54(2): Resonances from the Ampère Law

(1) The standard model Ampère Law is:

$$\underline{\nabla} \times \underline{H} = \underline{J} \quad -(1)$$

$$\underline{B} = \mu_0 (\underline{H} + \underline{M}) \quad -(2)$$

i.e.:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} + \underline{\nabla} \times \underline{M} \quad -(3)$$

$$:= \mu_0 \underline{J}_m$$

where $\underline{B}$ is magnetic flux density, $\underline{H}$ is magnetic field strength, μ_0 is vacuum permeability, and $\underline{M}$ is magnetization. Here $\underline{J}$ is current density. The current density $\underline{J}_m$ in eq. (3) has been defined to include $\underline{\nabla} \times \underline{M}$.

In ECE theory

$$\underline{\nabla} \times \underline{B}^a = \mu_0 \underline{J}_m^a \quad -(4)$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(5)$$

where $\underline{A}^a$ is the vector potential and $\underline{\omega}^a{}_b$ is the vector spin connection.

For simplicity of development (papers 60 and 61) we may omit the indices a and b in eqs. (4) and (5). In a computation, this need not be assumed. So:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J}_m \quad - (6)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad - (7)$$

In off resonance conditions we know that the standard Ampère Law is observed experimentally, so this means:

$$\underline{B} = \underline{\nabla} \times \underline{A} = -\underline{\omega} \times \underline{A} \quad - (8)$$

indicating that the spin connection doubles the magnetic flux density off resonance. So its presence is hidden. At resonance however it has a dramatic new effect, as follows.

From eq. (8) in eq. (6):

$$\boxed{\underline{\nabla} \times (\underline{\omega} \times \underline{A}) = -\mu_0 \underline{J}_m} \quad - (9)$$

Under well defined conditions this is a resonance equation. We keep things simple in the following for the sake of analytical illustration. A computation can deal with any degree of complexity or any particular design.

Consider ω_x and A_z, ~~then~~ :

3)

$$\underline{\omega} \times \underline{A} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & 0 & 0 \\ 0 & 0 & A_z \end{vmatrix} \quad -(10)$$

$$= -\omega_x A_z \underline{j}.$$

$$\underline{\nabla} \times (\underline{\omega} \times \underline{A}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & -\omega_x A_z & 0 \end{vmatrix} \quad -(11)$$

$$= \frac{\partial}{\partial z}(\omega_x A_z)\,\underline{i} + \frac{\partial}{\partial x}(\omega_x A_z)\,\underline{k}$$

So:

$$\frac{\partial}{\partial z}(\omega_x A_z) = -\mu_0 J_{mx} \quad -(12)$$

$$\frac{\partial}{\partial x}(\omega_x A_z) = -\mu_0 J_{mz} \quad -(13)$$

These are two resonance equations. For example:

$$\frac{\partial}{\partial z}(\omega_x A_z) = -\mu_0 J_{mx}(0) \sin(\kappa z) \quad -(14)$$

Differentiating both sides of eq (14) with respect to z: $\quad -(15)$

$$\omega_x \frac{\partial^2 A_z}{\partial z^2} + 2\left(\frac{\partial A_z}{\partial z}\right)\left(\frac{\partial \omega_x}{\partial z}\right) + \left(\frac{\partial^2 \omega_x}{\partial z^2}\right) A_z = \mu_0 \kappa J_{mx}(0) \cos(\kappa z)$$

The resonance spectrum is a graph of A_Z against Z. This is a damped driven oscillator equation. The **damping term** is $\frac{\partial}{\omega_x}\left(\frac{\partial \omega_x}{\partial Z}\right)$. The **Hooke term** is $\frac{1}{\omega_x}\left(\frac{\partial^2 \omega_x}{\partial Z^2}\right)$. The **driving term** is $\frac{1}{\omega_x} \cdot \mu_0 \kappa J_{mx}(0) \cos(\kappa Z)$.

At **resonance** A_Z is greatly amplified, and from (a), J_m is greatly amplified, producing an amplification of current density in a coil wound around a magnet. The conditions for tuning to resonance are determined by equations such as (15). This means tuning the κ to the Hooke term.