Notes 59(3): Resonance Structure of the Coulomb Law in General Relativity.

The Coulomb Law is:

$$\underline{\nabla}\cdot\underline{\nabla}A^{a0}+\frac{1}{c}\frac{\partial}{\partial t}\left(\underline{\nabla}\cdot\underline{A}^{a}\right)+\underline{\nabla}\cdot\left(\omega^{0a}{}_{b}\underline{A}^{b}\right)-\underline{\nabla}\cdot\left(\underline{\omega}^{a}{}_{b}A^{0b}\right)=-\mu_0\tilde{J}^{0a} \quad -(1)$$

and this may be rewritten as:

$$\nabla^2A^{a0}+\frac{1}{c}\underline{\nabla}\cdot\frac{\partial\underline{A}^{a}}{\partial t}+\left(\omega^{0a}{}_{b}\underline{\nabla}\cdot\underline{A}^{b}-\underline{\omega}^{a}{}_{b}\cdot\underline{\nabla}A^{0b}\right)+\left(\left(\underline{\nabla}\omega^{0a}{}_{b}\right)\cdot\underline{A}^{b}-A^{0b}\underline{\nabla}\cdot\underline{\omega}^{a}{}_{b}\right)=-\mu_0\tilde{J}^{0a} \quad -(2)$$

On the left hand side there is the second order term ∇^2A^{a0}, and the linear first order terms $\left(\omega^{0a}{}_{b}\underline{\nabla}\cdot\underline{A}^{b}-\underline{\omega}^{a}{}_{b}\cdot\underline{\nabla}A^{0b}\right)$. There is also the terms $\left(\left(\underline{\nabla}\omega^{0a}{}_{b}\right)\cdot\underline{A}^{b}-A^{0b}\underline{\nabla}\cdot\underline{\omega}^{a}{}_{b}\right)$. On the right hand time dependent term $\frac{1}{c}\underline{\nabla}\cdot\partial\underline{A}^{a}/\partial t$. On the right hand side there is the driving term $-\mu_0\tilde{J}^{0a}$. Therefore eq. (1) is a linear inhomogeneous differential equation capable of giving resonance solutions. The simplest type of resonance equation is:

$$\ddot{x}+2\beta\dot{x}+\omega_0^2x=A\cos\omega t \quad -(3)$$

and eq. (2) is a more complicated version of eq. (3).

The Q factor for eq. (3) is:

$$Q=\frac{\omega_R}{2\beta} \quad -(4)$$

2)

where the resonance frequency is:

$$\omega_R = \left(\omega_0^2 - 2\beta^2\right)^{1/2} \quad -(5)$$

The phase is

$$\delta = \tan^{-1}\left(\frac{2\omega\beta}{\omega_0^2 - \omega^2}\right) \quad -(6)$$

The particular solution is:

$$x_p(t) = D\cos(\omega t - \delta) \quad -(7)$$

where:

$$D = A\left((\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2\right)^{-1/2} \quad -(8)$$

Graph of D vs ω and δ vs ω are given as follows:

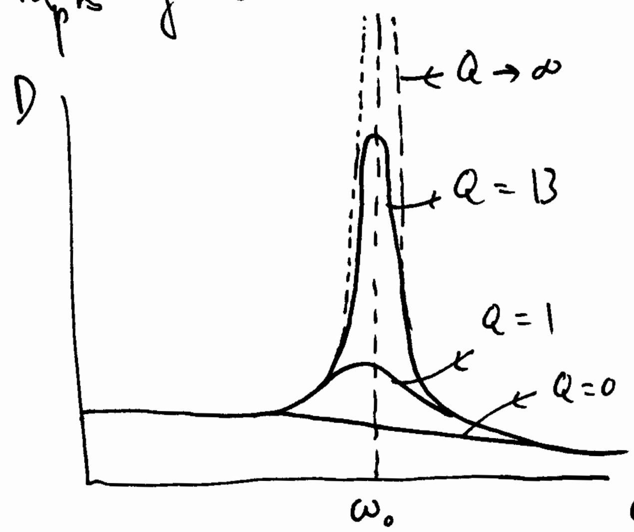

Fig (1)

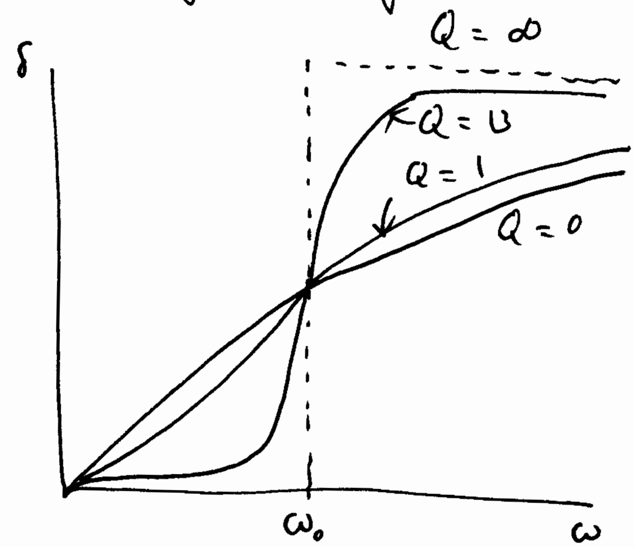

Fig (2)

Here ω_R is the frequency for amplitude resonance, i.e. D and x_p go to infinity at resonance for $Q = 0$. There is also kinetic energy resonance.

So at resonance the magnitude of the potential becomes very large for a given $\tilde{J}^{oa}$.

3)

It is now possible to obtain simple analytical solutions from eq. (2) using:

$$\omega^a{}_b = -\frac{g}{2}\epsilon^a{}_{bc}A^c \quad -(8)$$

This can be done using the method developed in previous papers but we first note that the usual standard model Coulomb Law is recovered when:

$$\omega^a{}_b \to 0. \quad -(9)$$

In this case eq. (2) reduces to:

$$\nabla^2 A^0 = -\mu_0 \tilde{J}^0 \quad -(10)$$

and there is no resonance. When the complete eq. (2) is considered, the system obviously has to be properly tuned before resonance occurs, this is true even if the spin connection is non-zero.

<u>Graphs from Eq. (2)</u>

Consider the H atom and graph resonances from eq. (2) similar to Figs (1) and (2). The charge density $\tilde{J}^{0a}$ must be considered as:

$$\tilde{J}^{0a} = J^a_{(0)} \cos \omega t \quad -(11)$$

where $J^a_{(0)}$ is an amplitude and ω some frequency of rotation or vibration of the H atom. With good tuning, the H atom breaks apart, freeing an electron.