1/72(2): Origin of Phase in the General Coordinate Transformation

It has been shown in paper 71 and in note 72(1) that the invariance of the tetrad postulate implies the existence of a dimensionless phase factor $e^{i\alpha}$ generated by coordinate transformation in general relativity. There are various kinds of phase now known - in electrodynamics for example the phase is conventionally expressed as:

$$\phi = x^{\mu} p_{\mu} = \omega t - \underline{\kappa} \cdot \underline{r} \quad -(1)$$

where ω is the angular frequency, t the time, $\underline{\kappa}$ the wavenumber vector and $\underline{r}$ the position vector. The Wu Yang phase was devised to explain the existence of the Aharonov Bohm effects, and uses the conventional Stokes theorem. There are also topological phases such as the Berry phase. In the standard model these are described by special relativity. In previous work in ECE theory and its precursor theories a common origin for these phases has been sought. In paper 71 it was shown that the general coordinate transformation produces the result:

$$\boxed{q^a_{\mu} \longrightarrow e^{i\alpha} q^a_{\mu}} \quad -(2)$$

where q^a_{μ} is the tetrad and where α is independent

2) of x^μ, i.e. independent of distance and time.

This means that α is "global" in nature whereas a phase such as ϕ of eq. (1) depends on x^μ and is "local" in nature. In note 72(1) it was concluded that any type of phase must consist of both a local and global component. Thus, the electrodynamical phase (1) for example is augmented by a global component and becomes:

$$\boxed{\phi = \underbrace{\omega t - \underline{\kappa} \cdot \underline{r}}_{\text{local}} + \underbrace{\alpha}_{\text{global}}} \quad -(3)$$

The complete phase factor is therefore:

$$\exp\left(i\left(\omega t - \underline{\kappa} \cdot \underline{r} + \alpha\right)\right). \quad -(4)$$

In quantum mechanics this implies that the action is:

$$S = \hbar\left(\omega t - \underline{\kappa} \cdot \underline{r} + \alpha\right) \quad -(5)$$

where $\hbar$ is the quantum of action or angular momentum. There therefore exists the global action:

$$\boxed{S(\text{global}) = \hbar \alpha} \quad -(6)$$

3) This is also a global angular momentum, which is generated by a general coordinate transformation as we have argued. In 72(1) it was argued that the phase is always of the type (3) because there is no preferred frame of reference, i.e. there is no reason to assume that α is zero.

This finding implies that wave-functions in quantum mechanics always consist of a local component that depends on x^μ and a global component. The latter is responsible for effects such as the Aharonov Bohm effects and quantum entanglement, one photon Young interferometric effects and "action at a distance" effects.

In ECE the existence of $\underline{\alpha}$ depends on the spinning of spacetime. The latter can be thought of as a "rotational transformation", a transformation that results in the rotation of the coordinate system itself. This is what is meant by "spinning of spacetime". Thus, $e^{i\alpha}$ is a rotation generator. In ECE theory the spinning of spacetime is described by the Cartan torsion:

4)

$$T^a = D \wedge q^a$$

$$= d \wedge q^a + \omega^a{}_b \wedge q^b \quad -(7)$$

where q^a is the tetrad and $\omega^a{}_b$ the spin connection.

If there is no spinning of spacetime then:

$$\omega^a{}_b = 0. \quad -(8)$$

When there is no spinning of spacetime:

$$q^a_\mu = q^a_\mu(0)\, e^{i(\omega t - \kappa Z)} \quad -(9)$$

for propagation in the Z axis. When there is spinning of spacetime:

$$q^a_\mu = q^a_\mu(0)\, e^{i(\omega t - \kappa Z + \alpha)} \quad -(10)$$

Therefore the appearance of α is related to the appearance of the spin connection $\omega^a{}_b$.

In the absence of spacetime spin the Cartan torsion is:

$$T^a_{\mu\nu} = d \wedge \left(q^a_\mu(0)\, e^{i(\omega t - \kappa Z)}\right) \quad -(11)$$

and is purely local. In electrodynamics

5) This is essentially the standard model result for each polarization indexed a:

$$F^a_{\mu\nu} = d \wedge \left(A^a_\mu(0) e^{i(\omega t - \kappa z)} \right) \qquad -(12)$$

For a standard model magnetic field for example:

$$\underline{B} = \underline{\nabla} \times \underline{A} \qquad -(13)$$

where:

$$\underline{A} = \frac{A^{(0)}}{\sqrt{2}} \left(\underline{i} - i\underline{j} \right) e^{i(\omega t - \kappa z)} \qquad -(14)$$

for a plane wave.

