1)

Notes 64(11): Simplifying the Spin Connection

The spin connection is the most important quantity in generally covariant electrodynamics and for practical applications it is important to simplify the theory and to identify the spin connection as precisely as possible. In the Coulomb Law for example the basic equations are:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(1)$$

and

$$\underline{E}^a = -\underline{\nabla}\phi^a - \underline{\omega}^a{}_b \phi^b . \quad -(2)$$

Only the scalar potential is being considered because this is electrostatics. Therefore eq. (1) becomes:

$$\phi^a_0 = \phi^{(0)} q^a_0 . \quad -(3)$$

From notes 64(8) the only spin connection elements are:

$$\left.\begin{aligned} \omega^1{}_{02} &= -\omega^2{}_{01} = \frac{\kappa}{2} q^0_0 \\ \omega^1{}_{03} &= -\omega^3{}_{01} = \frac{\kappa}{2} q^0_0 \\ \omega^2{}_{03} &= -\omega^3{}_{02} = \frac{\kappa}{2} q^0_0 \end{aligned}\right] \quad -(4)$$

Therefore there is only one spin connection element.

2)

The scalar potential $\phi^a_{\;0}$ must be a scalar in all frames of reference, and so must be the same quantity in all frames of reference. It follows that $q^a_{\;0}$ must be unity in all frames of reference. So:

$$\phi^a_{\;0} = \phi^{(0)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad q^a_{\;0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad -(5)$$

The product of $\underline{\omega}^a_{\;b}$ and ϕ^b must be:

$$\omega^a_{\;0b}\phi^b = \omega^a_{\;01}\phi^1 + \omega^a_{\;02}\phi^2 + \omega^a_{\;03}\phi^3 \quad -(6)$$

$$= \phi^{(0)}\left(\omega^a_{\;01} + \omega^a_{\;02} + \omega^a_{\;03}\right).$$

From eq. (4) it is seen that a must be 1, 2 or 3.

If $a = 1$:

$$\omega^a_{\;0b}\phi^b = \phi^{(0)}\left(\omega^1_{\;02} + \omega^1_{\;03}\right)$$

$$= \kappa\phi^{(0)} \quad -(7)$$

If $a = 2$: $$\omega^a_{\;0b}\phi^b = 0 \quad -(8)$$

If $a = 3$: $$\omega^a_{\;0b}\phi^b = -\kappa\phi^{(0)}. \quad -(9)$$

Thus:

$$\boxed{\begin{aligned} \underline{E}_1 &= -\underline{\nabla}\phi - \underline{\kappa}\phi \quad (a=1) \\ \underline{E}_0 &= -\underline{\nabla}\phi \quad (a=2) \\ \underline{E}_{-1} &= -\underline{\nabla}\phi + \underline{\kappa}\phi \quad (a=3) \end{aligned}}$$ $-(10)$, $-(11)$, $-(12)$

3) It is seen that the static electric field $\underline{E}$ of the standard model becomes $\underline{E}_1$, $\underline{E}_0$ and $\underline{E}_{-1}$ of general relativity (ECE theory). In other words $\underline{E}$ then becomes a vector boson, similar to that used in electroweak theory - the massive vector boson.

Off Resonance Condition

This is what is almost always observed experimentally. In this condition:

$$\phi = - \frac{e}{4\pi\epsilon_0 r} \quad - (13)$$

and

$$\underline{\nabla} \phi = \pm \underline{\kappa} \phi . \quad - (14)$$

In the radial direction:

$$\underline{\kappa} = \pm \frac{1}{r} \underline{e}_r \quad - (15)$$

where $\underline{e}_r$ is the radial unit vector.

Resonant Condition

The spin connection is still given by eq. (15), but ϕ becomes infinite.