69(2): Levels of Approximation in the IFE and RFR.

In general relativity both effects originate in the $\omega \wedge A$ term of:

$$F = d \wedge A + \omega \wedge A \quad -(1)$$

and there are various levels of approximation that can be used to evaluate $\omega \wedge A$: classical, semi-classical, special relativistic q.e.d., and general relativistic. For rigorously objective physics general relativity is always needed for any equation of physics. For electromagnetism free of any gravitational influence the spin connection ω is dual to the potential, defining the ECE spin field. The IFE and RFR follow directly from the spin field:

$$\underline{B}^{(3)*} = -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(2)$$

The inverse Faraday effect is the magnetization of matter by a circularly polarized electromagnetic field. In free space the latter's spin field is:

$$\underline{B}^{(3)} = B^{(0)}\,\underline{k} = \kappa A^{(0)}\,\underline{k} \quad -(3)$$

where

$$g = \kappa / A^{(0)}. \quad -(4)$$

The conjugate product is:

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = \underline{A} \times \underline{A}^{*} \quad -(5)$$

When $\underline{B}^{(3)}$ interacts with matter (e.g. an electron) then:

$$g \rightarrow g' \quad -(6)$$

where g' is to be determined from dynamics. The IFE is the magnetization:

$$\underline{m}^{(3)} = \frac{1}{\mu_0} \underline{B}^{(3)} = -ig' \underline{A}^{(1)} \times \underline{A}^{(2)}$$

$$\boxed{\underline{m}^{(3)} = g' A^{(0)2} \underline{k}} \quad -(7)$$

We now relate $A^{(0)2}$ of the e/m field to its power density I (watts m^{-2}) using the standard optical equation (e.g. Atkins, "Molecular Quantum Mechanics" (OUP, 1983, 2nd. ed.)):

$$A^{(0)2} = \frac{\mu_0}{c} \left(\frac{I}{\omega^2} \right) \quad -(8)$$

So:

$$\boxed{\underline{m}^{(3)} = g' \frac{\mu_0}{c} \left(\frac{I}{\omega^2} \right) \underline{k}} \quad -(9)$$

The factor g' must also be calculated from general relativity to be self-consistent

It must be calculated from the ECE wave equation in the presence of interaction between the electromagnetic field and, on the simplest level, one electron. This problem is described in chapter 21, pp. 374 ff. of M. W. Evans, "Generally Covariant Unified Field Theory" (Abramis, 2005), volume one. In general relativity the equation is:

$$\left(\gamma^a\left(i\hbar\, d_a - eA_a\right) - mc\right)\psi^c = 0 \quad -(10)$$

which in the absence of interaction of the fermion with the gravitational field reduces to the well known equation:

$$\left(\gamma^\mu\left(i\hbar\, \partial_\mu - eA_\mu\right) - mc\right)\psi = 0. \quad -(11)$$

Eq. (11) is the Dirac equation with minimal prescription, which is well known to introduce the half integral spin and to describe the Zeeman effect. In the non-relativistic limit, eq. (11) reduces to the Schrödinger - Pauli equation:

$$H\psi = E\psi \quad -(12)$$

where:
$$H = \frac{1}{2m}\,\underline{\sigma}\cdot\left(\underline{p} + e\underline{A}\right)\underline{\sigma}\cdot\left(\underline{p} + e\underline{A}^*\right)\underline{\sigma} + V \quad -(13)$$

4) For a static magnetic field (Evans & Crowell, pp. 27 ff):

$$H\phi = \frac{e\hbar}{2m}\, \underline{\sigma} \cdot \underline{B}\, \phi \quad -(14)$$

and for an electromagnetic field:

$$H\phi = \frac{\mu_0 c e^2}{2m} \frac{I}{\omega^2} \sigma_Z \phi \quad -(15)$$

Resonance Conditions

1) For electron spin resonance:

$$\hbar\omega_{res} = \frac{e\hbar}{2m}(1-(-1))B \quad -(16)$$

i.e.

$$\boxed{\omega_{res} = \frac{e}{m}B} \quad -(17)$$

ESR

2) For radiatively induced fermion resonance (RFR):

$$\hbar\omega_{res} = \frac{\mu_0 c e^2}{2m}\frac{I}{\omega^2}(1-(-1))$$

$$\boxed{\omega_{res} = \left(\frac{\mu_0 c e^2}{\hbar m}\right)\frac{I}{\omega^2}} \quad -(18)$$

RFR

The inverse Faraday effect is obtained in the classical relativistic limit of eq. (11)

5) which is the relativistic Hamilton Jacobi equation:

$$(p^\mu - eA^\mu)(p_\mu - eA_\mu)^* = m^2c^2 \quad -(19)$$

(see M. W. Evans and J.-P. Vigier, "The Enigmatic Photon", vols 1 and 3).

The solution of eq. (19) for N electrons in a sample volume V is:

$$\boxed{\underline{B}^{(3)}_{\text{in sample}} = \frac{N}{V}\,\frac{\mu_0 e^3 c^2}{2m\omega^2}\left(\frac{B^{(0)}}{(m^2\omega^2 + e^2 B^{(0)2})^{1/2}}\right)\underline{B}^{(3)}_{\text{free space}}} \quad -(20)$$

In the limit:

$$m\omega \ll eB^{(0)} \quad -(21)$$

$$\underline{B}^{(3)}_{\text{in sample}} \rightarrow \frac{N}{V}\left(\frac{\mu_0^2 e^3 c}{2m^2}\right)\frac{I}{\omega^2}\underline{k}, \quad -(22)$$

$$\underline{M}^{(3)}_{\text{in sample}} = \frac{1}{\mu_0}\underline{B}^{(3)}_{\text{in sample}}. \quad -(23)$$

Comparing eqs. (9) and (23):

$$g' = \frac{N}{V}\,\frac{e^3c^2}{2m^2} \quad -(24)$$

in this limit.