Notes 57(3): Solution of the Field Equations and Electromagnetic AB Effects.

The equations of the electromagnetic AB effects are:

$$F = 0, \quad D \wedge A = 0, \quad A \neq 0. \quad -(1)$$

In standard notation:

$$d \wedge A^a + \omega^a{}_b \wedge A^b = 0, \quad -(2)$$

$$\omega^a{}_b \neq 0. \quad -(3)$$

It is important to note that the spin connection $\omega^a{}_b$ must be non-zero, a fundamental requirement of general relativity. In order to obtain solutions of eq. (1) the spin connection must be given an analytical form. We know from the Faraday law of induction, contained in:

$$d \wedge F^a = 0, \quad -(4)$$

that:

$$j^a = 0. \quad -(5)$$

Eq. (5) holds in the laboratory under the usual experimental conditions. Eq. (5) implies that:

$$\omega^a{}_b = -\frac{\kappa}{2} \epsilon^a{}_{bc} q^c \quad -(6)$$

to a good approximation. The proportionality constant $-\kappa/2$ in eq. (6) has been assumed to be a scalar, the minus sign and factor half have been chosen for convenience. More generally, the proportionality factor in eq. (6) can be a tensor and so may have different components. Its units are inverse metres.

2)

Now use the ECE Ansatz:

$$A^a = A^{(0)} q^a \quad -(7)$$

to find that:

$$\omega^a{}_b = -\frac{g}{2} \epsilon^a{}_{bc} A^c \quad -(8)$$

where:

$$g = \frac{\kappa}{A^{(0)}} \quad -(9)$$

Therefore the AB effects are described by:

$$\boxed{d \wedge A^a = \frac{g}{2} \epsilon^a{}_{bc} A^c \wedge A^b} \quad -(10)$$

In the region defined by eq. (10):

$$F^a = 0. \quad -(11)$$

Eq (10) is determined to an excellent approximation by the Faraday law of induction (4). The latter is of course well proven experimentally under known lab conditions, but under resonance conditions, j^a may become non-zero as discussed in papers 52 and 53. As shown in paper 56 eq (10) is equivalent to:

$$d \wedge A^1 = g A^2 \wedge A^3 \quad -(12)$$

$$d \wedge A^2 = g A^3 \wedge A^1 \quad -(13)$$

$$d \wedge A^3 = g A^1 \wedge A^2 \quad -(14)$$

$$d \wedge A^0 = - d \wedge A^3. \quad -(15)$$

The mathematical and numerical task is to find

3) solutions to eqns (12) to (15).

First write out eq. (14) in tensor notation:

$$\partial_\mu A^3_\nu - \partial_\nu A^3_\mu = g\left(A^1_\mu A^2_\nu - A^1_\nu A^2_\mu\right) \quad -(16)$$

In the complex circular basis:

$$\partial_\mu A^{(3)}_\nu - \partial_\nu A^{(3)}_\mu = -ig\left(A^{(1)}_\mu A^{(2)}_\nu - A^{(1)}_\nu A^{(2)}_\mu\right) \quad -(17)$$

Eq (17) is, for example:

$$\partial_x A^{(3)}_y - \partial_y A^{(3)}_x = \kappa A^{(0)}, \quad -(18)$$

however:

$$A^{(3)}_y = A^{(3)}_x = 0, \; A^{(0)} \neq 0, \quad -(19)$$

so the only solution is

$$\kappa = 0. \quad -(20)$$

Similarly, eq. (17) gives:

$$\partial_0 A^{(3)}_Z - \partial_Z A^{(3)}_0 = -ig\left(A^{(1)}_0 A^{(2)}_Z - A^{(1)}_Z A^{(2)}_0\right) \quad -(21)$$

and again eq. (20) is the only solution, because:

$$A^{(2)}_Z = A^{(1)}_Z = 0, \; A^{(0)} \neq 0. \quad -(22)$$

In order to obtain a self-consistent solution to the simultaneous equations (12) - (15) it must be assumed that κ_i is a tensor:

$$\kappa_i = \begin{bmatrix} \kappa & 0 & 0 \\ 0 & \kappa & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad -(23)$$

so eqns (12) - (15) become:

