76(5): Derivation of The Spiral Equation

The angular velocity is:

$$\omega = \frac{d\theta}{dt} = \frac{v}{r}. \quad -(1)$$

For constant v_0:

$$\frac{d\theta}{dt} = \frac{v_0}{r} \quad -(2)$$

So:

$$\theta = v_0 \int_0^{\tau} \frac{dt}{r} = \frac{v_0 \tau}{r} \quad -(3)$$

This is the equation of a hyperbolic spiral:

$$r = a\,\theta^{1/n} \quad -(4)$$

where:

$$a = v_0 \tau, \quad n = -1. \quad -(5)$$

Newtonian Result (Marion and Thornton)

This is:

$$\frac{\alpha}{r} = 1 + \epsilon \cos\theta \quad -(7.41)$$

where:

$$v^2 = \frac{k}{\mu}\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(7.72)$$

On M & T p. 250 it is shown that if a particle moves on a logarithmic spiral:

$$r = k \exp(\alpha\theta) \quad -(6)$$

its force law must be inverse cubed. This is not a Newtonian force law.

Hyperbolic Spiral