THE RESONANT COULOMB LAW OF EINSTEIN CARTAN EVANS FIELD THEORY

by

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ABSTRACT

Einstein Cartan Evans (ECE) field theory is used to show that in general relativity the structure of the laws of electricity, magnetism and electromagnetism is changed fundamentally. This result is demonstrated analytically and numerically with the Coulomb Law. In ECE theory the electromagnetic field is spinning space-time, characterized by the spin connection of Cartan geometry. The spin connection is shown in this paper to change the Poisson equation into a differential equation capable of giving resonance. Off resonance, the standard Poisson equation is observed, and the standard Coulomb Law. At space-time resonance the scalar potential in volts is greatly amplified with fundamental consequences in the natural, engineering and life sciences.

Keywords: Einstein Cartan Evans (ECE) field theory, generally covariant unified field theory, electricity, magnetism, electromagnetism, classical electrodynamics, Coulomb Law, space-time resonance.

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1. INTRODUCTION

The need for objectivity in the natural, engineering and life sciences means that all the fundamental laws of physics must be laws of general relativity, where objectivity is represented by geometry. This includes the laws of classical electrodynamics, the laws of electricity, magnetism and electromagnetism. General relativity means that these laws must be generally covariant, must retain their form under any type of coordinate transformation. They must therefore be laws of a generally covariant unified field theory. Recently {1-19} the Einstein Cartan Evans (ECE) theory has been developed along these well known guidelines. Objectivity in ECE field theory is maintained through the use of standard Cartan geometry {20}. The electromagnetic field is represented as the Cartan torsion within a scalar factor $A^{(0)}$. Here $cA^{(0)}$ is a primordial voltage. This procedure follows a well known suggestion by Cartan to Einstein that the electromagnetic field be the Cartan torsion form, a vector valued two-form of differential geometry {1-20}. ECE theory applies Cartan's suggestion systematically to all the laws of physics.

The well known Maxwell Heaviside (MH) field theory is used in the standard model {21} to represent the laws of electricity, magnetism and electromagnetism. The MH theory is a nineteenth century theory of special relativity and is neither generally covariant nor unified with other fundamental fields such as gravitation. It is a Lorentz covariant theory in which the electromagnetic field is considered to be an entity separate from the frame of reference in a Minkowski space-time. The latter is often referred to as flat space-time, because it has neither curvature nor torsion. It is also a static space-time. The laws of gravitation on the other hand are described in the standard model by the Einstein Hilbert (EH) field theory of general relativity {1-20} and the gravitational field is the frame itself, not something separate from the frame as in MH theory. EH space-time has curvature but no torsion, and is a dynamic space-time. Objectivity in EH field theory is based on Riemann geometry with a Christoffel connection. This assumption implies a zero torsion tensor {20} and means that gravitation cannot be unified with electromagnetism in EH theory. As we have argued, electrodynamics cannot be unified with gravitation in MH theory. In ECE theory {1-19} a unified description of all fields has been developed straightforwardly using ECE space-time in which curvature and torsion are simultaneously non-zero. The generally covariant unified field of ECE theory is the frame itself, as required by general relativity and objectivity. Gravitation is described by curvature, the electromagnetic, weak and strong fields by torsion using the appropriate representation spaces (respectively O(3), SU(2) and SU(3)). The two Cartan structure equations and the two Bianchi identities of differential geometry control all the laws of physics {1-19}, including those of quantum mechanics through the tetrad postulate {20}. The latter is the fundamental requirement that the complete vector field in n dimensions be independent of the components and basis elements chosen to represent it. Thus ECE theory has unified quantum mechanics with general relativity and provides a generally covariant unified field theory. ECE theory has therefore been accepted as mainstream physics {22}.

In Section 2, the Coulomb law of electricity is developed with the spin connection incorporated as required by general relativity, by the fact that the complete electromagnetic field be spinning space-time. A spinning of space-time means a spinning of the frame of reference itself. This means that the spin connection must always be non-zero. The Coulomb Law is derived in ECE theory {1-19} from the first Cartan structure equation and the first Bianchi identity. Use of vector notation and some simplifying assumptions {1-19} lead to the initial equations of Section 2. These give a resonance equation whose properties are developed analytically to give the required resonance solution. The latter is fundamentally important in the natural, engineering and life sciences because the Coulomb law is the basis of all quantum chemistry, and therefore the basic law of computational quantum chemistry.