In the presence of spacetime spin the Cartan version is:

$$T^a_{\mu\nu} = d \wedge \left(q^a_\mu(0) e^{i(\omega t - \kappa z + \alpha)} \right) + \omega^a{}_b \wedge \left(q^a_\mu(0) e^{i(\omega t - \kappa z + \alpha)} \right) \qquad -(15)$$

and the magnetic field is:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a + \underline{\omega}^a{}_b \times \underline{A}^b. \qquad -(16)$$

For a plane wave solution, with:

$$a = (1), (2) \text{ and } (3) \qquad -(17)$$

6)

Then:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \left(\underline{i} - i\underline{j} \right) e^{i(\omega t - \kappa z + \alpha)} \quad -(18)$$

$$\underline{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}} \left(\underline{i} + i\underline{j} \right) e^{-i(\omega t - \kappa z + \alpha)} . \quad -(19)$$

In this case the spin connection is dual to the potential and:

$$\underline{B}^{(1)*} = \underline{\nabla} \times \underline{A}^{(1)*} - ig\, \underline{A}^{(2)} \times \underline{A}^{(3)} \quad -(20)$$

$$\underline{B}^{(2)*} = \underline{\nabla} \times \underline{A}^{(2)*} - ig\, \underline{A}^{(3)} \times \underline{A}^{(1)} \quad -(21)$$

$$\underline{B}^{(3)*} = \underline{\nabla} \times \underline{A}^{(3)*} - ig\, \underline{A}^{(1)} \times \underline{A}^{(2)} . \quad -(22)$$

where:

$$g = \frac{\kappa}{A^{(0)}} . \quad -(23)$$

The spin connection vector components are therefore $\underline{A}^{(1)}$, $\underline{A}^{(2)}$ and $\underline{A}^{(3)}$. Therefore eqs. (18) and (19) are the required relations between the spin connection and α.

These are also the complex conjugate

1) vector potentials. This exercise is a simple illustration for plane waves, but shows that the phase of the vector potential and magnetic field contains α in general. The α is due to spinning spacetime, and therefore due to the spin connection. Without the spin connection there is no α. Therefore the spin connection has a global component which is always non-zero.

This finding is self-consistent with previous work, where the Berry phase, AB effects, and quantum entanglement effects were explained with the spin connection. The electromagnetic phase in general was also shown to be local due to the ECE spin field. The simple phase (1) has basic inconsistencies such as the failure to describe reflection and interferometry. It was shown in previous work that the electromagnetic phase is described by a non-Abelian Stokes Theorem:

$$\oint_{\partial S} q = \int_S D \wedge q = \int_S T. \quad -(24)$$

in shorthand notation explained in previous work. So it is reasonable to assume that α also has this form in general:

$$\boxed{\alpha = \kappa \oint_{DS} q = \kappa \int_S T} \quad -(25)$$

where κ is a wavenumber.

Therefore the α factor produced by the general coordinate transformation is the surface integral over the Cartan torsion and the contour integral over the tetrad. It is reasonable to assume that the non-local, or global, α is derived from the spin connection, so:

$$\alpha = \kappa \int_S \omega \wedge q. \quad -(26)$$

The local part of the phase in the standard model is:

$$\phi = \kappa \int_S d \wedge q. \quad -(27)$$

In vol. 2, eq. (2.19), it was shown that the magnetic flux (weber) of ECE theory is:

$$\Phi^a = \int_S F^a = \oint A^a + \int_S \omega^a{}_b \wedge A^b \quad -(28)$$

If the magnetic flux density F^a is confined to an inner region in an AB experiment, the second, global term exists outside this region. The observed shift in the electron diffraction pattern of a Chambers type experiment is therefore:

$$\delta = x \int_S \omega^a{}_b \wedge A^b \quad -(29)$$

and is therefore a global phenomenon depending on α:

$$\boxed{\delta = \frac{x}{\kappa} A^{(0)} \alpha} \quad -(30)$$

Here x is a factor that is determined by the experimental configuration of the Chambers experiment.

The conclusion is that the AB effects are due to the invariance of the tetrad postulate under the general coordinate transformation.

