CANONICAL AND SECOND QUANTIZATION IN GENERALLY COVARIANT QUANTUM FIELD THEORY

by

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ABSTRACT

Einstein Cartan Evans (ECE) field theory is shown to be a rigorous quantum field theory in which the tetrad is both the eigenfunction or wave-function and a quantized field that is generally covariant. Unification of fields is achieved with standard Cartan geometry on both the classical and quantum levels. The fundamental commutators needed for canonical quantization of the field / eigenfunction are derived self consistently from the same Cartan geometry. Second quantization proceeds straightforwardly thereafter by expanding the tetrad in terms of creation and annihilation operators. The latter are used to define the number operator in the usual way, and a generally covariant multi particle field theory obtained. The theory is illustrated with a discussion of the electromagnetic Aharonov Bohm effect.

Keywords: Einstein Cartan Evans (ECE) unified field theory, quantum field theory, canonical quantization and second quantization, Aharonov Bohm effects.

57th paper

1. INTRODUCTION

Einstein Cartan Evans (ECE) unified field theory has been well developed analytically on the classical and single particle quantum levels {1-7}. In this paper it is shown that ECE field theory is a rigorous quantum field theory in which the tetrad is both the wave-function and the field. In Section 2, the fundamental commutators needed for canonical quantization {8,9} are introduced from Cartan geometry and developed and in section 3 second quantization {8,9} is developed straightforwardly by expanding the tetrad in a Fourier series to give creation and annihilation operators that define the number operator. Therefore ECE theory produces a rigorous quantum field theory and can be given a multi particle interpretation as required {8,9}. The theory in this paper is illustrated in Section 4 with the electromagnetic Aharonov Bohm effect.

Canonical quantization in quantum field theory is the name given to the construction of the Heisenberg commutators of the quantum field. Canonical quantization of the electromagnetic potential field, for example, runs into difficulties {8} in the contemporary standard model because of the assumption of a massless electromagnetic field with infinite range, and an identically zero photon mass. In special relativity this means that the special relativistic potential field A_μ can have only two physical components {8} and these are taken as the transverse components. However, A_μ must have four physical components to be manifestly covariant, and this is also a fundamental requirement of general relativity. Therefore there is a basic contradiction in the standard model, a contradiction that leads to well known difficulties {1-15}. It is shown in Section 2 that this contradiction is removed straightforwardly in ECE theory, in which the photon mass is identically non-zero as required. Without photon mass there can be no explanation of the Eddington experiment, contradicting the well known tests of the Einstein Hilbert (EH) field theory of gravitation, tests which are now known from NASA Cassini to be accurate to one part in one hundred thousand for light grazing the sun. So the existence of identically non-zero photon mass has been tested to this accuracy, because the Eddington experiment is explained in EH theory by considering the mass of the photon and the mass of the sun. If the photon mass is identically zero, the EH explanation makes no sense. Thus, photon mass is known very accurately to be identically non-zero. In Maxwell Heaviside field theory however, it is identically zero. This diametric contradiction is inherent in the standard model because gravitation is in that model a theory of general relativity and electromagnetism is a theory of special relativity (the Maxwell Heaviside (MH) field theory). Electromagnetism in the standard model is developed {8,9} in terms of gauge theory. The simplest example of gauge theory is illustrated as follows. In MH theory the electromagnetic field in tensor notation is an anti-symmetric rank two tensor independent of gravitation:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \quad -(1)$$

This field tensor is unchanged under the mathematical transform:

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda \quad -(2)$$

which is a simple example of a gauge transformation {8-15}. Here Λ is any scalar function and the invariance of the field under the gauge transformation is an example of the Poincaré Lemma. The field is said to be gauge invariant, and in gauge field theory this is a central hypothesis which leads to a debate concerning the nature of $F_{\mu\nu}$ and A_μ . One or the other is regarded as fundamental, there being proponents of both views. ECE theory has shown that this debate is superfluous, and ECE has replaced gauge theory with a generally covariant unified field theory {1-7} in which the fundamental transformation is the general coordinate transformation of general relativity. The debate in the gauge theory of the standard model was initiated to a large extent by the discovery of the magnetic Aharonov Bohm effect by

