83(6): Calculation of Light Deflection due to Gravitation

The relevant semi-classical ECE wave equation leads to:

$$(p^\mu + \epsilon G^\mu)(p_\mu + \epsilon G_\mu) = m^2 c^2 \quad -(1)$$

where p^μ is the photon energy-momentum and m the photon mass. The gravitational interaction is given by ϵG^μ, where ϵ is to be determined. From previous work the interaction kinetic energy is:

$$T_{int} = \frac{\epsilon G^{(0)2} c^2}{mc^2(1+\gamma) + \epsilon c G^{(0)}} \quad -(2)$$

$$\rightarrow \epsilon c G^{(0)} \quad -(3)$$

in the limit:

$$\epsilon G^{(0)} \gg mc(1+\gamma). \quad -(4)$$

Now identify:

$$\boxed{T_{int} = \epsilon c G^{(0)} = \frac{4GmM}{r}} \quad -(5)$$

where G is Newton's constant and M is the mass of a particle that attracts the photon mass. Here r is the distance between the particles.

By unit analysis:

$$T \quad T \quad (6)$$

2)

where ω is angular velocity and J is angular momentum. So:

$$\omega = \frac{d\theta}{dt} = \frac{4Gm M}{r}\frac{1}{J} \quad -(7)$$

$$\theta = \frac{4GmM}{r}\int \frac{1}{J}\,dt. \quad -(8)$$

Now identify:

$$\int \frac{1}{J}\,dt = \frac{1}{mc^2} = \frac{1}{\hbar\omega} \quad -(9)$$

to find

$$\boxed{\theta = \frac{4GM}{rc^2}}. \quad -(10)$$

It has been assumed that:

$$J = \hbar, \quad \int dt = \frac{1}{\omega}. \quad -(11)$$

Here

$$\hbar\omega = mc^2 \quad -(12)$$

is the de Broglie equation of the photon.

Eq (10) is the relativistic deflection of light due to gravity.