Notes for Paper 55 Part Three

Orbital and Intrinsic Torque

As seen in paper 55, notes 2, the orbital torque is

$$\underline{N}_L^{\,a} = -\frac{\partial \underline{J}^a}{\partial t} - c\,\underline{\nabla} J^{0a} - c\,\omega^{0a}{}_b\,\underline{J}^b + c\,\underline{\omega}^a{}_b\,J^{0b} \quad -(1)$$

and the intrinsic or spin torque is:

$$\underline{N}_S^{\,a} = c\left(\underline{\nabla} \times \underline{J}^a - \underline{\omega}^a{}_b \times \underline{J}^b\right). \quad -(2)$$

In general there is a combination of orbital and spin torque:

$$\underline{N}^a = \underline{N}_L^{\,a} + \underline{N}_S^{\,a}. \quad -(3)$$

If the angular momentum is irrotational:

$$\underline{\nabla} \times \underline{J}^a = \underline{0}. \quad -(4)$$

For pure rotational motion unaffected by gravitation:

$$\omega^a{}_b = -\frac{\kappa}{2}\,\epsilon^a{}_{bc}\,q^c \quad -(5)$$

$$R^a{}_b = -\frac{\kappa}{2}\,\epsilon^a{}_{bc}\,T^c \quad -(6)$$

where $R^a{}_b$ is the curvature form appropriate to pure rotation.

2) The angular velocity is defined as:

$$\Omega^a{}_b = c\,\omega^a{}_b = -\frac{\omega}{2}\epsilon^a{}_{bc}\, q^c \quad -(7)$$

where

$$\Omega^c = \omega\, q^c, \quad -(8)$$

$$\omega = \kappa c. \quad -(9)$$

So:

$$\Omega^a{}_b = -\frac{1}{2}\epsilon^a{}_{bc}\,\omega^c. \quad -(10)$$

It is known that the orbital torque is defined in the Newtonian limit by:

$$\underline{N}_L = \underline{r} \times \underline{F}, \quad -(11)$$

where $\underline{r}$ is distance and $\underline{F}$ is force. In ECE theory this relation is given by:

$$F^a{}_b = c\, J^{(0)} R^a{}_b. \quad -(12)$$

The orbital torque is:

$$N^c_L = c\, J^{(0)} T^c_L, \quad -(13)$$

so:

$$\boxed{N^c_L = -\frac{r}{2}\epsilon^{bc}{}_a F^a{}_b.} \quad -(14)$$

This is eqn. (11) in tensor notation. However, eq. (14) is philosophically different from eq. (11)

3) because eq. (14) originates in the spinning of spacetime itself. Eq. (11) originates in the non-relativistic Newtonian force $\underline{F}$. Eq. (14) is generally covariant but eq. (11) is not covariant. In Newtonian dynamics the spin torque does not exist. In Eulerian dynamics the term:

$$\underline{N}_S{}^a = -c\,\underline{\omega}^a{}_b \times \underline{J}^b \quad -(15)$$

is introduced by considering the spin of one frame with respect to another. In ECE this Eulerian term is part of the spin torque, and again due to the spinning of spacetime itself.

The conclusion is that there are many phenomena of rotational dynamics that are indicated by ECE theory but not by the classical Euler equations.