In Section 3, numerical solutions of the resonance Coulomb law are developed to illustrate a novel space-time resonance spectrum. Off resonance the standard Coulomb Law is recovered. The standard Coulomb law is well known {21} to be among the most precise laws of physics, so must be recovered from ECE theory in a given limit. This is achieved by identifying the radial spin connection as the one that gives the standard Coulomb law off resonance. In the off resonant condition the spin connection effectively doubles the value of the electric field, so its presence cannot be detected experimentally. At space-time resonance however the scalar potential in volts of the Coulomb Law is greatly amplified, leading to a surge in voltage that cannot be explained by MH theory. This phenomenon has been reported experimentally by several independent groups {23}. If this resonant amplification of the scalar potential occurs inside an atom or molecule, electrons may be released by ionization. In this Section the process is illustrated by the radial wave-functions of the hydrogen atom in anticipation of the systematic development of density functional code incorporating the resonant Coulomb law of ECE theory. A short review of density functional methods is also given in this section. Finally a discussion is given of how to induce space-time resonance in circuits and materials.

2. THE RESONANT COULOMB LAW

In the simplest instance {1-19} the Coulomb law in ECE theory is given by:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \qquad -(1)$$

where

$$\underline{E} = -(\underline{\nabla} + \underline{\omega})\phi . \qquad -(2)$$

Here ϕ is the scalar potential in volts, $\underline{\omega}$ is the vector spin connection in inverse meters,

$\underline{E}$ is the electric field strength in volts m^{-1}, ρ is the charge density in C m^{-3}, and is the S. I. vacuum permittivity:

$$\epsilon_0 = 8.854 \times 10^{-12} J^{-1} C^2 m^{-1}. \quad -(3)$$

Thus:

$$\underline{\nabla} \cdot ((\underline{\nabla} + \underline{\omega})\phi) = -\rho / \epsilon_0 \quad -(4)$$

i.e.

$$\nabla^2 \phi + \underline{\nabla} \cdot (\phi \underline{\omega}) = -\rho / \epsilon_0 . \quad -(5)$$

If there is no spin connection, Eq. (5) is the Poisson equation {21} of the standard model. Otherwise:

$$\nabla^2 \phi + \underline{\omega} \cdot \underline{\nabla} \phi + (\underline{\nabla} \cdot \underline{\omega}) \phi = -\rho / \epsilon_0 \quad -(6)$$

which is an equation capable of giving resonant solutions {1-19, 24} from the spin connection vector. The Poisson equation does not give resonant solutions. Eq. (6) is first developed in one (Z) dimension of the Cartesian coordinate system, and then for the radial component of the spherical polar coordinate system { 25 }.

In one Z dimension Eq. (6) becomes:

$$\frac{\partial^2 \phi}{\partial Z^2} + \omega_Z \frac{\partial \phi}{\partial Z} + \left(\frac{\partial \omega_Z}{\partial Z} \right) \phi = -\rho / \epsilon_0 . \quad -(7)$$

The spin connection in Eq. (7) must be:

$$\omega_Z = \frac{2}{Z} \quad -(8)$$

in order to recover the standard Coulomb law off resonance. This is because:

$$\phi = -\frac{e}{4\pi\epsilon_0 Z}, \qquad -(9)$$

$$\frac{\partial \phi}{\partial Z} = \frac{e}{4\pi\epsilon_0 Z^2} = -\frac{\omega_Z}{2}\phi$$

in the off resonant condition, giving Eq. (8). In the off resonant condition the role of the spin connection is to change the sign of the electric field according to Eq. (9). The way in which the field E and potential ϕ are related is changed, but this has no experimental effect since $\underline{E}$ is effectively replaced by $-\underline{E}$. With the spin connection (8) Eq. (7) becomes:

$$\frac{\partial^2 \phi}{\partial Z^2} + \frac{2}{Z}\frac{\partial \phi}{\partial Z} - \frac{2}{Z^2}\phi = -\frac{\rho}{\epsilon_0}. \qquad -(10)$$

