Notes for Paper 53: Coulomb's Law in ECE Resonant Form

The origin of Coulomb's Law in ECE field theory is the inhomogeneous field equation:

$$d \wedge \tilde{F} = \mu_0 J = A^{(0)} (\tilde{R} \wedge q - \omega \wedge \tilde{T}) \quad -(1)$$

which in tensorial notation becomes:

$$\partial_\mu F^{a\mu\nu} = \mu_0 J^{a\nu} = A^{(0)} (R^a{}_b{}^{\mu\nu} q^b_\mu - \omega^a_{\mu b} T^{b\mu\nu}). \quad -(2)$$

The notation is fully explained in previous notes and publications, in particular appendix K of volume 1.

The Coulomb Law is obtained by using $\nu = 0$

$$\partial_1 F^{a10} + \partial_2 F^{a20} + \partial_3 F^{a30} = \mu_0 J^{a0} \quad -(3)$$

where:

$$J^{a0} = \frac{A^{(0)}}{\mu_0} \Big(R^a{}_b{}^{10} q^b_1 + R^a{}_b{}^{20} q^b_2 + R^a{}_b{}^{30} q^b_3 - \omega^a_{1b} T^{b10} - \omega^a_{2b} T^{b20} - \omega^a_{3b} T^{b30} \Big) \quad -(4)$$

In vector notation eqn. (3) is:

$$\boxed{\underline{\nabla} \cdot \underline{E}^a = -c\, \underline{A}^{b\prime} \cdot \underline{R}^a{}_b - \underline{\omega}^a{}_b{}' \cdot \underline{E}^b}$$

2) Conventionally we have defined:

$$A^{a}_{\mu}{}' := g_{\mu\nu} A^{a\nu} \quad - (6)$$

$$\omega^{a}_{\mu b}{}' := g_{\mu\nu} \omega^{\nu a}{}_{b} \quad - (7)$$

i.e. in previous notes we have defined $A^{a\nu}$ and $\omega^{\nu a}{}_{b}$ as metric free. It follows that the $A^{a}_{\mu}{}'$ and $\omega^{a}_{\mu b}{}'$ quantities have to be defined as in eqs. (6) and (7), so we distinguish them with the tick. This is needed because $g_{\mu\nu}$ in general is no longer the Minkowski metric.

The resonant version of eq. (5) may now be developed as:

$$\underline{\nabla} \cdot \underline{E}^{a} + \underline{\omega}^{a}{}_{b}{}' \cdot \underline{E}^{b} = -c\, \underline{A}^{b}{}' \cdot \underline{R}^{a}{}_{b} \quad - (8)$$

where

$$\underline{E}^{a} = -\frac{\partial \underline{A}^{a}}{\partial t} - c\, \underline{\nabla} A^{0a} - c\, \omega^{0a}{}_{b} \underline{A}^{b} + c\, \underline{\omega}^{a}{}_{b} A^{0b} \quad - (9)$$

Eqs (8) and (9) give the linear inhomogeneous differential equation

$$\boxed{\begin{aligned} & c\, \underline{\nabla} \cdot \underline{\nabla} A^{0a} + \underline{\nabla} \cdot \frac{\partial \underline{A}^{a}}{\partial t} + c\, \underline{\nabla} \cdot \left(\omega^{0a}{}_{b} \underline{A}^{b} - \underline{\omega}^{a}{}_{b} A^{0b} \right) \\ & + \underline{\omega}^{a}{}_{b}{}' \cdot \left(\frac{\partial \underline{A}^{b}}{\partial t} + c\, \underline{\nabla} A^{0b} + c\, \omega^{0b}{}_{c} \underline{A}^{c} - c\, \underline{\omega}^{b}{}_{c} A^{0c} \right) \\ & = -c\, \underline{A}^{b}{}' \cdot \underline{R}^{a}{}_{b} \quad - (10) \end{aligned}}$$

3)

