1)

88(6): Relation between "the Second Bianchi Identity" and the Einstein Field Equation

The derivation by Einstein of his field equation in 1915 started from "the second Bianchi identity":

$$D \wedge R^a{}_b = 0 \qquad (1)$$

which in Riemann notation is:

$$D_\lambda R_{\rho\sigma\mu\nu} + D_\rho R_{\sigma\lambda\mu\nu} + D_\sigma R_{\lambda\rho\mu\nu} = 0 \qquad (2)$$

where:

$$R_{\rho\sigma\mu\nu} = g_{\rho\lambda} R^\lambda{}_{\sigma\mu\nu} \qquad (3)$$

with

$$g_{\mu\nu} = g_{\nu\mu}. \qquad (4)$$

It is now known that these equations are true if and only if:

$$T^a = 0, \qquad (5)$$

when

$$\Gamma^\kappa_{\mu\nu} = \Gamma^\kappa_{\nu\mu}. \qquad (6)$$

At the time (1915), the Cartan torsion (1922) was not known, and Einstein accepted eq. (2) as an identity. In fact it is not an identity, it depends on eq. (5). In 1922 and in later work, Cartan showed that in the presence of torsion:

$$D \wedge T^a = R^a{}_b \wedge q^b \qquad (7)$$

$$T^a = D \wedge q^a \qquad (8)$$

$$R^a{}_b = D \wedge \omega^a{}_b. \qquad (9)$$

2)

In general:

$$D \wedge T^a = R^a{}_b \wedge q^b \neq 0 \qquad -(10)$$

in the presence of torsion. Therefore in general:

$$\boxed{D \wedge R^a{}_b := D \wedge (D \wedge \omega^a{}_b) \neq 0} \qquad -(11)$$

and

$$\boxed{D \wedge (R^a{}_b \wedge q^b) := D \wedge (D \wedge T^a) \neq 0} \qquad -(12)$$

Cosmology and general relativity should therefore be developed from eqs. (11) and (12), not from eq. (1). For example, spiral galaxies are due to torsion, and not "dark matter". The latter does not exist in natural philosophy, and is akin to "phlogiston".

The actual field equation of Einstein (1915) is derived from the double index contraction:

$$g^{\nu\sigma} g^{\mu\lambda} \left(D_\lambda R_{\rho\sigma\mu\nu} + D_\rho R_{\sigma\lambda\mu\nu} + D_\sigma R_{\lambda\rho\mu\nu} \right) = 0 \qquad -(13)$$

The Einstein field equation is true only in the absence of torsion. In the presence of torsion the Noether Theorem is also extended.

). The following definitions (see Carroll) are then used in eq. (13):

$$D^\mu R_{\rho\mu} := -(g^{\mu\lambda} D_\lambda)(g^{\nu\sigma} R_{\rho\sigma\mu\nu}) \quad -(14)$$

$$D^\nu R_{\rho\nu} := -(g^{\nu\sigma} D_\sigma)(g^{\mu\lambda} R_{\sigma\lambda\mu\nu}) \quad -(15)$$

$$D_\rho R := D_\rho (g^{\nu\sigma} g^{\mu\lambda} R_{\sigma\lambda\mu\nu}) \quad -(16)$$

The convention used by Einstein is as follows. If indices are in the same order in the metric and in the tensor multiplied by the metric, then the resulting sign is positive. If indices are in the opposite order in the tensor to the index order in the metric, then the sign is negative. Thus:

$$D^\mu R_{\rho\mu} - D_\rho R + D^\nu R_{\rho\nu} = 0 \quad -(17)$$

i.e.

$$D^\mu R_{\rho\mu} - \frac{1}{2} D_\rho R = 0 \quad -(18)$$

or

$$R_{\rho\nu} = \frac{1}{4} R g_{\rho\nu}. \quad -(19)$$

Finally use:

$$D_\rho = g_{\rho\mu} D^\mu \quad -(20)$$

to obtain:

$$\boxed{D^\mu \left(R_{\rho\mu} - \frac{1}{2} R g_{\rho\mu}\right) = 0} \quad -(21)$$

