67(4): Electric Field as Vector Boson, Angular Momentum Theory

Angular momentum theory is developed in M. W. Evans and J.-P. Vigier, "The Enigmatic Photon", volume one, chapter five (Kluwer softback 2002). In ECE theory the electric field is a vector boson defined by:

$$\underline{E}_1 = -\underline{\nabla}\phi + \underline{\kappa}\phi \quad -(1)$$

$$\underline{E}_0 = -\underline{\nabla}\phi \quad -(2)$$

$$\underline{E}_{-1} = -\underline{\nabla}\phi - \underline{\kappa}\phi \quad -(3)$$

The components are written as $(-1, 0, 1)$ to emphasize the angular momentum property of the electric field. Angular momentum theory is very highly developed, and a short summary is given in the above reference. Angular momentum is used extensively throughout "The Enigmatic Photon" and in later publications.

In the simplest instance it can be developed by considering $O(3)$ vector relations. In the Cartesian basis:

$$\underline{i} \times \underline{j} = \underline{k} \quad -(4)$$

$$\underline{k} \times \underline{i} = \underline{j} \quad -(5)$$

$$\underline{j} \times \underline{k} = \underline{i}. \quad -(6)$$

Defining the operator:

$$\hat{J}_z = \underline{k} \times \quad -(7)$$

Then:

$$\hat{J}_z \underline{i} = 1 \underline{j} \quad -(8)$$

$$\hat{J}_z \underline{j} = -1 \underline{i} \quad -(9)$$

$$\hat{J}_z \underline{k} = 0 \underline{k} \quad -(10)$$

so the eigenvalues of $\hat{J}_z$ are -1, 0, and 1

Defining:

$$\underline{\kappa} = \underline{\kappa}_x + \underline{\kappa}_y + \underline{\kappa}_z \quad -(11)$$

where:

$$\underline{\kappa}_x = \kappa_x \underline{i} \quad -(12)$$

$$\underline{\kappa}_y = \kappa_y \underline{j} \quad -(13)$$

$$\underline{\kappa}_z = \kappa_z \underline{k} \quad -(14)$$

then:

$$\hat{J}_z \underline{\kappa}_x = 1 \underline{\kappa}_y \quad -(15)$$

$$\hat{J}_z \underline{\kappa}_y = -1 \underline{\kappa}_x \quad -(16)$$

$$\hat{J}_z \underline{\kappa}_z = 0 \underline{\kappa}_z. \quad -(17)$$

Complex Circular Basis

This is defined by the unit vectors:

$$\underline{e}^{(2)*} = \underline{e}^{(1)} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) \quad -(18)$$

$$\underline{e}^{(1)*} = \underline{e}^{(2)} = \frac{1}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right) \quad -(19)$$

$$\underline{e}^{(3)*} = \underline{e}^{(3)} = \underline{k}. \quad -(20)$$

3)

Thus $O(3)$ symmetry is represented by:

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i\underline{e}^{(3)*} \quad -(21)$$

$$\underline{e}^{(2)} \times \underline{e}^{(3)} = i\underline{e}^{(1)*} \quad -(22)$$

$$\underline{e}^{(3)} \times \underline{e}^{(1)} = i\underline{e}^{(2)*} . \quad -(23)$$

Now define the operator:

$$\hat{J}^{(3)} = i\underline{e}^{(3)} \times \quad -(24)$$

Then:

$$\hat{J}^{(3)}\underline{e}^{(1)} = 1\,\underline{e}^{(1)} \quad -(25)$$

$$\hat{J}^{(3)}\underline{e}^{(2)} = -1\,\underline{e}^{(2)} \quad -(26)$$

$$\hat{J}^{(3)}\underline{e}^{(3)} = 0\,\underline{e}^{(3)} \quad -(27)$$

Thus $\hat{J}^{(3)}$ is a type of angular momentum operator with eigenvalues $-1, 0, 1$. If:

$$\underline{\kappa} = \underline{\kappa}^{(1)} + \underline{\kappa}^{(2)} + \underline{\kappa}^{(3)} \quad -(28)$$

then:

$$\hat{J}^{(3)}\underline{\kappa}^{(1)} = 1\,\underline{\kappa}^{(1)} \quad -(29)$$

$$\hat{J}^{(3)}\underline{\kappa}^{(2)} = -1\,\underline{\kappa}^{(2)} \quad -(30)$$

$$\hat{J}^{(3)}\underline{\kappa}^{(3)} = 0\,\underline{\kappa}^{(3)} . \quad -(31)$$

The eigenvalues $(-1, 0, 1)$ signify the existence of $O(3)$ symmetry. The numbers