4)

$$d \wedge A^1 = g A^2 \wedge A^3 \quad -(24)$$
$$d \wedge A^2 = g A^3 \wedge A^1 \quad -(25)$$
$$d \wedge A^3 = - d \wedge A^0 = 0 \quad -(26)$$
$$A^1 \wedge A^2 \neq 0. \quad -(27)$$

The solutions are:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} - i\underline{j}) e^{i(\omega t - \kappa z)}, \quad -(28)$$
$$\underline{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}} (\underline{i} + i\underline{j}) e^{-i(\omega t - \kappa z)}. \quad -(29)$$

Thus:

$$\underline{\nabla} \times \underline{A}^{(1)} = \kappa \underline{A}^{(1)}, \quad -(30)$$
$$\underline{A}^{(1)} \times \underline{A}^{(2)} = i A^{(0)2} \underline{k}, \quad -(31)$$
$$\underline{\nabla} \times \underline{A}^{(1)*} = -ig \underline{A}^{(2)} \times \underline{A}^{(3)}, \quad -(32)$$
$$\underline{\nabla} \times \underline{A}^{(2)*} = -ig \underline{A}^{(3)} \times \underline{A}^{(1)}, \quad -(33)$$
$$\underline{\nabla} \times \underline{A}^{(3)*} = \underline{0}. \quad -(34)$$

Eqs (28) ~~and~~ to (34) are tetrad equations, with:

$$\underline{A}^{(3)} = A^{(0)} \underline{k}. \quad -(35)$$

Thus:

$$A^0 = - A^{(0)}. \quad -(36)$$

The complex circular basis is defined by the tetrad equations:

5)

$$\underline{q}^{(1)} \times \underline{q}^{(2)} = i\underline{q}^{(3)*} \quad -(37)$$

$$\underline{q}^{(2)} \times \underline{q}^{(3)} = i\underline{q}^{(1)*} \quad -(38)$$

$$\underline{q}^{(3)} \times \underline{q}^{(1)} = i\underline{q}^{(2)*} \quad -(39)$$

where:

$$\underline{q}^{(1)} = \underline{q}^{(2)*} = \frac{1}{\sqrt{2}}(\underline{i} - i\underline{j})e^{i(\omega t - kz)} \quad -(40)$$

$$\underline{q}^{(3)} = \underline{q}^{(3)*} = \underline{k} \, . \quad -(41)$$

These tetrads are the mechanism for defining the Cartan torsion and spinning spacetime responsible for electromagnetic potential.

Electromagnetic AB Effects

These are caused by $\underline{A}^{(1)}$, $\underline{A}^{(2)}$ and $\underline{A}^{(3)}$ in regions where $\underline{E}^a = \underline{0}$ and $\underline{B}^a = \underline{0}$. They occur for example in regions outside a radar beam or laser beam. The $\underline{A}^{(1)}$ and $\underline{A}^{(2)}$ components are rapidly oscillating, so:

$$\langle \underline{A}^{(1)} \rangle = \langle \underline{A}^{(2)} \rangle = \underline{0} \quad -(42)$$

on average, but:

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = iA^{(0)2}, \quad -(43)$$

and is non-zero on average.

The electromagnetic field components in these regions are zero, for example:

$$\underline{B}^{(1)*} = \underline{\nabla} \times \underline{A}^{(1)*} + ig\,\underline{A}^{(2)} \times \underline{A}^{(3)} = \underline{0}$$

$$\underline{B}^{(2)*} = \underline{\nabla} \times \underline{A}^{(2)*} + ig\,\underline{A}^{(3)} \times \underline{A}^{(1)} = \underline{0}. \quad -(44)$$

Therefore the beam intensity, defined by:

$$\underline{I} = -\frac{ic}{\mu_0} \left| \underline{B}^{(1)} \times \underline{B}^{(2)} \right| = 0 \quad -(45)$$

in watts per square metre.

The intensity or power density of the radar or laser beam is non-zero if and only if the electric and magnetic fields making up the beam are non-zero. Obviously, outside the beam there is no beam intensity. However, the conjugate product $\underline{A}^{(1)} \times \underline{A}^{(2)}$ still exists outside the beam because in these regions, $\underline{A}^{(1)} = \underline{A}^{(2)*} \neq \underline{0}$. Similarly the colour of the laser beam is due to the fact that its electric and magnetic fields are non-zero. Outside the laser beam, but

7) invisible, are regions where $\underline{A}^{(1)} = \underline{A}^{(2)*} \neq \underline{0}$. For a static magnetic field (Chambers experiment), $\underline{B}$ is non-zero inside the iron whisker, but is zero outside. Outside the iron whisker, $\underline{A}$ is non zero and causes the magnetic AB effect.