10) This gives a solid geometrical interpretation based on the fundamentals of general relativity.

It has also been shown in previous work that the electromagnetic phase in general is described by the ECE spin field $\underline{B}^{(3)}$ ~~through~~ the non-Abelian Stokes theorem as follows:

$$\phi = \exp\left(i g \oint \underline{A}^{(3)} \cdot d\underline{r} \right) \quad -(31)$$

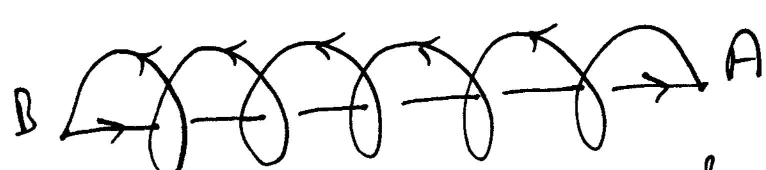
$$= \exp\left(i g \int \underline{B}^{(3)} \cdot \underline{k} \, dAr \right) \quad -(32)$$

where:

$$g = \frac{\kappa}{A^{(0)}}.$$

The area Ar is defined by that of a helix whose arc-length is the same as the circumference of the circle enclosing Ar. The integration is carried out along the following path:

Fig. (1)



from B to A. Only the line from B to A contributes, the integration around the helix vanishes.

Using the definition:

$$\underline{B}^{(3)*} = -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)} \quad \text{— (33).}$$

eq. (31) becomes:

$$\phi = \exp\left(i\kappa \oint \underline{k} \cdot d\underline{r}\right) \quad \text{— (34)}$$

$$= \exp\left(g^2 \int \underline{A}^{(1)} \times \underline{A}^{(2)} \cdot \underline{k}\, dAr\right).$$

The global part of this is the area integral, the local part is the line integral, which is B to A in Fig. (1). In the Chambers experiment the local line integral is along the length of the iron whisker, and is confined to the whisker (or solenoid) and the global area integral extends laterally outside the iron whisker, giving the AB effect.

In another context the line integral is the correct way to express the phase:

$$\kappa \oint \underline{k} \cdot d\underline{r} = \kappa Z \quad \text{— (35)}$$

because this has the correct parity inversion symmetry. This is the correct way to describe reflection and interferometry, and indeed all optical phenomena that depend on phase. This was shown, for example, in vol. 119 of "Advances in Chemical Physics".

In Section (11.4) of volume 1 eq. (34) was described as a new phase law:

$$\phi = \exp\left(i \oint \underline{\kappa} \cdot d\underline{r}\right) = \exp\left(i \int \kappa^2 dAr\right)$$

$$= \exp\left(i \underline{\Phi}_E\right) \quad -(36)$$

and it can now be seen that the global part of this ECE phase law is the area integral, the local part is the line integral. The Berry phase was identified by eq. (11.14) of volume (1):

$$\theta = \kappa \oint ds = R \int dAr \quad -(37)$$

where R is the scalar curvature. On pp. 223 ff of volume 1 the Tomita Chiao and Berry laws were explained with eq. (36).

13) Therefore it may now be seen that these topological phases are global / local in nature. The global part is independent of geometry, as observed experimentally, and is due to the area integral of eq. (36) and therefore, self-consistently, the are due to α. As we have argued, the latter is due to the invariance of the tetrad postulate under the general coordinate transform. Therefore the topological phases and the Tomita Chiao effect are due to Cartan geometry.

Therefore a unified description of several effects emerges from the invariance principle of paper 71. These include quantum entanglement, the AB effects, one photon Young interferometry, and the topological phase effects. These are due to the area integral of the ECE phase law, the global part of the phase law. The global and local parts of the phase law are linked by the non-Abelian Stokes Theorem. Fig (1) may represent a propagating e/m wave or a helically wound optical fibre of the Tomita Chiao experiment.

Spacetime spinning is an intrinsic part of such a description. The spinning of spacetime produces the global phase α and also the spin connection:

$$\boxed{\alpha = \kappa \int_S \omega \wedge q.} \quad -(38)$$

These considerations can be extended to quantum mechanics, where α is present in the wave-function, and gives rise to entanglement effects, or non-locality of the wave-function. In the one photon Young interferometer there is always a non-local part of the photon. So even with one photon there is an interferogram.
