GEODESICS AND THE AHARONOV BOHM EFFECTS IN ECE THEORY

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ABSTRACT

Geodesics are considered in the Einstein Cartan Evans (ECE) unified field theory. The concept of parallel transport along any curve is extended to the covariant exterior derivative of Cartan geometry. The Lorentz force is thus recognized as the covariant exterior derivative of the potential form along any path in ECE space-time. The class of Aharonov Bohm effects are due to parallel transport of the exterior covariant derivative. The equations of the electromagnetic and gravitational Aharonov Bohm effects are developed in ECE theory.

Keywords: Einstein Cartan Evans (ECE) unified field theory, geodesics, Lorentz force equation, Aharonov Bohm effects.

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1. INTRODUCTION

In Einsteinian philosophy every equation of physics must be rigorously objective to all observers. This is the most fundamental principle of relativity theory, the latter must be generally covariant, covariant under the general coordinate transformation. This principle must apply both on the classical and quantum levels. Recently {1-15}, a generally covariant unified field theory has been developed which meets these fundamental philosophical requirements, and is known as Einstein Cartan Evans (ECE) field theory. It has been applied to many aspects of physics {1, 2} and has been tested extensively against experimental data. It also meets the fundamental philosophical requirement known as Okham's Razor, that a theory of physics must be as simple as possible. In this respect the ECE theory needs only the four dimensions of the original theory of relativity, and is preferred to string theory. The latter uses many unphysical dimensions as is well known. ECE theory is preferred to gauge theory because the latter is not generally covariant in three of its sectors: the electromagnetic, weak and strong sectors. Gauge theory in these sectors superimposes an abstract set of numbers on a Minkowski spacetime, while ECE theory is rigorously covariant in all sectors {1-15}, being based directly on standard Cartan geometry. Gauge theory runs into trouble in trying to explain the well known Aharonov Bohm (AB) effects, while ECE theory explains them straightforwardly. The standard model runs into trouble in quantum mechanics, because of the lack of objectivity in its quantum mechanical sector, while ECE theory rigorously retains objectivity on the classical and quantum levels. Therefore ECE theory is preferred to the standard model, and explains data in a generally covariant manner under all circumstances.

In Section 2 the theory of geodesics in ECE theory is developed by extending the concept of parallel transport to the covariant exterior derivative of Cartan geometry. It is shown that the potential form {1-15} is parallel transported along any curve in ECE theory. It follows that the Cartan structure equations are also parallel transported along any curve, and that the Lorentz force equation is the covariant exterior derivative of the potential form along any path in ECE space-time. All AB effects are due to parallel transport of the exterior covariant derivative. These concepts extend the concept of the geodesic in relativity theory. The geodesic is the path that parallel transports it own tangent vector {15} and in gravitational relativity (Einstein Hilbert (EH) field theory) is the path followed by unaccelerated particles. In Section 3 the equations of the electromagnetic AB effect are given, and in Section 4, the equations are given of the gravitational AB effect.

2. PARALLEL TRANSPORT AND GEODESICS IN ECE THEORY, APPLICATIONS TO THE LORENTZ FORCE EQUATION AND AHARONOV BOHM EFFECTS.

In standard general relativity (EH theory {15}) the geodesic is defined as the path that parallel transports its own tangent vector. If $x^\mu(\lambda)$ is any curve, the tangent vector is defined {15} as:

$$t^\mu = \frac{dx^\mu}{d\lambda} . \qquad (1)$$

The covariant derivative along the path is defined {15} as:

$$\frac{D}{d\lambda} := \frac{dx^\mu}{d\lambda} D_\mu \qquad (2)$$

where D_μ is the covariant derivative. The parallel transport of any tensor is then defined {15} as:

$$\left(\frac{DT}{d\lambda}\right)^{\mu_1\mu_2\cdots\mu_k}_{\nu_1\nu_2\cdots\nu_l} = \frac{dx^\sigma}{d\lambda} D_\sigma T^{\mu_1\mu_2\cdots\mu_k}_{\nu_1\nu_2\cdots\nu_l} = 0. \qquad (3)$$

In EH theory the connection is always metric compatible {15}:

$$D_\mu g_{\rho\nu} = 0 \quad -(4)$$

where $g_{\rho\nu}$ is the symmetric metric of EH field theory. Such a connection is always parallel transported {15}:

$$\frac{D g_{\mu\nu}}{d\lambda} = \frac{dx^\sigma}{d\lambda} D_\sigma g_{\mu\nu} = 0. \quad -(5)$$