Static Electric Field

In the standard model a static electric field is defined by:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(11)$$

$$\underline{\nabla} \times \underline{E} = \underline{0} \quad -(12)$$

$$\nabla^2 \Phi = -\frac{\rho}{\epsilon_0} \quad -(13)$$

Eq (11) is Coulomb's Law in the standard model, and eq (13) is the Poisson equation. Eq. (12) implies that:

$$\underline{E} = -\underline{\nabla}\, \underline{\Phi}. \quad -(14)$$

The above is part of the Maxwell Heaviside theory of special relativity. In general relativity the Coulomb law is unified with gravitation (ECE). More generally in the standard model:

$$\underline{E} = -\underline{\nabla}\, \underline{\Phi} - \frac{\partial \underline{A}}{\partial t} \quad -(15)$$

so a static electric field means that:

$$\frac{\partial \underline{A}}{\partial t} = \underline{0}. \quad -(16)$$

Therefore to investigate the effect of a static electric field configuration on gravitation eq. (10) can be approximated

4)

If we assume for the sake of approximation that

$$\underline{\nabla} \cdot \underline{A}^{b} = 0 \quad -(17)$$

the eq. (10) simplifies further to:

$$\underline{\nabla} \cdot \underline{\nabla} A^{oa} - \underline{\nabla} \cdot \left(\underline{\omega}^{a}{}_{b} A^{ob} \right) + \underline{\omega}^{a}{}_{b}{}' \cdot \underline{\omega}^{ob}{}_{c} \underline{A}^{c} - \underline{\omega}^{a}{}_{b}{}' \cdot \underline{\omega}^{b}{}_{c} A^{oc}$$

$$+ \underline{\omega}^{a}{}_{b}{}' \cdot \underline{\nabla} A^{ob}$$

$$= - \underline{A}^{b\prime} \cdot \underline{R}^{a}{}_{b} = c\mu_0 J^{oa}$$

$$-(18)$$

Limit of Weak Interaction

In the limit of weak interaction between e/m and gravitation eq. (1) simplifies to:

$$d \wedge \tilde{F} = \mu_0 J \longrightarrow A^{(0)} (\tilde{R} \wedge q)_{grav} \quad -(19)$$

(see appendices of volume one). This is because:

$$(\tilde{R} \wedge q)_{em} \sim (\omega \wedge \tilde{T})_{em} \quad -(20)$$

and:

$$\tilde{T}_{grav} \sim 0. \quad -(21)$$

The structure of eq (5) simplifies to:

$$\underline{\nabla} \cdot \underline{E}^{a} \sim \bar{\mathcal{L}} c \underline{A}^{b\prime} \cdot \underline{R}^{a}{}_{b} \quad -(22)$$

and the spin connection in eq. (9) is

5) approximately dual to the tetrad, because it is the spin connection for e/m, governed by eq. (20). So the resonant equation simplifies to:

$$\underline{\nabla} \cdot \left(-\frac{\partial \underline{A}^a}{\partial t} - c\,\underline{\nabla} A^{0a} - c\,\omega^{0a}{}_b \underline{A}^b + c\,\underline{\omega}^a{}_b A^{0b} \right) \sim \overline{\underline{\lceil c \underline{A}}}^{b\prime} \cdot \underline{R}^a{}_b \quad -(23)$$

with:

$$\omega^a{}_b \sim -\frac{\kappa}{2} q^c \epsilon^a{}_{bc} \quad -(24)$$

If we use a static electric field and assume eqn (17), eq. (23) simplifies to:

$$\boxed{\underline{\nabla} \cdot \underline{\nabla} A^{0a} - \underline{\nabla} \cdot \left(\underline{\omega}^a{}_b A^{0b} \right) \sim -\underline{A}^{b\prime} \cdot \underline{R}^a{}_b \quad -(25)}$$

If we do not assume eq. (17):

$$\boxed{\underline{\nabla} \cdot \underline{\nabla} A^{0a} + \underline{\nabla} \cdot \left(\omega^{0a}{}_b \underline{A}^b \right) - \underline{\nabla} \cdot \left(\underline{\omega}^a{}_b A^{0b} \right) \sim -\underline{A}^{b\prime} \cdot \underline{R}^a{}_b \quad -(26)}$$

Eq. (25) is a Hooke's Law type eqn. with driving force, eqn. (26) has a damping term. In both cases resonance occurs, i.e. gravitation is resonantly affected by an electric field.