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This is a re-arrangement of eq. (1). Eq. (21) is true if and only if the torsion is zero. The usual nomenclature is as follows. The Einstein tensor is:

$$G_{\rho\mu} = R_{\rho\mu} - \frac{1}{2} R g_{\rho\mu} \quad -(22)$$

where $R_{\rho\mu}$ is the Ricci tensor, R is the scalar curvature, and $g_{\rho\mu}$ is the metric. Thus:

$$\boxed{D^{\mu} G_{\rho\mu} = 0.} \quad -(23)$$

The conventional (torsion free) Noether theorem is:

$$\boxed{D^{\mu} T_{\rho\mu} = 0} \quad -(24)$$

where $T_{\rho\mu}$ is the canonical energy-momentum tensor.

Due to the neglect of torsion

$$G_{\rho\mu} = G_{\mu\rho} \quad -(25)$$

$$T_{\rho\mu} = T_{\mu\rho} \quad -(26)$$

To obtain the famous field equation, Einstein made eq. (23) proportional to eq. (24).

$$D^\mu G_{\rho\mu} = k D^\mu T_{\rho\mu} = 0 \quad \text{— (27)}$$

and so:

$$\boxed{G_{\rho\mu} = k T_{\rho\mu}} \quad \text{— (28)}$$

where k is the Einstein constant.

Modern cosmology is all based on eq. (28). It completely neglects torsion.

Eq. (28) can equally well be written as:

$$D \wedge R^a{}_b = k D \wedge T^a{}_b \quad \text{— (29)}$$
$$= 0$$

where:

$$R^a{}_b := R^a{}_{b\mu\nu} \quad \text{— (30)}$$

is the Cartan curvature form and:

$$T^a{}_b = T^a{}_{b\mu\nu} \quad \text{— (31)}$$

is the Noether form. In the presence of torsion:

$$D \wedge R^a{}_b = k D \wedge T^a{}_b \neq 0 \quad \text{— (32)}$$

and

$$\boxed{R^a{}_{b\mu\nu} = k T^a{}_{b\mu\nu}}. \quad \text{— (33)}$$

In general:

$$R^a{}_{b\mu\nu} \neq R^b{}_{a\mu\nu} \quad \text{— (34)}$$
$$T^a{}_{b\mu\nu} \neq T^b{}_{a\mu\nu}. \quad \text{— (35)}$$

6) The usual equation:

$$R = -kT \quad (36)$$

first derived by Einstein uses the assumption again that torsion is zero. Einstein defined:

$$R := g^{\mu\nu} R_{\mu\nu} \quad (37)$$

$$T := g^{\mu\nu} T_{\mu\nu} \quad (38)$$

and used

$$g^{\mu\nu} g_{\mu\nu} = 4. \quad (39)$$

To obtain eq. (36) he started from:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu} \quad (40)$$

and multiplied both sides by $g^{\mu\nu}$ to obtain eq. (36).

Eq. (36) in ECE Theory

In the ECE, R is obtained from the Lemma and tetrad postulate:

$$\square q^a_\mu = R q^a_\mu \quad (41)$$

where:

$$\boxed{R := \frac{1}{4} q^\lambda_a \partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a_\nu - \omega^a_{\mu b} q^b_\lambda \right)} \quad (42)$$

In ECE:

$$R := -kT \quad (43)$$

where R is given by eq. (42). Eq. (43) is true in the presence of torsion.

7) The most general Noether Theorem is therefore:

$$T = -\frac{R}{k} \quad - (44)$$

where R is defined by eq. (42). The ECE wave equation is then:

$$(\square + kT)\, q^a_\mu = 0 \quad - (45)$$

where T is defined by eq. (44).