

In general, gravitation influences $F^{a'}_{\mu'\nu'}$ through the spin connection. This is the rigorous covariant form of the law is $F^{a'}_{\mu'\nu'}$