The equation of the geodesic in EH theory is {15}:

$$\frac{D}{d\lambda}\left(\frac{dx^\mu}{d\lambda}\right) = 0 \quad -(6)$$

and parallel transports the tangent vector. Eq. (6) can be rewritten as:

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0 \quad -(7)$$

which is the best known form of the geodesic equation of EH field theory. The geodesic in EH field theory is the path followed by unaccelerated particles. In flat space-time the connection vanishes, and Eq. (7) becomes:

$$\frac{d^2 x^\mu}{d\lambda^2} = 0 \quad -(8)$$

which is the equation of a straight line. Newton's first law is therefore regained, unaccelerated particles move in a straight line. The parameter λ can be related {15} to the proper time τ by:

$$\lambda = a\tau + b. \quad -(9)$$

An affine parameter is related to the proper time in this way. If:

$$\lambda = \tau \quad -(10)$$

the geodesic equation becomes:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\tau} \frac{dx^\sigma}{d\tau} = 0 \quad -(11)$$

and in the Newtonian limit this equation means that:

$$\underline{a} = \underline{0} \quad -(12)$$

where $\underline{a}$ denotes acceleration. Therefore in the Newtonian limit the geodesic equation is the Newton force law for:

$$\underline{f} = m\underline{a} = \underline{0}. \quad -(13)$$

The Newton force law is therefore the limit of a more general geometry than Euclidean geometry. For EH theory this is well known to be Riemann geometry. For ECE theory Riemann geometry is generalized to the well known Cartan geometry {1-15}.

When there is a force present the right hand side of Eq. (11) is no longer zero. The standard model Lorentz force for example (15} is:

$$f^\mu = e U^\lambda F_\lambda{}^\mu = e F^\mu{}_\nu \frac{dx^\nu}{d\tau} \quad -(14)$$

where $F^\mu{}_\nu$ is the standard model electromagnetic field tensor and U^λ is the four velocity. In the presence of a Lorentz force, the electron no longer moves along a geodesic. The standard model Lorentz force, however, is Lorentz covariant, and is therefore the result of a theory of special relativity, not of general relativity as required. In this section the required generally covariant Lorentz force equation is developed with ECE theory. In so doing the effect of gravitation on the Lorentz force may be estimated. This is not possible in the standard model.

Parallel transport is defined for the arbitrary tensor T (indices suppressed for clarity) as:

$$\frac{dT}{d\lambda} = \frac{dx^\mu}{d\lambda}\frac{\partial T}{\partial x^\mu} = 0. \qquad -(15)$$

This result comes from the rule for differentiation {16} of a function of a function. If:

$$T = T\left(x^\mu(\lambda)\right) \qquad -(16)$$

then:

$$\frac{dT}{d\lambda} = \frac{dT}{dx^\mu}\frac{dx^\mu}{d\lambda}. \qquad -(17)$$

The rule (17) may be extended to the covariant derivative of a tensor of any rank:

$$\frac{DT}{d\lambda} = \frac{dx^\mu}{d\lambda} D_\mu T \qquad -(18)$$

where T is defined by Eq. (16). The rule (18) is extended in this Section to the covariant exterior derivative of Cartan {1-15}, used in the first and second Cartan structure equations:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \qquad -(19)$$

and

$$R^a{}_b = D \wedge \omega^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b. \qquad -(20)$$

Here T^a is the torsion form, $R^a{}_b$ is the Riemann or curvature form, q^a is the tetrad form, $\omega^a{}_b$ is the spin connection, D ^ denotes the covariant exterior derivative and d ^ denotes the exterior derivative. It will be proven that:

$$\frac{dx^\mu}{d\lambda}(D \wedge A)_{\mu\nu} = \frac{dx^\mu}{d\lambda} F^a{}_{\mu\nu} \qquad -(21)$$

is the origin of the generally covariant Lorentz force and also the origin of the class of AB effects. Here $F^{a}_{\mu\nu}$ is the generally covariant electromagnetic field tensor of ECE theory, defined in standard notation {1-15} as:

$$F^{a} = D \wedge A^{a}. \quad -(22)$$

Consider:

$$A^{a}_{\mu} = A^{a}_{\mu}\left(x^{\nu}(\lambda)\right) \quad -(23)$$

and

$$A^{a}_{\nu} = A^{a}_{\nu}\left(x^{\mu}(\lambda)\right) \quad -(24)$$

with:

$$F^{a}_{\mu\nu} = \partial_{\mu} A^{a}_{\nu} - \partial_{\nu} A^{a}_{\mu} + \omega^{a}_{\mu b} A^{b}_{\nu} - \omega^{a}_{\nu b} A^{b}_{\mu}. \quad -(25)$$

