4. COMPARISON OF DIRECT INTEGRATION AND HAMILTON JACOBI METHODS.

The angular momentum magnitude from the direct integration method gives:

$$J = \frac{1}{\gamma m \omega}\left(\gamma m \omega r_0 + eA^{(0)}\right)\left(\gamma m v_0 + eA^{(0)}\right) \quad -(41)$$

where r_0 and v_0 are the initial position and velocity of the electron. The Hamilton Jacobi method {1-12} gives:

$$J_{int} = \frac{c e^2 A^{(0)2}}{\omega\left(m^2c^2 + e^2A^{(0)2}\right)^{1/2}} . \quad -(42)$$

The interaction terms of Eq. (41) are:

$$J_{int} = eA^{(0)}\left(r_0 + \frac{v_0}{\omega}\right) + \frac{e^2A^{(0)2}}{\gamma m \omega} \quad -(43)$$

which can be compared directly with Eq. (42).

In the non-relativistic limit:

$$\gamma \rightarrow 1, \quad mc \gg eA^{(0)}, \quad -(44)$$

and both Eqs. (42) and (43) give:

$$J_{int} \rightarrow \frac{e^2A^{(0)2}}{m\omega} . \quad -(45)$$

So in this limit both methods give the same result. In the opposite hyper-relativistic limit:

$$eA^{(0)} \gg mc \quad -(46)$$

Eq. (42) becomes:

$$J_{int} \rightarrow \frac{ecA^{(0)}}{\omega}. \quad - (47)$$

If the second order term in Eq. (43) is neglected then:

$$J_{int} \sim eA^{(0)}\left(r_0 + \frac{v_0}{\omega} \right) \quad - (48)$$

so:

$$r_0 + \frac{v_0}{\omega} \longrightarrow \frac{c}{\omega} \quad - (49)$$

i.e.

$$v_0 \rightarrow c, \quad \frac{v_0}{\omega} \gg r_0 \quad - (50)$$

in the hyper-relativistic limit.

In general, comparing Eqs. (42) and (43):

$$eA^{(0)}\left(r_0 + \frac{v_0}{\omega}\right) + \frac{e^2 A^{(0)2}}{\gamma m \omega} = \frac{ce^2 A^{(0)2}}{\omega\left(m^2c^2 + e^2 A^{(0)2}\right)^{1/2}}. \quad - (51)$$

For an initially stationary electron ~~a~~t the origin:

$$v_0 = 0, \quad r_0 = 0 \quad - (52)$$

we obtain:

$$\gamma = \frac{1}{mc}\left(m^2c^2 + e^2 A^{(0)2}\right)^{1/2} \quad - (53)$$

which is precisely the expression used in the original Hamilton Jacobi method of volume one of "The Enigmatic Photon" (1-12}. In these equations the factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(54)$$

is used to find that:

$$1 - \frac{v^2}{c^2} = \left(1 + \left(\frac{eA^{(0)}}{mc}\right)^2\right)^{-1}. \quad -(55)$$

For v << c:

$$v \sim \frac{e^2 A^{(0)2}}{mc^2} \quad -(56)$$

i.e. if

$$eA^{(0)} << mc. \quad -(57)$$

Eq. (56) means that the initially stationary electron has attained an orbital velocity of $e^2 A^{(0)2}/mc^2$ from the applied electromagnetic field. This is therefore a self-consistent analysis. The important result for the IFE and radiatively induced fermion resonance (RFR) {1-12} is Eq. (45). Therefore IFE and RFR have been self-consistently derived in many ways in this paper and in previous work {1-12}, from classical to fully quantum levels. These theoretical methods assume a circularly polarized plane wave of type (5), so must be tested experimentally under the same conditions, using a circularly polarized radio frequency plane wave and an electron beam. In the non-relativistic limit:

$$T \rightarrow \frac{e^2 A^{(0)2}}{2m} \quad -(58)$$

and comparing Eqs. (45) and (58):

$$T = \frac{1}{2}\omega J = \frac{1}{2}\Omega J \quad -(59)$$

so in the non-relativistic limit:

$$\Omega = \omega \qquad - (60)$$

i.e. the angular velocity ω of the electromagnetic field is fully imparted to the electron. This same result is obtainable from Eq. (29) of the Hamilton Jacobi method {1-12}. RFR is then derived from Eq. (58) using the SU(2) basis {1-12} to give:

$$\hbar \omega_{res} = \frac{e^2 A^{(0)2}}{m} \qquad - (61)$$

as the difference of energy levels of the Z Pauli matrix as in ESR or NMR. It is therefore deduced that gravitation will also affect the RFR resonance frequency because RFR is the detection of IFE using resonance instead of induction, and as argued already, IFE is affected by gravitation.