Now assume that the charge density is initially oscillatory:

$$\rho = \rho(0)\cos(\kappa Z) \qquad -(11)$$

where κ is a wave-number. Thus:

$$\frac{\partial^2 \phi}{\partial Z^2} + \frac{2}{Z}\frac{\partial \phi}{\partial Z} - \frac{2}{Z^2}\phi = -\rho(0)\cos(\kappa Z). \qquad -(12)$$

Since ϕ depends only on Z the partial derivatives can be replaced by total derivatives to give an ordinary differential equation {24, 26}:

$$\frac{d^2 \phi}{dZ^2} + \frac{2}{Z}\frac{d\phi}{dZ} - \frac{2}{Z^2}\phi = -\rho(0)\cos(\kappa Z) \qquad -(13)$$

using the well known Euler method { 24, 26} this equation can be reduced to an undamped oscillator equation that has resonant solutions. Define a change of variable {24, 26} by:

$$\kappa Z = \exp(i\kappa x). \quad -(14)$$

Thus:

$$\frac{dx}{dZ} = -\frac{i}{\kappa Z}. \quad -(15)$$

Now use:

$$\frac{d\phi}{dZ} = \frac{d\phi}{dx}\frac{dx}{dZ} = -\frac{i}{\kappa Z}\frac{d\phi}{dx} \quad -(16)$$

and construct the second derivative:

$$\frac{d^2\phi}{dZ^2} = \frac{i}{\kappa Z^2}\frac{d\phi}{dx} - \frac{i}{\kappa Z}\frac{d}{dZ}\left(\frac{d\phi}{dx}\right). \quad -(17)$$

Isotropy means that:

$$\frac{d}{dZ}\left(\frac{d\phi}{dx}\right) = \frac{d^2\phi}{dZ\,dx} = \frac{d^2\phi}{dx\,dZ} = \frac{d}{dx}\left(\frac{d\phi}{dZ}\right) \quad -(18)$$

so

$$\frac{d^2\phi}{dZ^2} = \frac{i}{\kappa Z^2}\frac{d\phi}{dx} - \frac{1}{\kappa^2 Z^2}\frac{d^2\phi}{dx^2}. \quad -(19)$$

Thus:

$$Z\frac{d\phi}{dZ} = -\frac{i}{\kappa}\frac{d\phi}{dx}, \quad -(20)$$

$$Z^2\frac{d^2\phi}{dZ^2} = \frac{i}{\kappa}\frac{d\phi}{dx} - \frac{1}{\kappa^2}\frac{d^2\phi}{dx^2}. \quad -(21)$$

Now substitute Eqs. (20) and (21) in Eq. (13) to give:

$$\frac{d^2\phi}{dx^2} + 2\kappa^2\phi = \frac{\rho(0)}{\epsilon_0}\,\mathrm{Real}\left(e^{2i\kappa x}\cos\left(e^{i\kappa x}\right)\right) \quad -(22)$$

which the undamped oscillator equation {24,26}:

$$\frac{d^2\phi}{dx^2} + 2\kappa^2\phi = \frac{\rho(0)}{\epsilon_0}\Big(\cos(2\kappa x)\cos(\cos(\kappa x))\cosh(\sin(\kappa x)) + \sin(2\kappa x)\sin(\cos(\kappa x))\sinh(\sin(\kappa x))\Big). \quad -(23)$$

Assume that the particular integral of this equation is:

$$\phi_p(x) = \frac{A\rho(0)}{\epsilon_0}\left(\frac{\cos(\kappa' x)}{2\kappa^2 - \kappa'^2}\right) \quad -(24)$$

where κ' is defined by the identity:

$$A\cos(\kappa' x) := \mathrm{Real}\left(e^{2i\kappa x}\cos\left(e^{i\kappa x}\right)\right). \quad -(25)$$

From Eqs. (22) to (25) it is found that:

$$\frac{d^2 f}{dx^2} + 2\kappa^2 f = \cos \kappa' x \quad -(26)$$

where:

$$f = \frac{\cos \kappa' x}{2\kappa^2 - \kappa'^2} . \quad -(27)$$

Eq. (27) is, self-consistently, the solution of Eq. (26), Q.E.D.

