

# 61(5): Resonant Equations for Torque and Force

## Torque Equations

The resonant torque equations are given in terms very similar to the resonant electrodynamical equations (pages 55) and are as follows:

$$\underline{\nabla} \cdot \underline{T}_S^a = \underline{j}^{0a} / c \quad - (1)$$

$$\underline{\nabla} \times \underline{T}_L^a + \frac{1}{c} \frac{\partial \underline{T}_S^a}{\partial t} = \underline{j}^a \quad - (2)$$

$$\underline{\nabla} \cdot \underline{T}_L^a = \underline{j}^{0a} \quad - (3)$$

$$\underline{\nabla} \times \underline{T}_S^a - \frac{1}{c^2} \frac{\partial \underline{T}_L^a}{\partial t} = \underline{j}^a \quad - (4)$$

$$\underline{T}_S^a = \underline{\nabla} \times \underline{q}^a - \underline{\omega}^a_b \times \underline{q}^b \quad - (5)$$

$$\underline{T}_L^a = - \frac{\partial \underline{q}^a}{\partial t} - c \underline{\nabla} \cdot \underline{q}^{0a} - c \underline{\omega}^{0a}_b \underline{q}^b + c \underline{q}^{0b} \underline{\omega}^a_b \quad - (6)$$

The various resonant equations are constructed by substituting eqns. (5) and (6) into eqns. (1) to (4). It is seen that the spin part of the torque plays the part of the magnetic field, the orbital part of the torque plays the part of the electric field. The tetrad plays the part of the potential field in electrodynamics.

So the various resonant electrodynamical equations obtained for electrodynamics can be translated directly into resonant torque equations. See pages 52, 53, 55, 59 and 60.

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## Force Equations

The resultant force equations are obtained from the Riemann form equations of page 55:

$$\underline{\nabla} \cdot \underline{R}(\text{spin}) = \underline{\tilde{j}}^0 \quad - (1)$$

$$\underline{\nabla} \times \underline{R}(\text{orbital}) + \frac{1}{c} \frac{\partial \underline{R}(\text{spin})}{\partial t} = \underline{\tilde{j}} \quad - (2)$$

$$\underline{\nabla} \cdot \underline{R}(\text{orbital}) = \underline{j}^0 \quad - (3)$$

$$\underline{\nabla} \times \underline{R}(\text{spin}) - \frac{1}{c} \frac{\partial \underline{R}(\text{orbital})}{\partial t} = \underline{j} \quad - (4)$$

where:

$$\underline{R}(\text{orbital}) = R^{0,01} \underline{i} + R^{0,02} \underline{j} + R^{0,03} \underline{k} \quad - (5)$$

$$\underline{R}(\text{spin}) = R^{2,23} \underline{i} + R^{1,3,31} \underline{j} + R^{1,2,12} \underline{k} \quad - (6)$$

The force is defined by:

$$\underline{F} = \frac{c^2}{\hbar R} \underline{R}(\text{orbital}) \quad - (7)$$

from dimensional analysis. Here  $R$  is the scalar curvature and  $\hbar$  the Planck constant. Therefore eqs. (1) to (4) are force equations.

The Riemann form is defined in terms of the spin convention by the second Cartan

3) structure equation:

$$R^a_b = d \wedge \omega^a_b + \omega^a_c \wedge \omega^c_b. \quad - (8)$$

Thus:

$$\underline{R}^a_b(\text{spi}) = \underline{\nabla} \times \underline{\omega}^a_b - \underline{\omega}^a_c \times \underline{\omega}^c_b \quad - (9)$$

$$\underline{R}^a_b(\text{orbital}) = -\frac{\partial \underline{\omega}^a_b}{\partial t} - c \underline{\nabla} \omega^{0a}_b - c \omega^{0a}_c \underline{\omega}^c_b + c \omega^{0c}_b \underline{\omega}^a_c \quad - (10)$$

Thus:

$$\underline{R}^0_{1^{01}}(\text{orbital}) = R^{01}_{1^{01}} \underline{i} = R^0_{1x} \underline{i} \quad - (11)$$

$$\underline{R}^0_{2^{02}}(\text{orbital}) = R^{02}_{2^{02}} \underline{j} = R^0_{2y} \underline{j} \quad - (12)$$

$$\underline{R}^0_{3^{03}}(\text{orbital}) = R^{03}_{3^{03}} \underline{k} = R^0_{3z} \underline{k} \quad - (13)$$

$$\text{end: } \underline{R}(\text{orbital}) = R^0_{1x} \underline{i} + R^0_{2y} \underline{j} + R^0_{3z} \underline{k} \quad - (14)$$

Using eq. (10):

$$\underline{R}^0_1(\text{orbital}) = R^0_{1x} \underline{i}$$

$$= -\frac{\partial \underline{\omega}^0_1}{\partial t} - c \underline{\nabla} \omega^{00}_1 - c \omega^{00}_c \underline{\omega}^c_1 + c \omega^{0c}_1 \underline{\omega}^0_c \quad - (15)$$

$$\underline{R}^0_2(\text{orbital}) = R^0_{2y} \underline{j}$$

$$= -\frac{\partial \underline{\omega}^0_2}{\partial t} - c \underline{\nabla} \omega^{00}_2 - c \omega^{00}_c \underline{\omega}^c_2 + c \omega^{0c}_2 \underline{\omega}^0_c \quad - (16)$$

$$\underline{R}^0_3(\text{orbital}) = R^0_{3z} \underline{k}$$

$$= -\frac{\partial \underline{\omega}^0_3}{\partial t} - c \underline{\nabla} \omega^{00}_3 - c \omega^{00}_c \underline{\omega}^c_3 + c \omega^{0c}_3 \underline{\omega}^0_c \quad - (17)$$

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## Newton Inverse Square Law and Resonance

The Newton inverse square law is obtained from:

$$\underline{\nabla} \cdot \underline{R} (\text{orbital}) = \underline{J}^{\circ} \quad - (18)$$

Using eq. (7) this is:

$$\underline{\nabla} \cdot \underline{F} = \frac{c^2}{RR} \underline{J}^{\circ} \quad - (19)$$

and using eqns (14) to (17) it is a resonance law. Its direct analogy with pages 52, 53, 59 and 60 resonance equations may be developed from eq. (19) involving the divergence of force.

Using eq. (4) a resonance equation may be developed using the time derivative of force.

## Conclusion

Rotational and translational dynamics are resonant phenomena.