79(4): Lorentz Force Equation from ECE Theory

This is derived from the transformation properties of the Cartan torsion. As in eq. (25) of paper 71, the general mixed index tensor transforms as:

$$T^{a'\mu'}_{b'\nu'} = \Lambda^{a'}_{a} \frac{\partial x^{\mu'}}{\partial x^{\mu}} \Lambda^{b}_{b'} \frac{\partial x^{\nu}}{\partial x^{\nu'}} T^{a\mu}_{b\nu}, \quad -(1)$$

so the Cartan torsion transforms as:

$$T^{a'}_{\mu'\nu'} = \Lambda^{a'}_{a} \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} T^{a}_{\mu\nu}. \quad -(2)$$

Here $\Lambda^{a'}_{a}$ denotes the Lorentz transformation and $\partial x^{\mu}/\partial x^{\mu'}$ and $\partial x^{\nu}/\partial x^{\nu'}$ denote the general coordinate transformation, which in the limit of Minkowski spacetime become Lorentz transformations.

In the special case where rotation and translation are mutually independent, the ECE field equations are:

$$d \wedge F^{a}_{\mu\nu} = 0 \quad -(3)$$

$$d \wedge \tilde{F}^{a}_{\mu\nu} = \mu_0 J^{a}_{\rho\mu\nu} \quad -(4)$$

For each polarization index a these are the Maxwell Heaviside (MH) field equations:

$$d \wedge F_{\mu\nu} = 0 \quad -(5)$$

$$d \wedge \tilde{F}_{\mu\nu} = \mu_0 J_{\rho\mu\nu} \quad -(6)$$

It is well known that $F_{\mu\nu}$ in eqn. (5) transforms as:

$$F_{\mu'\nu'} = \Lambda^{\mu}{}_{\mu'} \Lambda^{\nu}{}_{\nu'} F_{\mu\nu} \qquad -(7)$$

for each state of polarization a. Eq (7) is the limit of eq. (2) where:

$$\frac{\partial x^{\mu}}{\partial x^{\mu'}} \rightarrow \Lambda^{\mu}{}_{\mu'} \; ; \; \frac{\partial x^{\nu}}{\partial x^{\nu'}} \rightarrow \Lambda^{\nu}{}_{\nu'} \qquad -(8)$$

for each fixed a.

The Lorentz force equation is obtained from eq (7) as follows. From eq. (7):

$$\underline{E}' = \gamma(\underline{E} + \underline{v} \times \underline{B}) - \dots \text{ higher order terms}$$

$$\underline{B}' = \gamma\left(\underline{B} - \frac{\underline{v}}{c^2} \times \underline{E}\right) - \dots \text{ "} \qquad -(9)$$

and

$$\underline{F}' = e\gamma(\underline{E} + \underline{v} \times \underline{B}) \dots \qquad -(10)$$

In the non-relativistic limit:

$$\gamma \rightarrow 1 \qquad -(11)$$

$$\boxed{\underline{F}' = e(\underline{E} + \underline{v} \times \underline{B})} \qquad -(12)$$

3)

Therefore for each sense of polarization:

$$a = (1), (2), (3) \quad -(13)$$

there is a corresponding Lorentz force:

$$\underline{F}^{(1)\prime} = e\left(\underline{E}^{(1)} + \underline{v} \times \underline{B}^{(1)}\right) \quad -(14)$$

$$\underline{F}^{(2)\prime} = e\left(\underline{E}^{(2)} + \underline{v} \times \underline{B}^{(2)}\right) \quad -(15)$$

$$\underline{F}^{(3)\prime} = e\left(\underline{E}^{(3)} + \underline{v} \times \underline{B}^{(3)}\right) \quad -(16)$$

In eq. (16) there is no radiated $\underline{E}^{(3)}$, so:

$$\underline{F}^{(3)\prime} = e\,\underline{v} \times \underline{B}^{(3)}$$

$$= -ieg\underline{v} \times \left(\underline{A}^{(1)} \times \underline{A}^{(2)}\right). \quad -(17)$$

For a plane wave:

$$\underline{A}^{(1)} = \underline{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi}. \quad -(18)$$

For a static magnetic field of the standard model:

$$\underline{F} = e\,\underline{v} \times \left(\underline{\nabla} \times \underline{A}\right) \quad -(19)$$

Eqs. (14) and (15) are first order forces and eq. (16) is a second order force.

+) These first and second order force are also present in the Hamilton - Jacobi method.

Due to the Lorentz invariance of the B Cyclic Theorem:

$$\underline{B}^{(1)} \times \underline{B}^{(2)} = i B^{(0)} \underline{B}^{(3)*} \quad — (20)$$

et cyclicum

the polarizations (1), (2) and (3) are invariant under a frame transformation. The Lorentz invariance of eq. (20) is obvious from the fact that it is:

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i \underline{e}^{(3)*} \quad — (21)$$

et cyclicum

where:

$$\underline{e}^{(1)} = \underline{e}^{(2)*} = \frac{1}{\sqrt{2}} (\underline{i} - i\underline{j})$$

$$\underline{e}^{(3)} = \underline{e}^{(3)*} = \underline{k} \quad — (22)$$

The unit vectors $\underline{i}$, $\underline{j}$ and $\underline{k}$ are not affected by a Lorentz transformation. For example the relativistic momentum is:

$$\underline{p} = \gamma m \underline{v} \quad — (23)$$

e.g.

$$p_Z \underline{k} = \gamma m v_Z \underline{k} \quad — (24)$$

and $\underline{k}$ is the same.