Chambers {8, 9}.It was shown experimentally by Chambers that in regions where $F_{\mu\nu}$ is zero, A_μ has a physical effect. The standard classical interpretation had been that $F_{\mu\nu}$ is physical but A_μ is unphysical. This interpretation was based directly on classical gauge theory, first proposed by Weyl and his contemporaries. In the ensuing forty year debate concerning the Aharonov Bohm (AB) effects some proponents have held the view that they are purely quantum effects, and in quantum theory the minimal prescription applies:

$$p_\mu \rightarrow p_\mu + eA_\mu \qquad -(3)$$

Here p_μ is the four-momentum of special relativity and -e is the charge on the electron. In this view, used for example in the Dirac equation to explain the Stern Gerlach effect, A_μ is a physical property. In gauge theory it is held that A_μ can be transformed to $A_\mu + \partial_\mu \Lambda$ without affecting the field $F_{\mu\nu}$, so in consequence A_μ is unphysical. The philosophical basis of gauge theory is therefore questionable, the assumption that A_μ is unphysical and that $F_{\mu\nu}$ is physical is untenable and confusing even to the experts, thus a protracted forty year debate that reveals this confusion. In the standard model of the late twentieth century, gauge invariance was elevated to a central hypothesis of the electromagnetic, weak and strong fields and has been applied to the gravitational field, where it is clearly superfluous by Okham's Razor. The general coordinate transformation is already sufficient for gravitational field theory and there is no need for the further postulate of gauge invariance. ECE theory has shown {1-7} that this is also true for unified field theory. The difficulties of using gauge theory outweigh any of its advantages, the latter being increasingly difficult to find as ECE is increasingly developed and accepted {16}. It has been shown {1-7} that ECE theory leads to a unified field theory in which the gravitational, electromagnetic, weak and strong fields are represented by the tangent space-time at point P to the base manifold in standard Cartan geometry. The various fields are represented by various representation spaces in the tangent space-time {1-7}. Gauge theory is superfluous in this context.

The difficulty inherent in the fundamental assumption of gauge invariance can be illustrated as follows using differential form notation {1-7, 17}, where Eqs. (1) and (2) become:

$$A \rightarrow A + d\Lambda \qquad -(4)$$

$$F = d \wedge A = d \wedge (A + d\Lambda) \qquad -(5)$$

because:

$$d \wedge d\Lambda := 0. \qquad -(6)$$

Eq. (6) is the Poincaré Lemma in differential form notation. However, ECE theory now shows that the generally covariant foundation of electrodynamics unifies the latter with the other fields, notably the gravitational field. The standard model is unable to do this despite many attempts throughout the twentieth century. In ECE theory the relevant differential form equations of the electromagnetic sector are {1-7}:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b, \qquad -(7)$$

$$d \wedge F^a = \mu_0 j^a, \qquad -(8)$$

$$d \wedge \tilde{F}^a = \mu_0 J^a. \qquad -(9)$$

Here $\omega^a{}_b$ is the spin connection, which is identically non-zero because the electromagnetic field is always a spinning frame, not a static frame as in the standard model. The homogeneous current j^a is in general non-zero, it vanishes if and only if the electromagnetic and gravitational fields become independent and do not influence each other {1-7}. The presence of j^a (however tiny in magnitude) is of key importance, because it may be amplified by resonance {1-7} producing easily measurable electric power from ECE space-time, a new source of energy. In the standard model j^a does not exist, there is no concept of j^a in the standard model because electromagnetism there is a concept of special relativity superimposed on a flat Minkowski frame. Eq. (9) is the Hodge dual of Eq. (8) and here μ_0 is the vacuum permeability in S.I. units. The electromagnetic potential field in ECE theory is

