64(6): Resonance Faraday Induction Law

The ECE Faraday law of induction is:

$$\underline{\nabla} \times \underline{E}^a + \frac{\partial \underline{B}^a}{\partial t} = \mu_0 \tilde{\underline{j}}^a \quad -(1)$$

where:

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} - c\underline{\nabla} A^{0a} - c\omega^{0a}{}_b \underline{A}^b + c\,\underline{\omega}^a{}_b A^{0b} \quad -(2)$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b. \quad -(3)$$

Assume that the scalar potential is zero and that far off-resonance:

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} = -c\omega^{0a}{}_b \underline{A}^b. \quad -(4)$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a = -\underline{\omega}^a{}_b \times \underline{A}^b. \quad -(5)$$

Method

Eqs (4) and (5) are used to find $\omega^{0a}{}_b$ and $\underline{\omega}^a{}_b$, and ~~thus~~:

$$\omega^a{}_{\mu b} = \left(\omega^{0a}{}_b, -\underline{\omega}^a{}_b\right) \quad -(6)$$

appropriate to the Faraday law of induction. Then these are assumed to be basic space-time quantities and used to find resonance from eqs (1) to (3).

2) Thus:

$$\underline{E}^a = -\frac{\partial \underline{A}^a}{\partial t} - c\omega^{0a}{}_b \underline{A}^b \quad -(7)$$

and

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(8)$$

with $\omega^a{}_{\mu b}$ determined as above.

From (7) and (8) in (1):

$$\boxed{c\omega^{0a}{}_b \underline{\nabla} \times \underline{A}^b + \frac{\partial}{\partial t}\left(\underline{\omega}^a{}_b \times \underline{A}^b\right) = -\mu_0 \underline{j}^a} \quad -(9)$$

Various types of resonance equations can be constructed from eq (9), for example by differentiating either side with respect to t: $\quad -(10)$

$$c\omega^{0a}{}_b \frac{\partial}{\partial t}\left(\underline{\nabla} \times \underline{A}^b\right) + \frac{\partial^2}{\partial t^2}\left(\underline{\omega}^a{}_b \times \underline{A}^b\right) = -\mu_0 \frac{\partial \underline{j}^a}{\partial t}$$

to give:

$$\frac{\partial}{\partial t}\left(\frac{\partial \underline{\omega}^a{}_b}{\partial t} \times \underline{A}^b + \underline{\omega}^a{}_b \times \frac{\partial \underline{A}^b}{\partial t}\right) + c\omega^{0a}{}_b \frac{\partial}{\partial t}\left(\underline{\nabla} \times \underline{A}^b\right)$$

$$= -\mu_0 \frac{\partial \underline{j}^a}{\partial t}. \quad -(11)$$

i.e.

3)

$$\frac{\partial^2 \underline{\omega}^a{}_b}{\partial t^2} \times \underline{A}^b + 2\frac{\partial \underline{\omega}^a{}_b}{\partial t} \times \frac{\partial \underline{A}^b}{\partial t} + \underline{\omega}^a{}_b \times \frac{\partial^2 \underline{A}^b}{\partial t^2}$$

$$+ c\omega^{0a}{}_b \underline{\nabla} \times \frac{\partial \underline{A}^b}{\partial t} = -\mu_0 \frac{\partial \underline{j}^a}{\partial t} \quad -(12)$$

This is a damped oscillator equation in the vector potential:

$$\boxed{\underline{\omega}^a{}_b \times \frac{\partial^2 \underline{A}^b}{\partial t^2} + \left(2\frac{\partial \underline{\omega}^a{}_b}{\partial t} + c\omega^{0a}{}_b \underline{\nabla} \right) \times \frac{\partial \underline{A}^b}{\partial t} + \frac{\partial^2 \underline{\omega}^a{}_b}{\partial t^2} \times \underline{A}^b = -\mu_0 \frac{\partial \underline{j}^a}{\partial t}} \quad -(13)$$

From experience gained in paper 63 it can be seen that this equation will give many resonant peaks in $\underline{A}^b$.

So from eqs (7) and (8):

$$\left.\begin{array}{l} \underline{E}^a \rightarrow \infty \\ \underline{B}^a \rightarrow \infty \end{array}\right\} -(14)$$

at resonance. The latter depends on the existence of the homogeneous current $\underline{j}^a$.