$$\frac{d\psi_0}{dr} = \frac{1}{4a^{5/2}}\left(1-\frac{r}{2a}\right)e^{-r/(2a)}$$

$$\frac{d^2\psi}{dr^2} = -\frac{1}{8a^{7/2}}e^{-r/(2a)} - \left(1-\frac{r}{2a}\right)\frac{1}{8a^{7/2}}e^{-r/(2a)}$$

$$= -\frac{1}{4a^{7/2}}e^{-r/(2a)} + \frac{r}{16a^{9/2}}e^{-r/(2a)}$$

$$\frac{d^2\psi}{dr^2} = \frac{1}{4a^{7/2}}e^{-r/(2a)}\left(\frac{r}{4a}-1\right)$$

$$\nabla^2\psi = \frac{e^{-r/(2a)}}{4a^{7/2}}\left(\frac{r}{4a}-1\right) + \frac{1}{2ra^{5/2}}\left(1-\frac{r}{2a}\right)e^{-r/(2a)}$$

$$= \frac{e^{-r/(2a)}}{a^{5/2}}\left(\frac{1}{4a}\left(\frac{r}{4a}-1\right) + \frac{2}{r}\left(1-\frac{r}{2a}\right)\right)$$

$$= -\frac{4\pi c}{\ell}\frac{1}{r_{vac}}\frac{1}{4}\frac{r}{a^{5/2}}e^{-r/(2a)}$$

$$\frac{1}{r_{vac}} = -\frac{\ell}{mc}\frac{1}{r}\left(\frac{1}{4a}\left(\frac{r}{4a}-1\right) + \frac{2}{r}\left(1-\frac{r}{2a}\right)\right)$$

$$= \frac{\ell}{mca}\left(\frac{1}{4r}\left(1-\frac{r}{4a}\right) + \frac{2a}{r^2}\left(\frac{r}{2a}-1\right)\right)$$

$$= \frac{\ell}{mca}\left(\frac{1}{4r}\left(1-\frac{r}{4a}\right) + \frac{1}{r} - \frac{2a}{r^2}\right)$$

$$\frac{E}{2\pi\epsilon a}\left(\frac{2}{r}\left(1-\frac{r}{2a}\right)-\frac{3}{4a}\left(1-\frac{r}{2a}\right)\right)\Big/\left(2-\frac{r}{a}\right)$$

~~$$\frac{E}{2\pi\epsilon a}\left(\frac{2}{r}-\right.$$~~

$$\frac{E}{2\pi\epsilon a}\left(\frac{1}{r}\left(2-\frac{r}{2a}\right)-\frac{1}{4a}\left(3-\frac{r}{2a}\right)\right)/(2$$

$$\text{Num} = \frac{2}{r}-\frac{1}{2a}-\frac{3}{4a}+\frac{r}{8a^2}$$

$$= \frac{2}{r}-\frac{5}{4a}+\frac{r}{8a^2}$$ ✓

$$\psi_0(2p_z) = \frac{1}{4}\frac{r}{a^{5/2}}e^{-r/(2a)}$$

$$\frac{d\psi_0}{dr} = \frac{e^{-r/(2a)}}{4a^{5/2}} - \frac{1}{8a^{7/2}}e^{-r/2a}$$

$$= \frac{1}{4a^{5/2}}\left(1-\frac{r}{2a}\right)e^{-r/(2a)}$$ ✓

~~$$\frac{d^2\psi_0}{dr^2} = -\frac{1}{8a^{7/2}}e^{-r/(2a)} \quad \frac{r}{4} - \frac{1}{8a^{7/2}}e^{-r/(2a)}$$~~

~~$$= -\frac{1}{4}e^{-r/(2a)}$$~~