67(3) Scalar Potential Structure from ECE Ansatz

The Ansatz is:

$$A^a_\mu = A^{(0)} q^a_\mu . \quad -(1)$$

The guide from MH to the scalar potential is:

$$\left.\begin{array}{l} A_\mu = (\phi/c, -\underline{A}) \\ \rightarrow (\phi/c, \underline{0}) \end{array}\right\} -(2)$$

In ECE it is possible that:

$$A^a_\mu \rightarrow (A^a_0, \underline{0}) \text{ or } (A^a_\mu, \underline{0}) \quad -(3)$$

To Prove

That the two definitions in eq. (3) are equivalent.

Proof

Use the fundamental definitions:

$$U^a = q^a_0 U^0 \quad -(4)$$

$$U^0 = q^0_\mu U^\mu \quad -(5)$$

Assume that $\quad U^0 U_0 = 1 \quad -(6)$

for simplicity of argument.

Multiply both sides of eqns. (4) and (5) by U_0 to obtain:

$$U_0 U^a = q^a_0 \quad -(7)$$

and

$$q^0_\mu U^\mu U_0 = 1 . \quad -(8)$$

The fundamental tetrad normalization is:

$$q^a_\mu q^\mu_a = 1 \quad -(9)$$

so

$$q^0_\mu q^\mu_0 = 1 \quad -(10)$$

and:

$$q^{a}{}_{0}\, q^{0}{}_{a} = 1 \qquad -(11)$$

Using eqns. (7), (8) and (11):

$$\boxed{q^{0}{}_{a}\, U^{a} = q^{0}{}_{\mu}\, U^{\mu}} \qquad -(12)$$

The role of the a and μ indices is equivalent and interchangeable if one index of the tetrad is fixed at zero.

<u>Conclusion</u>

A scalar potential is representable either by $A^{0}{}_{\mu}$ or by $A^{a}{}_{0}$, QED.

<u>Notes</u>

1) We have $A^{a}{}_{0} = A^{(0)}\, q^{a}{}_{0}$ $\quad -(13)$

and

$$A^{0}{}_{\mu} = A^{(0)}\, q^{0}{}_{\mu} \qquad -(14)$$

where:

$$\phi^{(0)} = c A^{(0)}. \qquad -(15)$$

The actual scalar is $\phi^{(0)}$ in volts. The tetrads $q^{a}{}_{0}$ and $q^{0}{}_{\mu}$ are <u>mixed index rank two tensors.</u>

2) The equivalence (12) means that one frame spinning with respect to a second is indistinguishable from the second frame spinning with respect to the first.

3) This equivalence is used in the development of the static electric field as a vector boson in 67(2)