Therefore Eq. (24) is a valid particular integral of Eq. (23) if κ' is defined by:

$$\kappa' = \frac{1}{x} \cos^{-1} \left(\frac{1}{A} \operatorname{Real} \left(e^{2i\kappa x} \cos \left(e^{i\kappa x} \right) \right) \right) . \quad -(28)$$

At resonance:

$$2\kappa^2 = \kappa'^2 \quad -(29)$$

and ϕ_p becomes theoretically infinite. For a given A, resonance in ϕ occurs when x is defined by:

$$\begin{aligned} A \cos (\kappa x / \sqrt{2}) &= \cos (2\kappa x) \cos (\cos (\kappa x)) \cosh (\sin (\kappa x)) \\ &+ \sin (2\kappa x) \sin (\cos (\kappa x)) \sinh (\sin (\kappa x)) . \end{aligned} \quad -(30)$$

It is seen that A can be greater than unity because $\cosh y$ and $\sinh y$ can be greater than unity.

Secondly consider the radial component r in three dimensions of the spherical polar coordinate system. In this system { 24 }:

$$\left.\begin{aligned} \nabla^2 \phi &= \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r}, \\ \underline{\omega} \cdot \underline{\nabla} \phi = \omega_r \frac{\partial \phi}{\partial r}, \quad (\underline{\nabla} \cdot \underline{\omega}) \phi &= \frac{\phi}{r^2} \frac{\partial}{\partial r} \left(r^2 \omega_r \right) \end{aligned}\right\} \quad -(31)$$

so Eq. (6) becomes:

$$\frac{\partial^2 \phi}{\partial r^2} + \left(\frac{2}{r} + \omega_r \right) \frac{d\phi}{dr} + \frac{\phi}{r^2} \left(2r\omega_r + r^2 \frac{\partial \omega_r}{\partial r} \right) = -\rho / \epsilon_0 . \quad -(32)$$

Now choose a radial spin connection:

$$\omega_r = -\frac{1}{r} \quad -(33)$$

to obtain:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} - \frac{1}{r^2} \phi = -\frac{\rho}{\epsilon_0} . \quad -(34)$$

This equation has the same mathematical structure as Eq. (13), and since ϕ is a function only of r can be written as an ordinary differential equation:

$$\frac{d^2 \phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} - \frac{1}{r^2} \phi = -\frac{\rho}{\epsilon_0} . \quad -(35)$$

Assume now that the initial charge is oscillatory as follows:

$$\rho = \rho(0) \cos(\kappa_r r) \quad -(36)$$

and use the change of variable:

$$\kappa_r r = \exp(i\kappa_r R) \quad -(37)$$

to obtain the undamped oscillator equation:

$$\frac{d^2\phi}{dR^2} + \kappa_r^2 \phi = \frac{\rho(0)}{\epsilon_0} \, \mathrm{Real}\left(e^{2i\kappa_r R} \cos\left(e^{i\kappa_r R}\right)\right) \quad -(38)$$

where

$$R = \frac{1}{\kappa_r} \cos^{-1}(\kappa_r r). \quad -(39)$$

Now define:

$$A \cos(\kappa' R) := \mathrm{Real}\left(e^{2i\kappa_r R} \cos\left(e^{i\kappa_r R}\right)\right) \quad -(40)$$

to obtain the particular integral:

$$\phi_p(R) = \frac{A\rho(0)}{\epsilon_0} \frac{\cos(\kappa' R)}{\kappa_r^2 - \kappa'^2}. \quad -(41)$$

Resonance occurs at:

$$\kappa_r = \kappa' \quad -(42)$$

where:

$$A\cos(\kappa_r R) = \cos(2\kappa_r R)\cos(\cos(\kappa_r R))\cosh(\sin(\kappa_r R)) + \sin(2\kappa_r R)\sin(\cos(\kappa_r R))\sinh(\sin(\kappa_r R)). \quad -(43)$$

