11(3): The Torsion Based Field Equation

Einstein derived his field equation from the equation

$$D \wedge R = k D \wedge N = 0 \quad -(1)$$

in the tensor form:

$$D^{\mu} G_{\nu\mu} = k D^{\mu} T_{\nu\mu}. \quad -(2)$$

From paper 88 it is now known that eq. (1) is a s[pecial]

case of

$$D \wedge T := R \wedge q. \quad -(3)$$

In this special case:

$$R \wedge q = 0 \quad -(4)$$

which can be written as:

$$R^{a}{}_{\sigma\mu\nu} + R^{a}{}_{\nu\sigma\mu} + R^{a}{}_{\mu\nu\sigma} = 0. \quad -(5)$$

Introducing the S torsion this can be written as:

$$D_{\sigma} S^{a}{}_{\mu\nu} + D_{\nu} S^{a}{}_{\sigma\mu} + D_{\mu} S^{a}{}_{\nu\sigma} = 0 \quad -(6)$$

This equation can be written as:

$$\boxed{D_{\mu} \tilde{S}^{a\mu\nu} = 0}, \quad -(7)$$

where

$$\tilde{S}^{a\mu\nu} = -\tilde{S}^{a\nu\mu} \quad -(8)$$

We now introduce the two-form:

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$$\tilde{N}^{a\mu\nu} = - \tilde{N}^{a\nu\mu} \quad - (9)$$

From paper 88 this Noether form is defined by:

$$D \wedge N^{a} := N^{a}{}_{b} \wedge q^{b} \quad - (10)$$

and has the units of mass per unit area. In the special case we are considering:

$$D \wedge N^{a} = 0 \quad - (11)$$

i.e.

$$D_{\sigma} N^{a}_{\mu\nu} + D_{\nu} N^{a}_{\sigma\mu} + D_{\mu} N^{a}_{\nu\sigma} = 0 \quad - (12)$$

which is

$$D_{\mu} \tilde{N}^{a\mu\nu} = 0 \quad - (13)$$

So comparing eqs. (7) and (13)

$$\boxed{D_{\mu} \tilde{S}^{a\mu\nu} = k D_{\mu} \tilde{N}^{a\mu\nu} = 0} \quad - (14)$$

where:
$$k = \frac{8\pi G}{c^{2}} = 1.86595 \times 10^{-26} \text{ m kgm}^{-1} \quad - (15)$$

is Einstein's constant where G is Newton's constant. A possible solution of eq. (14) is

$$\boxed{\tilde{S}^{a\mu\nu} = k \tilde{N}^{a\mu\nu}} \quad - (16)$$

Units Check

$\tilde{S} = m^{-1}, \quad k = m / \text{kgm}, \quad N = \text{kgm}\, m^{-2}$ ✓✓

Eq. (14) can be written as:

$$D_\sigma S^a_{\mu\nu} + D_\nu S^a_{\sigma\mu} + D_\mu S^a_{\nu\sigma} \quad -(17)$$

$$= k \left(D_\sigma N^a_{\mu\nu} + D_\nu N^a_{\sigma\mu} + D_\mu N^a_{\nu\sigma} \right)$$

$$= 0$$

i.e.:

$$\boxed{D \wedge S^a = k D \wedge N^a = 0} \quad -(18)$$

in differential form notation. Written out in full, eq. (18) is:

$$d \wedge S^a + \omega^a{}_b \wedge S^b = k \left(d \wedge N^a + \omega^a{}_b \wedge N^b \right)$$

$$= 0, \quad -(19)$$

a particular solution of which is:

$$S^a_{\mu\nu} = k N^a_{\mu\nu} \quad -(20)$$

However there are more general solutions of eq. (19) than that of eq. (20).

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The particular solution (20) means that, if there is no mass anywhere in the universe:

$$N^a_{\mu\nu} = 0 \quad \text{--- (21)}$$

which means that there is no curvature:

$$R^a_{\mu\nu\sigma} = D_\mu S^a_{\nu\sigma} = 0. \quad \text{--- (22)}$$

The more general equation (18) means that it is possible to write:

$$D \wedge S^a = 0 \quad \text{--- (23)}$$

$$D \wedge N^a = 0 \quad \text{--- (24)}$$

Eq. (23) is the Bianchi equation and eq. (24) is the conservation of energy. In order to relate physics to geometry it is necessary to write:

$$D \wedge S^a = k(D \wedge N^a) = 0 \quad \text{--- (25)}$$

Eqs. (20) and (25) offer two different torsion based cosmologies. The first step towards a development is to deduce the Newtonian limit. This will be done in the next note.