A SIMPLE EXAMPLE OF THE ECE LEMMA

The simplest form of the lemma is:

$$\square q^a_\lambda = R q^a_\lambda \quad -(1)$$

where

$$R := q^\sigma_a \partial^\mu \left(\Gamma^\nu_{\mu\sigma} q^a_\nu - \omega^a_{\mu b} q^b_\sigma \right). \quad -(2)$$

The scalar curvature in eq. (2) may be expressed as:

$$R = q^\nu_a R^a_\nu \quad -(3)$$

where

$$R^a_\nu = \partial^\mu \left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \right) \quad -(4)$$

and:

$$\Gamma^a_{\mu\lambda} = \Gamma^\nu_{\mu\lambda} q^a_\nu, \quad \omega^a_{\mu\lambda} = \omega^a_{\mu b} q^b_\lambda. \quad -(5)$$

It is seen that R^a_ν is the generalization of the Ricci tensor of Riemann geometry to Cartan geometry.

Now assume that the tetrad is a travelling wave in the Z axis:

$$q^a_\lambda = q^a_\lambda(0)\, e^{i(\omega t - \kappa Z)} \quad -(6)$$

2) and let:

$$\kappa = \frac{\omega}{v} \quad -(7)$$

where v is the phase velocity. Then:

$$\Box q^a_\lambda = \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial Z^2} \right) q^a_\lambda(0)\, e^{i(\omega t - \kappa Z)}$$

$$= \left(\kappa^2 - \frac{\omega^2}{c^2} \right) q^a_\lambda . \quad -(8)$$

So:

$$R = \kappa^2 - \frac{\omega^2}{c^2} = \omega^2 \left(\frac{1}{v^2} - \frac{1}{c^2} \right) - (9)$$

and

$$R^a_\mu = \omega^2 \left(\frac{1}{v^2} - \frac{1}{c^2} \right) q^a_\mu \quad -(10)$$

From eq. (4):

$$\left(\Gamma^a - \omega^a \right)_{03} = R \int q^a_0(0)\, e^{i(\omega t - \kappa Z)}\, dZ$$

$$= \frac{iR}{\kappa} q^a_0(0)\, e^{i(\omega t - \kappa Z)} \quad -(11)$$

$$\text{Real}\left(\left(\Gamma^a - \omega^a \right)_{03} \right) = -\frac{R}{\kappa} q^a_0(0) \sin(\omega t - \kappa Z) . \quad -(12)$$

These results are useful to understand the Dirac equation. The latter is obtained when:

$$R = -\left(\frac{mc}{\hbar} \right)^2 . \quad -(12)$$

The negative sign in eq. (12) is a matter of convention, and originates in:

$$R = -kT \quad -(13)$$

3) The modulus, or positive value, of R is:

$$|R| = \left(\frac{mc}{\hbar}\right)^2 \quad \text{— (14)}$$

From eqs. (9) and (13):

$$\kappa^2 - \frac{\omega^2}{c^2} = -\frac{m^2c^2}{\hbar^2} \quad \text{— (15)}$$

i.e.

$$\frac{\omega^2}{c^2} = \kappa^2 + \frac{m^2c^2}{\hbar^2} \quad \text{— (16)}$$

Now use:

$$En = \hbar\omega, \quad p = \hbar\kappa \quad \text{— (17)}$$

and eq. (16) becomes ~~the~~ <u>Einstein equation</u>:

$$En^2 = p^2 + m^2c^4. \quad \text{— (18)}$$

The Compton wavelength is:

$$\lambda_c = \frac{\hbar}{mc}. \quad \text{— (19)}$$

The Dirac equation is:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right)q^a_{\sim} = 0 \quad \text{— (20)}$$

where:

$$q^a_{\sim} = q^a_{\sim}(0)\exp\left(i(\omega t - \kappa Z)\right)$$

$$:= q^a_{\sim}(0)\exp\left(ix_\mu p^\mu\right) \quad \text{— (21)}$$

By convention (Ryder page 50, 2nd. ed.) the positive energy plane wave spinor is:

4)

$$q^a = q^a(0) \exp(-i x_\mu p^\mu) \quad -(22)$$

and the negative energy plane wave spinor is:

$$q^a = q^a(0) \exp(i x_\mu p^\mu) \quad -(23)$$

So the Ricci type curvature for the Dirac electron is:

$$R^a = \left(\kappa^2 - \frac{\omega^2}{c^2}\right) q^a \quad -(24)$$

and the connection element in eq. (12) is given for the Dirac electron and any fermion by using eq. (9) for R.

Conclusion

The Dirac electron is a manifestation of ECE space-time in the particular case defined by eq. (9) for R and eq. (24) for R^a. The Einstein equivalence principle states that this particular case is the limit when the fermion field has become independent of the gravitational field. This is the limit of special relativity. More generally in ECE theory it is the case where the fermion field has become independent of all other fields, not only of the gravitational field.