So the experiment to detect the electromagnetic AB effect must be set up to observe the inverse Faraday effect in regions outside the radar or laser beam. As described in Appendix F of volume three of M.W. Evans and J.-P. Vigier, "The Enigmatic Photon", the inverse Faraday effect in an electron gas is given by:

$$\underline{B}^{(3)}_{\text{in sample}} = \frac{N}{V} \frac{\mu_0 e^3 c^2}{2m\omega^2} \left(\frac{B^{(0)}}{(m^2\omega^2 + e^2 B^{(0)2})^{1/2}} \right) \underline{B}^{(3)}_{\text{free space}} \quad -(46)$$

At visible frequencies (laser),

$$\left| \underline{B}^{(3)}_{\text{in sample}} \right| \rightarrow \frac{N}{V} \left(\frac{\mu_0 e^3 c^2}{2m^2\omega^3} \right) B^{(0)2} \quad -(47)$$

At radar frequencies:

$$\left| \underline{B}^{(3)}_{\text{in sample}} \right| \rightarrow \frac{N}{V} \left(\frac{\mu_0 e^2 c^2}{2m\omega^2} \right) B^{(0)} \quad -(48)$$

In terms of intensity I eq. (47) is:

$$\left| \underline{B}^{(3)}_{\text{in sample}} \right| = \frac{N}{V} \left(\frac{\mu_0^2 e^3 c}{2m^2} \right) \frac{I}{\omega^3} . \quad - (49)$$

For a power density of $I = 5.5 \times 10^{12}$ watts m^{-2} and a Nd-YAG frequency of 1.77×10^{16} rad s^{-1}, $|\underline{B}^{(3)}_{\text{in sample}}| \sim 10^{-9}$ tesla $= 10^{-5}$ gauss, assuming $N/V = 10^{26} m^{-3}$. This is in good agreement with experimental results, e.g. van der Ziel et al. in the original inverse Faraday effect experiment at Harvard in the mid sixties.

Eq. (46) is obtained from the relativistic Hamilton Jacobi equation as in volume one of "The Enigmatic Photon".

The electromagnetic AB effect at laser frequencies is, from eq. (47), the magnetization due to $\underline{A}^{(1)} \times \underline{A}^{(2)}$ in regions where there are no electric or magnetic fields of the laser beam. It is given by the same equation (49), but the interpretation of I is different. It is the intensity of the laser beam transmitted to regions outside the beam by the spin connection, i.e. by the spinning of spacetime. Similarly the $\underline{B}$ field of the iron whisker in the Chambers experiment is transmitted to regions outside the whisker by the spinning of spacetime.

These are all effects of the generally covariant electromagnetic field. The latter is always defined in general relativity by:

$$F = D \wedge A. \quad \text{— (50)}$$

The electromagnetic AB experiment therefore has to produce the conditions:

$$F = D \wedge A = 0, \quad A \neq 0. \quad \text{— (51)}$$

This has already been done for the static magnetic field, and static electric field, but has not yet been done for the electromagnetic field. These experiments define what is meant by a "field" in generally covariant electrodynamics. It must be a field of force, accompanied by kinetic energy. When the field of force is zero, the potential energy may still be non-zero, and there is a potential for the creation of a force field.

In the Chambers experiment for example there is no force field outside the iron whisker, but there is a potential for the creation of a force field. The potential is contained within the spinning of spacetime itself. The gravitational Aharonov Bohm effect has also been observed experimentally, and is due to the potential for force generated by the curving of spacetime itself, in regions of zero gravity, i.e. zero Riemann curvature locally, but non-zero spin connection:

$$R = D \wedge \omega = 0, \quad \omega \neq 0. \quad -(52)$$

So the usual properties of an electromagnetic beam are due to non-zero electric and magnetic fields of force. These properties include intensity, colour (i.e. spectrum), transmission of signals, and so on. In general relativity (ECE theory) there exist properties, the AB effects, due to the potential for the creation of a field of force where the field itself is zero. The potential is the tetrad, the transmission is due to the spin connection.