In the SU(2) basis the interaction kinetic energy is:

$$T = \frac{1}{2m} \underline{\sigma} \cdot (\underline{p} + e\underline{A}) \underline{\sigma} \cdot (\underline{p} + e\underline{A}^*)$$

$$= \frac{p^2}{2m} + \frac{ie^2}{2m} \underline{\sigma} \cdot \underline{A} \times \underline{A}^* + \ldots \qquad - (62)$$

where $\underline{A}^*$ is the complex conjugate of $\underline{A}$. The RFR term is then given by:

$$\hbar \omega_{res} = \frac{2e^2}{2m} i \, |\underline{A} \times \underline{A}^*| . \qquad - (63)$$

The resonance occurs between the states of:

$$\sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad - (64)$$

and introduces the factor 2 in the numerator of Eq. (63), as in ESR theory. The conjugate product is:

$$i\,\underline{A} \times \underline{A}^* = A^{(0)2}\,\underline{k} \qquad -(65)$$

so:

$$i\,\underline{\sigma} \cdot \underline{A} \times \underline{A}^* = A^{(0)2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \qquad -(66)$$

A photon $\hbar\omega_{res}$ of a probe beam is absorbed from the interaction kinetic energy:

$$T = \begin{bmatrix} \frac{e^2 A^{(0)2}}{2m} & 0 \\ 0 & -\frac{e^2 A^{(0)2}}{2m} \end{bmatrix} = \begin{bmatrix} T_{spin\ up} & 0 \\ 0 & T_{spin\ down} \end{bmatrix}. \qquad -(67)$$

Thus:

$$T_{spin\ up} - T_{spin\ down} = \hbar\omega_{res} = \frac{e^2 A^{(0)2}}{2m} - \left(-\frac{e^2 A^{(0)2}}{2m}\right) = \frac{e^2 A^{(0)2}}{m}. \qquad -(68)$$

So the RFR frequency is:

$$f_{res} = \frac{\omega_{res}}{2\pi} = \frac{e^2 A^{(0)2}}{2\pi\hbar m}. \qquad -(69)$$

At this frequency the probe beam's photon $\hbar\omega_{res}$ is absorbed, and one absorption line is observed in a spectrometer. Using Eqs. (33) and (38) the RFR resonance frequency can be expressed in terms of the pump beam's power density I in watts per square meter:

$$\delta_{res} = \left(\frac{e^2 \mu_0 c}{2\pi \hbar m} \right) \frac{I}{\omega^2} . \quad -(70)$$

The pump beam should be a circularly polarized, radio frequency, plane wave for accurate comparison with this theory, and the sample should be N electrons in an electron beam. The great advantage of RFR over ESR and NMR is that RFR does not use magnets and by adjusting I and ω , has a much greater spectral resolution. RFR also has its own chemical shift pattern {1-12}, so if developed would be a powerful new fermion resonance technique.

The Clifford algebra needed to prove:

$$\underline{\sigma} \cdot \underline{A}\, \underline{\sigma} \cdot \underline{A}^* = \underline{A} \cdot \underline{A}^* \sigma_0 + i\, \underline{A} \times \underline{A}^* \cdot \sigma_Z \underline{k} \quad -(71)$$

is as follows. For plane waves:

$$A_Z = 0 \quad -(72)$$

so:

$$\underline{\sigma} \cdot \underline{A}\, \underline{\sigma} \cdot \underline{A}^* = \begin{bmatrix} 0 & A_X - iA_Y \\ A_X + iA_Y & 0 \end{bmatrix} \begin{bmatrix} 0 & A_X^* - iA_Y^* \\ A_X^* + iA_Y^* & 0 \end{bmatrix}$$

$$= A^{(0)2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + A^{(0)2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(73)$$

where:

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(74)$$

Q.E.D.