The particular integral of eq. (38) may be obtained by first assuming that the solution has the form:

$$\phi = A\frac{\rho(0)}{\epsilon_0}\,\mathrm{Real}\left(e^{2ik_rR}\cos\left(e^{ik_rR}\right)\right) \quad (44)$$

where A is to be determined. Substituting Eq. (44) in Eq. (38) gives:

$$A = \frac{\mathrm{Real}\left(e^{2ik_rR}\cos\left(e^{ik_rR}\right)\right)}{k^2\left(-3e^{2ik_rR}\cos\left(e^{ik_rR}\right) + 5e^{3ik_rR}\sin\left(e^{ik_rR}\right) + e^{4ik_rR}\cos\left(e^{ik_rR}\right)\right)}$$

$$= \frac{r^2\cos(k_r r)}{k_r^4 r^4\cos(k_r r) + 5k_r^3 r^3\sin(k_r r) - 3k_r^2 r^2\cos(k_r r)} \quad (45)$$

Therefore the particular integral is:

$$\phi = \frac{\rho(0)}{\epsilon_0}\left(\frac{k_r^2 r^4\cos^2(k_r r)}{k_r^4 r^4\cos(k_r r) + 5k_r^3 r^3\sin(k_r r) - 3k_r^2 r^2\cos(k_r r)}\right) \quad (46)$$

which has the correct S.I. units of volts = J C^{-1}. Resonance occurs in the scalar potential in volts of eq. (46) when:

$$k_r^4 r^4\cos(k_r r) + 5k_r^3 r^3\sin(k_r r) = 3k_r^2 r^2\cos(k_r r) \quad (47)$$

If

$$x := \kappa_r r \qquad (48)$$

the structure of Eq. (47) is as follows:

$$\left.\begin{aligned}(x^2-3)\cos x + 5x\sin x &= 0,\\ \text{i.e.}\quad \tan x &= (3-x^2)/(5x),\end{aligned}\right\} \qquad (49)$$

and in general will show peaks as a function of x. The analytical solution (46) also shows many sharp peaks (Section 3), all of which denote surges in voltage (scalar potential). These peaks in voltage can be used in the equivalent circuits of Eqs. (35) or (38) to produce new power:

$$\phi \rightarrow \infty \text{ or maximized.} \qquad (50)$$

These equations are analyzed numerically in Section 3. To obtain this result it has been assumed that the initial driving charge density oscillates according to a cosinal function on the right hand side of Eq. (36). A more complicated initial driving function may be used according to circuit design or similar. The important result is that resonance occurs in the voltage, and this surge in voltage is caused by the spin connection of space-time. The voltage obtained in this way may be used for new energy.

In the limit:

$$r \rightarrow \infty, \qquad (51)$$

$$\kappa = \text{constant}, \qquad (52)$$

eq. (35) reduces to the Poisson equation:

$$\frac{d^2\phi}{dr^2} = -\frac{\rho}{\epsilon_0} \qquad (53)$$

used in the standard model. Eq. (46) may be rewritten as:

$$\phi = \frac{\rho(0)}{\epsilon_0} \frac{\cos^2(\kappa r)}{\left(\kappa^2 \cos(\kappa r) + \frac{5\kappa}{r} \sin(\kappa r) - \frac{3}{r^2} \cos(\kappa r) \right)} \quad -(54)$$

and in infinite r limit this equation becomes:

$$\phi \xrightarrow[r \to \infty]{} \frac{\rho(0)}{\epsilon_0} \left(\frac{\cos(\kappa r)}{\kappa^2} \right) \quad -(55)$$

so that:

$$\frac{d^2 \phi}{d r^2} = -\frac{\rho(0)}{\epsilon_0} \cos(\kappa r) = -\frac{\rho}{\epsilon_0} \quad -(56)$$

Q.E.D. In this limit is known that the scalar potential is { 21 }:

$$\phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\underline{r}')}{|\underline{r} - \underline{r}'|} d^3 r' \quad -(57)$$

so:

$$\left(\frac{\cos(\kappa r)}{\kappa^2} \right) \xrightarrow[r \to \infty]{} \frac{1}{4\pi\rho(0)} \int \frac{\rho(r')}{|\underline{r} - \underline{r}'|} d^3 r' . \quad -(58)$$

This is a mathematical check on the self-consistency of the analytical solution (46) of the resonance equation (35). Physically however the spin connection cannot vanish unless r becomes the radius of the universe. This is because the electromagnetic field is always spinning space-time in ECE theory. Similarly, the gravitational field is always curving space-time. MH theory (standard model) has no conception of the spin connection.

