87(5): Direct Response Equation in r(vac)

We start with the response equation of paper 63 with $\rho(0)$ and no assumed radial dependence:

$$\frac{d^2\phi}{dR^2} + \kappa^2\phi = \frac{\rho(0)}{\epsilon_0}\exp(2i\kappa R) \quad -(1)$$

where $$\exp(i\kappa R) = \kappa r \quad -(2)$$

Now we have the relative coordinate:

$$r_1 = r + r(vac) \quad -(3)$$

and $$\kappa_1 r_1 = \exp(i\kappa_1 R_1) \quad -(4)$$

so:

$$\boxed{\frac{d^2\phi}{dR_1^2} + \kappa_1^2\phi = \frac{\rho(0)}{\epsilon_0}\exp(2i\kappa_1 R_1)} \quad -(5)$$

with:

$$R_1 = \frac{1}{\kappa_1}\cos^{-1}\left(\kappa_1(r + r(vac)\right) \quad -(6)$$

which gives resonance in ϕ because of the oscillating of $r(vac)$.
