1) Notes 81(2), Third Cross Check a Self-Consistency

The relativistic momentum is defined by:

$$\underline{p} = m\frac{d\underline{r}}{d\tau} = m\frac{d\underline{r}}{dt}\frac{dt}{d\tau} \quad -(1) \qquad \underline{r} = \frac{1}{\gamma}\int\frac{\underline{p}}{m}dt \quad -(1a)$$

where:

$$\frac{dt}{d\tau} = \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(2)$$

so

$$\underline{r} = \frac{1}{m}\int\underline{p}\,d\tau = \frac{\gamma}{m}\int\underline{p}\,dt = \gamma\int\underline{v}\,dt \quad -(3)$$

If we now consider relativistic interaction momentum:

$$\underline{p} = e\underline{A} \quad -(4)$$

then

$$\underline{r} = \frac{e}{m}\int\underline{A}\,dt \quad -(5)$$

Now assume the plane wave:

$$\underline{A} = A^{(0)}(\underline{i}\cos\phi + \underline{j}\sin\phi) \quad -(6)$$

then:

$$p_x = eA^{(0)}\cos\phi, \quad p_y = eA^{(0)}\sin\phi$$

$$r_x = \frac{eA^{(0)}}{\omega}\sin\phi, \quad r_y = -\frac{eA^{(0)}}{\omega}\cos\phi$$

So:

$$p = (p_x^2 + p_y^2)^{1/2} = eA^{(0)} \quad -(7)$$

$$r = (r_x^2 + r_y^2)^{1/2} = \frac{1}{m\omega}eA^{(0)} \quad -(8)$$

$$J = rp = \frac{e^2A^{(0)2}}{m\omega} = \frac{e^2c^2B^{(0)2}}{m\omega^3} \quad -(9)$$

$\boxed{\text{LARMOR RADIUS}}$

DOUBLE CROSS CHECKED IN THREE WAYS

More generally:

$$\underline{p} = \gamma m \underline{v} + e\underline{A} \quad -(10)$$

$$\underline{r} = \frac{1}{m}\left(\frac{1}{\gamma}\int m\underline{v}\,dt + \int e\underline{A}\,dt\right) \quad -(11)$$

$$\underline{r} = \frac{1}{\gamma}\int \underline{v}\,dt + \frac{e}{m}\int \underline{A}\,dt \quad -(12)$$

and

$$\underline{J} = \underline{r} \times \underline{p}. \quad -(13)$$

- the relativistic velocity is:

$$\underline{v} = \gamma \underline{v} + \frac{e}{m}\underline{A} \quad -(14)$$

and the angular velocity of the electron in ~~the~~ e/m field is:

$$\Omega = \left|\frac{\underline{v}}{|\underline{r}|}\right|, \quad -(15)$$

while its total kinetic energy is:

$$T = \frac{\gamma^2 m^2 v^2 c^2 + e^2 c^2 A^2}{mc^2(1+\gamma) + e\phi}. \quad -(16)$$

<u>Hyper-Relativistic Limit</u>

This is the limit:

$$e\phi \gg mc^2(1+\gamma) \quad -(17)$$

so

$$T_{int} = \frac{ec^2 A^2}{\phi} \quad -(18)$$

If we assume that:

$$|A| = A^{(0)}, \quad |\phi| = cA^{(0)} \quad -(19)$$

Then:

$$T_{int} = ecA^{(0)} \quad -(20)$$

using the de Broglie wave-particle duality:

$$eA^{(0)} = \hbar \kappa \quad -(21)$$

then:

$$T_{int} = \hbar c \kappa = \hbar \omega \quad -(22)$$

which is the quantum of energy of a photon.