Then:

$$\frac{DA^{a}_{\mu}}{d\lambda} = \frac{dx^{\mu}}{d\lambda} D_{\nu} A^{a}_{\mu}, \quad -(26)$$

$$\frac{DA^{a}_{\nu}}{d\lambda} = \frac{dx^{\nu}}{d\lambda} D_{\mu} A^{a}_{\nu}. \quad -(27)$$

Using the tetrad postulate {1-15}:

$$D_{\nu} A^{a}_{\mu} = D_{\mu} A^{a}_{\nu} = 0, \quad A^{a}_{\mu} = A^{(0)} q^{a}_{\mu}, \quad -(28)$$

it follows that:

$$\frac{DA^{a}_{\mu}}{d\lambda} = \frac{DA^{a}_{\nu}}{d\lambda} = 0 \quad -(29)$$

a result which means that

$$A^{a}_{\mu} = A^{(0)} q^{a}_{\mu} \quad -(30)$$

is parallel transported along any curve $x^\mu(\lambda)$. It follows that:

$$\frac{dx^\mu}{d\lambda}\left(D_\nu A^a_\mu - D_\mu A^a_\nu\right) = 0. \quad -(31)$$

The exterior covariant derivative is defined as:

$$(D \wedge A)^a_{\mu\nu} = F^a_{\mu\nu} \quad -(32)$$

with:

$$D_\mu A^a_\nu - D_\nu A^a_\mu = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \omega^a_{\mu b} A^b_\nu - \omega^a_{\nu b} A^b_\mu - F^a_{\mu\nu}$$
$$= 0. \quad -(33)$$

It follows from Eqs. (31) and (33) that the Cartan structure equation is parallel transported along any curve $x^\mu(\lambda)$:

$$\frac{dx^\mu}{d\lambda}\left(F^a - D \wedge A^a\right)_{\mu\nu} = 0 \quad -(34)$$

and so:

$$\frac{dx^\mu}{d\lambda}\left(D \wedge A^a\right)_{\mu\nu} = \frac{dx^\mu}{d\lambda} F^a_{\mu\nu} \quad -(35)$$

Q.E.D. The generally covariant Lorentz force is the right hand side of Eq. (35) multiplied by e, the charge on the electron.

In the absence of the Lorentz force and thus in the absence of acceleration:

$$\frac{dx^\mu}{d\lambda} F^a_{\mu\nu} = \frac{dx^\mu}{d\lambda}\left(D \wedge A^a\right)_{\mu\nu} = 0 \quad -(36)$$

and the exterior covariant derivative of Cartan {1-15} is parallel transported along any curve.

Similarly:

$$\frac{dx^\mu}{d\lambda} R^a{}_{b\mu\nu} = \frac{dx^\mu}{d\lambda}\left(D \wedge \omega^a{}_b\right)_{\mu\nu} \quad -(37)$$

and in the absence of force, the exterior covariant derivative of the spin connection is parallel transported along any curve:

$$\frac{dx^\mu}{d\lambda} R^a{}_{b\mu\nu} = \frac{dx^\mu}{d\lambda} \left(D \wedge \omega^a{}_b \right)_{\mu\nu} = 0. \quad -(38)$$

The class of AB effects may now be defined in terms of these equations. The electromagnetic AB effects are defined by Eq. (36). When there is no field there may still be a potential present in ECE theory, a potential defined by:

$$A^{(0)} \left(d \wedge q^a + \omega^a{}_b \wedge q^b \right) = 0, \quad -(39)$$

where $cA^{(0)}$ is a primordial voltage. Eq (39) is true for all $A^{(0)}$ and all paths $x^\mu(\lambda)$. The Cartan geometry of the electromagnetic AB effects is therefore always:

$$D \wedge q^a = 0. \quad -(40)$$

Similarly the Cartan geometry of the gravitational AB effects is always:

$$D \wedge \omega^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b = 0. \quad -(41)$$

All AB effects are therefore due to parallel transport of the exterior covariant derivative of Cartan geometry. They are effects of a generally covariant unified field theory {1, 2}, of spinning and curving space-time. In the standard model there are no AB effects because in the standard model {1, 2}:

$$F_{\mu\nu} = (d \wedge A)_{\mu\nu} \quad -(42)$$

and if F is zero so is A and vice-versa.