1)

65(2) : Basic Resonance Equations of Magnetostatics

In general there are four basic equations:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(1)$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(2)$$

$$\underline{\nabla} \cdot \underline{B}^a = \mu_0 j^{0a} \quad -(3)$$

$$\underline{\nabla} \times \underline{B}^a = \mu_0 \underline{J}_m^a . \quad -(4)$$

There is no scalar potential, so ~~the~~ space-like elements of A^a_μ are needed:

$$A^a_\mu = \left(0, -\underline{A}^a\right). \quad -(5)$$

This implies, from eq. (1):

$$q^a_\mu = \left(0, -\underline{q}^a\right). \quad -(6)$$

From eq. (2):

$$\underline{B}^1 = \underline{\nabla} \times \underline{A}^1 - \left(\underline{\omega}^1{}_0 \times \underline{A}^0 + \underline{\omega}^1{}_2 \times \underline{A}^2 + \underline{\omega}^1{}_3 \times \underline{A}^3\right) \quad -(7)$$

etc.

for $a = 1, 2, 3$.

If electromagnetism and gravitation are independent:

$$j^{0a} = 0. \quad -(8)$$

Following paper 56, eq. (63), eqn. (8) implies:

2)

$$\underline{B}^1 = \underline{\nabla} \times \underline{A}^1 - g\, \underline{A}^2 \times \underline{A}^3 \quad -(9)$$

$$\underline{B}^2 = \underline{\nabla} \times \underline{A}^2 - g\, \underline{A}^3 \times \underline{A}^1 \quad -(10)$$

$$\underline{B}^3 = \underline{\nabla} \times \underline{A}^3 - g\, \underline{A}^1 \times \underline{A}^2 \quad -(11)$$

$$\underline{\nabla} \times \underline{A}^0 = - \underline{\nabla} \times \underline{A}^3 \quad -(12)$$

From eqns (3) and (8):

$$\underline{\nabla} \cdot \underline{B}^a = 0. \quad -(13)$$

The basic equations of magnetostatics in general relativity are therefore eqn. (13) and:

$$\underline{\nabla} \times \underline{B}^a = \mu_0 \underline{J}_m^a \quad -(14)$$

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - g\, \underline{A}^b \times \underline{A}^c. \quad -(15)$$

Here:

$$g = \frac{\kappa}{A^{(0)}}. \quad -(16)$$

The resonance equation is:

$$\boxed{\underline{\nabla} \times \left(\underline{\nabla} \times \underline{A}^a - g\, \underline{A}^b \times \underline{A}^c \right) = \mu_0 \underline{J}_m^a} \quad -(17)$$

with:

$$\underline{\nabla} \cdot \left(\underline{\nabla} \times \underline{A}^a - g\, \underline{A}^b \times \underline{A}^c \right) = 0 \quad -(18)$$

i.e. $$\boxed{\underline{\nabla} \cdot \underline{A}^b \times \underline{A}^c = 0} \quad -(19)$$