If we define:

$$g = \frac{e}{\hbar} = \frac{\kappa}{A^{(0)}} \quad -(23)$$

then:

$$\boxed{\underline{B}^{(3)*} = -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)}} \quad -(24)$$

In this limit, the definition of g defines the ECE spin field in free space. The $eA^{(0)}$ interaction momentum is equal to the photon momentum $\hbar\kappa$. A photon of e/m radiation has transferred all its momentum and energy to the electron. This is therefore an elastic collision between photon and an electron. In the opposite (inelastic) limit:

$$\underline{m}^{(3)} = \frac{e}{2m}\underline{J}^{(3)} = -\frac{ie^3}{2m^2\omega}\underline{A}^{(1)} \times \underline{A}^{(2)}$$

$$\boxed{\underline{B}^{(3)*} = -i\left(\frac{\mu_0}{V}\frac{e^3}{2m^2\omega}\right)\underline{A}^{(1)} \times \underline{A}^{(2)}} \quad -(25)$$

4)

Calculation of Angular Momentum and Energy

Assuming that:

$$\underline{v} = v^{(0)}\left(\underline{i}\cos\phi + \underline{j}\sin\phi\right) \quad -(26)$$

the relativistic angular momentum can be calculated as follows from eq. (13). We have:

$$r_x = \frac{1}{\omega}\left(\frac{1}{\gamma}v_x + \frac{e}{m}A^{(0)}\right)\sin\phi \quad -(27)$$

$$r_y = -\frac{1}{\omega}\left(\frac{1}{\gamma}v_y + \frac{e}{m}A^{(0)}\right)\cos\phi \quad -(28)$$

$$p_x = \left(\gamma m v_x + eA^{(0)}\right)\cos\phi \quad -(29)$$

$$p_y = \left(\gamma m v_y + eA^{(0)}\right)\sin\phi \quad -(30)$$

$$J_z = r_x p_y - r_y p_x, \quad -(31)$$

$$J_z = \frac{m v^{(0)2}}{\omega} + \frac{e v^{(0)}A^{(0)}}{\gamma\omega} + \frac{e^2 A^{(0)2}}{m\omega} \quad -(32)$$

So the rotational kinetic energy is:

$$T = \frac{1}{2}\left(m v^{(0)2} + \frac{e v^{(0)}}{\gamma}A^{(0)} + \frac{e^2 A^{(0)2}}{m}\right)$$

$$= \frac{1}{2}\omega J_z \quad -(33)$$

Comparison with Hamilton - Jacobi Equation

In the non-relativistic limit the same second order interaction term is recovered:

5)

$$T = \frac{1}{2} \frac{e^2 A^{(0)2}}{m} = \frac{e^2 c^2}{2 m \omega^2} B^{(0)2} \quad -(34)$$

CHECKED IN FOUR WAYS

The first order term is:

$$T_1 = \frac{1}{2} e v^{(0)} \left(1 - \frac{u^2}{c^2} \right)^{1/2} A^{(0)} \quad -(35)$$

In the limit:

$$v^{(0)} \rightarrow c, \quad u \rightarrow 0 \quad -(36)$$

$$T_1 \rightarrow \frac{1}{2} e c A^{(0)} = \frac{e c^2}{2 \omega} B^{(0)} \quad -(37)$$

which is the ultra-relativistic limit of the Hamilton - Jacobi method. In this limit the velocity of the orbiting electron approaches the speed of light, at which no frame can move faster, so $u \rightarrow 0$. So this method gives the same results as the Hamilton - Jacobi method. The latter therefore applies to an electron orbiting according to eq. (26).

Finally the angular velocity of the electron in the e/m field is given by:

5)

$$\Omega = \frac{d\theta}{dt} = \frac{v}{r_L} \quad -(38)$$

where:

$$r_L = \left(r_x^2 + r_y^2\right)^{1/2} \quad -(39)$$

$$v = \left(v_x^2 + v_y^2\right)^{1/2} \quad -(40)$$

The Larmor radius is:

$$r_L = \left(\left(\frac{v^{(0)}}{\omega\gamma}\right)^2 + \left(\frac{eA^{(0)}}{m\omega}\right)^2\right)^{1/2} \quad -(41)$$

and the net orbital velocity is:

$$v = \left(\left(\gamma v^{(0)}\right)^2 + \left(\frac{eA^{(0)}}{m}\right)^2\right)^{1/2} \quad -(42)$$

In the non-relativistic limit:

$$\gamma \rightarrow 1 \quad -(43)$$

and:

$$\Omega \rightarrow \left(\frac{\left(v^{(0)}\right)^2 + \left(\frac{eA^{(0)}}{m}\right)^2}{\left(\frac{v^{(0)}}{\omega}\right)^2 + \left(\frac{eA^{(0)}}{m\omega}\right)^2}\right)^{1/2}$$

$$= \omega \quad -(43)$$

<u>which is the result of paper 78.</u>