

(2): Vector Basis Structure of Electric Field :
Some Details

$$\underline{E}^a = -\underline{\nabla} \phi^a + \underline{\omega}^a{}_b \phi^b \quad - (1)$$

$a, b = 0, \dots, 3$

Assume for simplicity only that:

$$\phi = \phi^0 = \phi^1 = \phi^2 = \phi^3 \quad - (2)$$

Then:

$$\underline{E}^a = -\underline{\nabla} \phi^a + (\underline{\omega}^a{}_0 + \underline{\omega}^a{}_1 + \underline{\omega}^a{}_2 + \underline{\omega}^a{}_3) \phi \quad - (3)$$

i.e.

$$\begin{aligned} E_1^a &= -\frac{\partial}{\partial x_1} \phi^a + (\omega^a{}_{10} + \omega^a{}_{11} + \omega^a{}_{12} + \omega^a{}_{13}) \phi \\ E_2^a &= -\frac{\partial}{\partial x_2} \phi^a + (\omega^a{}_{20} + \omega^a{}_{21} + \omega^a{}_{22} + \omega^a{}_{23}) \phi \\ E_3^a &= -\frac{\partial}{\partial x_3} \phi^a + (\omega^a{}_{30} + \omega^a{}_{31} + \omega^a{}_{32} + \omega^a{}_{33}) \phi \end{aligned} \quad - (4)$$

The elements are defined by:

$$\omega_{\mu 2}^1 = \omega_{\mu 3}^1 = \omega_{\mu 3}^2 = \frac{\kappa}{2} \mathcal{V}_{\mu}^0, \quad - (5)$$

$\omega_{\mu 1}^2 = -\omega_{\mu 2}^1$ etc.

For $a=1$

$$E_1^1 = -\frac{\partial}{\partial x_1} \phi + (\omega^1{}_{10} + \omega^1{}_{11} + \omega^1{}_{12} + \omega^1{}_{13}) \phi$$

(II)

$$= -\frac{\partial}{\partial x_2} \phi + (\omega'_{12} + \omega'_{13}) \phi$$

$$= -\frac{\partial}{\partial x_1} \phi + \kappa \phi$$

+1

$$\boxed{\underline{E}^1 = -\underline{\nabla} \phi + \underline{\kappa} \phi} \quad - (6)$$

a=2

$$E_1^2 = -\frac{\partial}{\partial x_2} \phi + (\omega^2_{10} + \omega^2_{11} + \omega^2_{12} + \omega^2_{13}) \phi$$

$$= -\frac{\partial}{\partial x_2} \phi + (\omega^2_{11} + \omega^2_{13}) \phi$$

$$= -\frac{\partial}{\partial x_2} \phi + (\omega^2_{13} - \omega'_{12})$$

$$= -\frac{\partial}{\partial x_2} \phi$$

0

$$\boxed{\underline{E}^2 = -\underline{\nabla} \phi} \quad - (7)$$

a=3

$$E_1^3 = -\frac{\partial \phi}{\partial x_3} + (\omega^3_{10} + \omega^3_{11} + \omega^3_{12} + \omega^3_{13}) \phi$$

$$= -\frac{\partial}{\partial x_3} \phi + (\omega^3_{11} + \omega^3_{12}) \phi$$

$$= -\frac{\partial}{\partial x_3} \phi - (\omega'_{13} + \omega^2_{13}) \phi$$

$$= -\frac{\partial}{\partial x_3} \phi - \kappa \phi$$

iii)

$$\boxed{\underline{E}^3 = -\underline{\nabla} \phi - \underline{\kappa} \phi} \quad - (9)$$

The Isa indices are $(1, 0, -1)$. The electric field develops an internal space, which originates in the structure of ECE spacetime. The spi connection reduces to $(\underline{\kappa}, 0, -\underline{\kappa})$. For the first and third of these resonance occurs, but not for the second, in which the spi connection vanishes. So the spi connection is also a vector Isa.

This is a rigorously objective development of the static electric field using the principles of general relativity in ECE theory. In fact, general relativity is ^{practically} much more important to physics when applied to electrostatics than when applied to gravitation. This is because of the obvious practical importance of electricity.
