1)

Simplified Summary Sheet

Relative Wave-number Shift

$$\frac{\Delta \tilde{\nu}}{\tilde{\nu}} = \frac{\alpha}{2\pi} \frac{a}{r_{vac}} \qquad (1)$$

1s Orbital

$$\frac{1}{r_{vac}}(1s) = \frac{1}{2\pi^{1/2}} \cdot \frac{\hbar}{2mca} \left(\frac{1}{r} - \frac{1}{2a} \right) \qquad (2)$$

2s Orbital

$$\frac{1}{r_{vac}}(2s) = \frac{1}{2\pi^{1/2}} \frac{\hbar}{2mca} \left(\frac{1}{r} - \frac{1}{8a} \right) \qquad (3)$$

$2p_z$ Orbital at $\cos\theta = 1$

$$\frac{1}{r_{vac}}(2p_z) = \frac{1}{2\pi^{1/2}} \frac{\hbar}{2mca} \left(\frac{1}{r} - \frac{1}{8a} - \frac{a}{r^2} \right) \qquad (4)$$

Lamb Shift

$$\boxed{\frac{\Delta\Delta \tilde{\nu}}{\tilde{\nu}} = \frac{\alpha}{4\pi^{3/2}} \frac{\hbar}{mc} \left(\frac{a}{r} \right)^2 = 4.096 \times 10^{-7} \text{ (experimental)}} \qquad (5)$$

Notes

We can find r from eq. (5), remembering that the result is for the $2p_z$ orbital with $\cos\theta = 1$ (the maximum of $\cos\theta$)