For multi electron systems and in three space dimensions, consider the equation:

$$\underline{E} = -\underline{\nabla}\phi = -\underline{\omega}\phi. \quad -(59)$$

The electric field from this equation is {21}:

$$\underline{E}(\underset{\sim}{r}) = -\frac{1}{4\pi\epsilon_0}\underline{\nabla}\int\frac{\rho(\underset{\sim}{r}')}{|\underset{\sim}{r}-\underset{\sim}{r}'|}d^3r' \quad -(60)$$

and the scalar potential is:

$$\phi = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\underset{\sim}{r}')}{|\underset{\sim}{r}-\underset{\sim}{r}'|}d^3r'. \quad -(61)$$

Thus:

$$\int\underline{\nabla}\phi\, d^3r' = \int\phi\,\underline{\omega}\, d^3r' \quad -(62)$$

where:

$$-\underline{\nabla}\left(\frac{1}{|\underset{\sim}{r}-\underset{\sim}{r}'|}\right) = \frac{\underset{\sim}{r}-\underset{\sim}{r}'}{|\underset{\sim}{r}-\underset{\sim}{r}'|^3} \quad -(63)$$

so:

$$\frac{\underset{\sim}{r}-\underset{\sim}{r}'}{|\underset{\sim}{r}-\underset{\sim}{r}'|^3} = -\frac{\underline{\omega}}{|\underset{\sim}{r}-\underset{\sim}{r}'|} \quad -(64)$$

and the three dimensional spin connection for an n electron system is:

$$\underline{\omega} = -\frac{(\underset{\sim}{r}-\underset{\sim}{r}')}{|\underset{\sim}{r}-\underset{\sim}{r}'|^2}. \quad -(65)$$

The resonance equation for n electrons is therefore:

$$\nabla^2 \phi + \underline{\omega} \cdot \underline{\nabla} \phi + (\underline{\nabla} \cdot \underline{\omega}) \phi = -\frac{\rho}{\epsilon_0} \quad -(66)$$

where the spin connection is given by Eq. (65) and where the charge density is defined in terms of the three dimensional Dirac delta function {21} as follows:

$$\rho(\underline{r}) = \sum_{i=1}^{n} q_i \, \delta(\underline{r} - \underline{r}_i). \quad -(67)$$

The electric field is {21}:

$$\underline{E}(\underline{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} q_i \frac{(\underline{r} - \underline{r}_i)}{|\underline{r} - \underline{r}_i|^3}$$

$$= \frac{1}{4\pi\epsilon_0} \int \rho(\underline{r}') \frac{(\underline{r} - \underline{r}')}{|\underline{r} - \underline{r}'|^3} d^3 r'. \quad -(68)$$

If Δq is the charge in a small volume

$$d^3 r = \Delta x \, \Delta y \, \Delta z \quad -(69)$$

then:

$$\Delta q = \rho(\underline{r}') \Delta x \, \Delta y \, \Delta z \quad -(70)$$

so the three dimensional resonance equation for n electrons is:

$$\nabla^2 \phi + \frac{(\underline{r} - \underline{r}_i)}{|\underline{r} - \underline{r}_i|^2} \cdot \underline{\nabla} \phi + \left(\underline{\nabla} \cdot \frac{(\underline{r} - \underline{r}_i)}{|\underline{r} - \underline{r}_i|^2} \right) \phi = -\frac{1}{\epsilon_0} \left(\sum_{i=1}^{n} q_i \, \delta(\underline{r} - \underline{r}_i) \right) \quad -(71)$$

where the spin connection is:

$$\omega_i = -\frac{(\underline{r}-\underline{r}_i)}{|\underline{r}-\underline{r}_i|^2} \, . \qquad -(72)$$

Therefore each electron and proton in an atom or molecule has its spin connection.

