

86(8) : Explanation for the Relatively Long Lifetime of  
ortho-positronium.

It was shown in 86(5) that the allowed wave-functions of ortho-positronium are:

$$\begin{aligned} \psi &= \psi_{-}(1, 2) |1, 1\rangle \\ &= \psi_{-}(1, 2) (|1, 0\rangle + |1, -1\rangle) \\ &= \psi_{-}(1, 2) |0, 0\rangle \end{aligned} \quad \left. \vphantom{\psi} \right\} - (1)$$

where:  $\psi_{-}(1, 2) = \frac{1}{2} (a(1)b(2) - b(1)a(2))$  - (2)

and:

$$\begin{aligned} a(1) &= \psi_{n, l, m_{l1}} \text{ (electron)} \\ a(2) &= \psi_{n, l, m_{l2}} \text{ (electron)} \\ b(1) &= \psi_{n, l, m_{l1}} \text{ (positron)} \\ b(2) &= \psi_{n, l, m_{l2}} \text{ (positron)} \end{aligned} \quad \left. \vphantom{a(1)} \right\} - (3)$$

In ortho-positronium therefore the electron and positron cannot be found in the same place, i.e. if

$$a(1)b(2) = b(1)a(2) \quad - (4)$$

then  $\psi_{-}(1, 2) = 0$  - (5)

In para-positronium the electron and positron can be found in the same place, so the process

2)  $p^+ + e^- = 2\gamma - (5)$

is easier to form positronium than the positronium, because the former the electron and positron can theoretically be found in the same place. The closer they approach the greater the probability of transmuting into photons. Positronium can be synthesized only in high energy collisions and both forms have only a fleeting existence, the latter form being larger lived.

The first few H-like orbitals of positronium are, in normalized form, as follows

	$n$	$l$	$m_l$	$\psi_{nlm_l}(r, \theta, \phi)$
1s	1	0	0	$\exp\left(-\frac{r}{a}\right)$
2s	2	0	0	$\left(2 - \frac{r}{a}\right) \exp\left(-\frac{r}{2a}\right)$
2p <sub>z</sub>	2	1	0	$\frac{r}{a} \cos \theta \exp\left(-\frac{r}{2a}\right)$
2p <sub>x</sub>	2	1	1	$-\frac{r}{a} \sin \theta \exp(i\phi) \exp\left(-\frac{r}{2a}\right)$
2p <sub>y</sub>	2	1	-1	$\frac{r}{a} \sin \theta \exp(-i\phi) \exp\left(-\frac{r}{2a}\right)$

Table 1

3) The allowed states of the positronium must be built up from two H like orbitals in such a way that:

$$a(1)b(2) \neq a(2)b(1), \quad - (6)$$

where a denotes electron and 1 denotes position.

For example:

$$\left. \begin{aligned} a(1) &= \psi_{1s}(r_1) \\ a(2) &= \psi_{2s}(r_1) \\ b(1) &= \psi_{1s}(r_2) \\ b(2) &= \psi_{2s}(r_2) \end{aligned} \right\} - (7)$$

In order for the complete wavefunction  $\psi$  of eq. (1) to exist, the electron must be at  $r_1$  and the positron at  $r_2$ . Other examples must be built up from Table 1 in such a way that eq. (6) applies.

For example:

$$\left. \begin{aligned} a(1) &= \psi_{2p_z}(r_1) \\ a(2) &= \psi_{2p_x}(r_1) \\ b(1) &= \psi_{2p_z}(r_2) \\ b(2) &= \psi_{2p_x}(r_2) \end{aligned} \right\} - (8)$$

etc.