-1, 0 and 1 appear again in the matrix equivalents of $\underline{i}$, $\underline{j}$ and $\underline{k}$:

$$\underline{i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \quad \underline{j} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \underline{k} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad -(32)$$

In the theory of irreducible tensorial sets, the rotation operators in Euclidean space are first rank irreducible tensor operators and $\hat{T}$ operators, which are irreducible tensor operators and under rotations transform into linear combinations of each other. The $\hat{T}$ operators are directly proportional to the scalar spherical harmonic operators. The rotation generators $\hat{J}$ of the full rotation group are related to the $\hat{T}$ operators as follows:

$$\hat{T}^1_{-1} = i\hat{J}^{(1)}, \quad \hat{T}^1_1 = i\hat{J}^{(2)}, \quad \hat{T}^1_0 = i\hat{J}^{(3)}. \quad -(33)$$

The spherical harmonic operators are defined by:

$$\hat{Y}^1_{-1} = \frac{i}{r}\left(\frac{3}{4\pi}\right)^{1/2}\hat{J}^{(1)}, \quad -(34)$$

$$\hat{Y}^1_1 = \frac{i}{r}\left(\frac{3}{4\pi}\right)^{1/2}\hat{J}^{(2)}, \quad -(35)$$

$$\hat{Y}^1_0 = \frac{i}{r}\left(\frac{3}{4\pi}\right)^{1/2}\hat{J}^{(3)}. \quad -(36)$$

The generator $\hat{J}^{(3)}$ is also an intrinsic spin and in quantum theory is <u>the</u> <u>intrinsic</u> <u>spin</u> <u>of</u> <u>a massive</u> <u>boson</u>.

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For a classical vector field its intrinsic or spin angular momentum is identifiable with its transformation properties under rotations, and within $\underline{\mathcal{L}}$, the rotation generators $\hat{J}$ are spin angular momentum generators of the spin one boson.

If we define:

$$\underline{e}_1 := -\underline{e}^{(2)}, \quad \underline{e}_{-1} := \underline{e}^{(1)}, \quad \underline{e}_0 := \underline{e}^{(3)} \quad -(37)$$

then the vector spherical harmonics and Clebsch-Gordan coefficients are defined by:

$$Y^{L}_{Ml1} = \sum_{m\,n} \langle l\,1\,m\,n \,|\, l\,1\,L\,M\rangle\, Y^{l}_{m}\, \underline{e}_{m} \quad -(38)$$

and it is found that:

$$\underline{e}^{(3)} = 2\,Y^{1}_{001} \,/\, Y^{1}_{0} \quad -(39)$$

$$= \frac{\sqrt{2}}{2}\, a\left(\underline{e}^{(1)} + \underline{e}^{(2)}\right) + \underline{b}$$

$$= -\frac{\sqrt{2}}{2}\, c\left(\underline{e}^{(1)} - \underline{e}^{(2)}\right) + \underline{d}$$

where:

$$a = \frac{2}{\sqrt{2}}\left(\frac{Y^{1}_{0}}{Y^{1}_{1} - Y^{1}_{-1}}\right), \quad c = -\frac{2}{\sqrt{2}}\left(\frac{Y^{1}_{0}}{Y^{1}_{1} + Y^{1}_{-1}}\right),$$

$$\underline{b} = \sqrt{2}\left(\frac{Y^{1}_{111} + Y^{1}_{-111}}{Y^{1}_{1} - Y^{1}_{-1}}\right), \quad \underline{d} = \frac{\sqrt{2}}{1}\left(\frac{Y^{1}_{111} - Y^{1}_{-111}}{Y^{1}_{1} + Y^{1}_{-1}}\right)$$

6)

Thus $\underline{e}^{(1)}$, $\underline{e}^{(2)}$ and $\underline{e}^{(3)}$ are linearly related. This also means that the components $\underline{E}_1$, $\underline{E}_0$ and $\underline{E}_{-1}$ are linearly related. They all must exist in general relativity, and all must be observable.

In volume one of "The Enigmatic Photon" by Vigier and myself this argument was used to ~~~~ show that if $\underline{B}^{(1)}$ and $\underline{B}^{(2)}$ exist, ~~then~~ $\underline{B}^{(3)}$ exists in Euclidean space. This is one of ~~the~~ many arguments for $\underline{B}^{(3)}$ used in that five volume book. I wrote them but Vigier gave constant encouragement. He worked with Louis de Broglie for many years and was invited to become Albert Einstein's assistant.

In eqs. (1) to (3) one may think of $\underline{E}_1$, $\underline{E}_0$ and $\underline{E}_{-1}$ as angular momentum operators giving eigen-values $\underline{K}$, $\underline{0}$ and $-\underline{K}$. This is a direct result of g.r. The spin connections are angular momentum eigen-values.