In order to clarify the meaning of these equations prior to coding in density function packages (Section 3), some detail is given as follows. To prove the vector equation {21}:

$$-\underline{\nabla}\left(\frac{1}{|\underline{r}-\underline{r}_i|}\right) = \frac{\underline{r}-\underline{r}_i}{|\underline{r}-\underline{r}_i|^3} \qquad -(73)$$

write it as:

$$-\underline{\nabla}\left(|\underline{r}-\underline{a}|\right)^{-1} = \frac{\underline{r}-\underline{a}}{|\underline{r}-\underline{a}|^3} \qquad -(74)$$

where

$$|\underline{r}-\underline{a}| = \left((x-a_x)^2+(y-a_y)^2+(z-a_z)^2\right)^{1/2} \qquad -(75)$$

and

$$-\underline{\nabla}\left(\frac{1}{|\underline{r}-\underline{a}|}\right) = -\frac{\partial}{\partial x}\left(\frac{1}{|\underline{r}-\underline{a}|}\right)\underline{i} - \frac{\partial}{\partial y}\left(\frac{1}{|\underline{r}-\underline{a}|}\right)\underline{j} - \frac{\partial}{\partial z}\left(\frac{1}{|\underline{r}-\underline{a}|}\right)\underline{k} \, . \qquad -(76)$$

Consider terms such as:

$$\frac{\partial}{\partial x}\left((x-a_x)^2+(y-a_y)^2+(z-a_z)^2\right)^{-1/2} = -\frac{(x-a_x)}{\left((x-a_x)^2+(y-a_y)^2+(z-a_z)^2\right)^{3/2}} \qquad -(77)$$

to find Eqs. (73) and (74), Q.E.D. In the resonance equation (71) there occurs the term:

$$f(\underline{r}) = \underline{\nabla} \cdot \frac{(\underline{r} - \underline{r}_i)}{|\underline{r} - \underline{r}_i|^2} \quad -(78)$$

and this may be developed as follows. Write:

$$f(\underline{r}) = \underline{\nabla} \cdot \frac{(\underline{r} - \underline{a})}{|\underline{r} - \underline{a}|^2} \quad -(79)$$

where:

$$|\underline{r} - \underline{a}|^2 = (x - a_x)^2 + (y - a_y)^2 + (z - a_z)^2 \quad -(80)$$

and

$$\underline{r} - \underline{a} = (x - a_x)\underline{i} + (y - a_y)\underline{j} + (z - a_z)\underline{k} \quad -(81)$$

so:

$$f(\underline{r}) = \frac{\partial}{\partial x}\frac{(x - a_x)}{|\underline{r} - \underline{a}|^2} + \frac{\partial}{\partial y}\frac{(y - a_y)}{|\underline{r} - \underline{a}|^2} + \frac{\partial}{\partial z}\frac{(z - a_z)}{|\underline{r} - \underline{a}|^2}$$

$$= \frac{\partial}{\partial x}\frac{(x - a_x)}{(x - a_x)^2 + (y - a_y)^2 + (z - a_z)^2} + \ldots \quad -(82)$$

Using the rules of differentiation:

$$f(\underline{r}) = \left((x - a_x)^2 + (y - a_y)^2 + (z - a_z)^2\right)^{-1} - \frac{2(x - a_x)^2}{\left((x - a_x)^2 + (y - a_y)^2 + (z - a_z)^2\right)^2} + \ldots$$

$$= \frac{1}{|\underline{r} - \underline{a}|^2} - \frac{2}{|\underline{r} - \underline{a}|^2}$$

$$= -\frac{1}{|\underline{r} - \underline{a}|^2} . \quad -(83)$$

Therefore:

$$\underline{\nabla} \cdot \frac{(\underline{r} - \underline{r}_i)}{|\underline{r} - \underline{r}_i|^2} = -\frac{1}{|\underline{r} - \underline{r}_i|^2} \quad -(84)$$

so the resonance equation is:

$$\nabla^2 \phi + \frac{(\underline{r} - \underline{r}_i)}{|\underline{r} - \underline{r}_i|^2} \cdot \underline{\nabla}\phi - \frac{1}{|\underline{r} - \underline{r}_i|^2}\phi = -\frac{1}{\epsilon_0}\left(\sum_{i=1}^{n} q_i \, \delta(\underline{r} - \underline{r}_i)\right). \quad -(85)$$

