1)

Notes 77(6): Limiting Cases

In this note we derive limiting cases of:

$$\theta = \int \left(\left(\frac{v_0}{r} \right)^2 + \left(\frac{eB^{(0)}}{m} \right)^2 \right)^{1/2} dt . \quad -(1)$$

This is just expressed as:

$$\theta = \left(\left(\frac{v_0}{r} \right)^2 + \left(\frac{eB^{(0)}}{m} \right)^2 \right)^{1/2} \tau \quad -(2)$$

where:

$$\tau := \int dt . \quad -(3)$$

This assumes that there is no implicit time dependence in v_0, $B^{(0)}$ and r.

Limiting Case One

In this case it is assumed that:

$$\omega = \frac{v_0}{r} \gg \frac{eB^{(0)}}{m} \quad -(4)$$

so:

$$\boxed{\theta \rightarrow \frac{v_0 \tau}{r}} . \quad -(5)$$

This is the equation of the hyperbolic spiral considered in ~~77(~~ paper 76. This can be considered to be the ultra-relativistic limit of synchrotron and pulsar radiation.

Limiting Case Two

This is considered to be:

$$x := \frac{mv_0}{eB^{(0)} r} \ll 1. \quad -(6)$$

Eq. (2) is:

$$\theta \rightarrow \frac{eB^{(0)} \tau}{m} \left(x + 1 \right)^{1/2} \quad -(7)$$

where:

$$x := \left(\frac{mv_0}{eB^{(0)}} \frac{1}{r} \right)^2 \quad -(8)$$

so:

$$\boxed{\theta \rightarrow \frac{eB^{(0)} \tau}{m} \left(1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots \right)} \quad -(9)$$

which is a sum of spirals of different types.

This is the limit:

$$\omega = \frac{v_0}{r} \ll \frac{eB^{(0)}}{m} \quad -(10)$$

which describes the central bulge - motions ~~are~~ become non-relativistic as ω becomes much less than $eB^{(0)}/m$. These are also limits of the IFE.