$$A^a = A^{(0)} q^a \quad -(10)$$

where $cA^{(0)}$ is the primordial voltage and q^a the tetrad field. Thus F^a is constructed from A^a through the first Cartan structure equation (7), and F^a obeys the first Bianchi identity, eq (8), and its Hodge dual (9). These are the classical field equations. The vector valued one-form A^a_μ and the vector valued two-form $F^a_{\mu\nu}$ are covariant under the general coordinate transformation {1-7, 17} according to the well known rules of standard Cartan geometry. If we attempt a "gauge transformation":

$$A^a \rightarrow A^a + d\Lambda^a \quad -(11)$$

then:

$$F^a \rightarrow d \wedge A^a + \omega^a{}_b \wedge A^b + \omega^a{}_b \wedge d\Lambda^b = F^a + \omega^a{}_b \wedge d\Lambda^b \quad -(12)$$

and F^a is not invariant: it must be coordinate covariant, not gauge invariant. Another fundamental problem of gauge theory in the standard model is that it uses a hypothesis superfluous to general relativity, the indices a of gauge theory are abstract mathematical concepts, whereas in Cartan geometry a is the index of the tangent space-time and thus well defined by geometry as required by relativity theory. In ECE theory the space-time that defines the electromagnetic field is the same as the space-time that defines all fields, including the gravitational field, in four physical dimensions, ct, X, Y and Z. Thus ECE is preferred to gauge theory by Okham's Razor of philosophy, which asserts the use of the minimum number of concepts. In ECE theory a is already inherent in Cartan geometry, in gauge theory the label a is introduced as an extra mathematical assumption, i.e. of Yang Mills theory. Similarly ECE theory is preferred to string theory by Okham's razor, because string theory uses superfluous parameters which are asserted arbitrarily to be dimensions. There is no experimental evidence for string theory, and for this reason string theory has been described as pre-Baconian. The experimental evidence for ECE theory is given in ref. (1) to (7) in a representative cross section of contemporary physics.

In ECE theory the fundamental wave equation of electrodynamics is {1-7}:

$$(\square + kT) A^a_\mu = 0 \quad -(13)$$

where k is the Einstein constant and T is the index contracted canonical energy-momentum density of the unified field. The Einstein Ansatz asserts that:

$$R = -kT \quad -(14)$$

where R is the scalar curvature. Using the correspondence principle of Einstein, the Proca equation emerges from Eq. (13) in the well defined limit:

$$kT \rightarrow \left(\frac{mc}{\hbar}\right)^2 \quad -(15)$$

where m is the identically non-zero photon mass indicated by the Eddington experiment, and $\hbar$ is the Planck constant. Note carefully that the d'Alembert wave equation of the standard model does not emerge from ECE theory, indicating that the d'Alembert wave equation is incomplete because it asserts identically zero photon mass. In the standard model the interpretation of the Proca equation is self-contradictory {8} because the Lagrangian needed to derive it is not gauge invariant. So for this reason the Proca equation is not used for canonical quantization in the standard model. The basic weak point here is again the assumption of gauge invariance, which has so long been thought of as the strength of gauge theory. In ECE theory there is no problem with the Proca equation because as we have seen, gauge invariance has been replaced by coordinate covariance in the unified field. Therefore the photon mass is identically non-zero as required by general relativity (photon mass was first proposed by Einstein) and the electromagnetic field is manifestly covariant with four physical components: time-like and three space-like, two transverse and one longitudinal. In the standard model the time-like and longitudinal components are dubiously removed by the Gupta Bleuler method {1-15}. The latter procedure is incorrect in general relativity, which prohibits the existence of a massless field. A massless field would mean no curvature R, and nothing at all (no field, no particles). Thus A^{a}_{μ} is manifestly covariant in ECE theory and can be canonically quantized in a rigorously correct way.

2 CANONICAL QUANTIZATION OF THE ECE FIELD.