This can be written as:

$$|\underline{r} - \underline{r}_i|^2 \nabla^2 \phi + (\underline{r} - \underline{r}_i) \cdot \underline{\nabla}\phi - \phi = -\frac{1}{\epsilon_0} |\underline{r} - \underline{r}_i|^2 \sum_{i=1}^{n} q_i \, \delta(\underline{r} - \underline{r}_i) \quad -(86)$$

and incorporated in density functional code for the Coulomb potential. In Eq. (86) the

Dirac delta function is defined as usual {21} by:

$$\delta(\underline{r} - \underline{r}_1) = \delta(x_1 - X_1)\,\delta(y_1 - Y_1)\,\delta(z_1 - Z_1) \quad -(87)$$

and the charge density is defined by:

$$\rho(\underline{r}) = \sum_{i=1}^{n} e_i \, \delta(\underline{r} - \underline{r}_i). \quad -(88)$$

Using:

$$\underline{r} - \underline{r}_i = (x - x_i)\underline{i} + (y - y_i)\underline{j} + (z - z_i)\underline{k} \quad -(89)$$

and:

$$|\underline{r} - \underline{r}_i|^2 = (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 \quad -(90)$$

the spin connection in eq. (72) can be developed as:

$$\underline{\omega}_i = -\frac{\left((x - x_i)\underline{i} + (y - y_i)\underline{j} + (z - z_i)\underline{k}\right)}{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}. \quad -(91)$$

With these definitions the resonance equation in three dimensions and for n electrons and protons in an atom or molecule is therefore:

$$\nabla^2 \phi + \frac{(\underline{r} - \underline{r}_i)}{|\underline{r} - \underline{r}_i|^2} \cdot \underline{\nabla} \phi - \frac{1}{|\underline{r} - \underline{r}_i|^2} \phi = -\frac{1}{\epsilon_0} \sum_{i=1}^{n} q_i \, \delta(\underline{r} - \underline{r}_i). \quad -(92)$$

The potential in volts from this equation can be used to build up the Hartree term in density functional code. This term describes electron electron repulsion through the Coulomb interaction and is (Section 3):

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{e^2 n_s(\underline{r})}{|\underline{r} - \underline{r}'|} \, d^3 r' \quad -(93)$$

where $n_s(r)$ is a number density. Therefore the scalar potential in volts is:

$$\phi_H = \frac{e}{4\pi\epsilon_0}\int n_s(r)\, d^2r'. \quad -(94)$$

When there are peaks in ϕ_H, the effect is to greatly amplify the number density and to greatly increase the electron electron repulsion, resulting in ionization of the atom or molecule into free electrons which can be used for new energy.

The most direct method for acquiring new energy from the spin connection is to construct the circuits equivalent to Eqs. (35) or (38). The type of circuit needed for Eq. (38), whose analytical solution is Eq. (46), is that of the undamped oscillator. The simplest undamped oscillator equation {24} is:

$$m\ddot{x} + kx = F \quad -(95)$$

where m is mass, x is displacement, k is Hooke's constant and F is driving force. Its equivalent circuit equation is {24}:

$$L\ddot{q} + \frac{q}{C} = E \quad -(96)$$

where q is charge, L is inductance, C is capacitance and E is electromotive force. Eq (96) describes an electromotive force in series with a capacitor and induction coil. A material may be incorporated inside the induction coil. Therefore the circuit equivalent to the undamped oscillator (38) is the same design, but the electromotive force is synthesized to be the same as the right hand side of Eq. (38), i.e. made up of circular and hyperbolic functions. The exact solution (46) shows many sharp peaks of voltage (Section 3), so the equivalent circuit also shows many sharp peaks of voltage for a small initial electromotive force. These peaks of voltage can be used for new power and fed to the grid from a power plant.