90(3), now renumbered 92(3) : Properties of the Spin Connection

In note 90(1) it was shown that the basic equation

$$\frac{\partial^2 \phi}{\partial r^2} + \left(\frac{2}{r} + \omega_r\right)\frac{\partial \phi}{\partial r} + \frac{\phi}{r^2}\left(2r\omega_r + r^2\frac{\partial \omega_r}{\partial r}\right) = -\frac{\rho}{\epsilon_0} \quad -(1)$$

reduces to:

$$\frac{\partial^2 \phi}{\partial r^2} + 2\beta\frac{\partial \phi}{\partial r} + \kappa_0^2 \phi = -\frac{\rho}{\epsilon_0} \quad -(2)$$

where:

$$\omega_r = \kappa_0^2 r - 4\beta \log_e r - \frac{4}{r} \quad -(3)$$

a) In the limit:

$$\kappa_0 \rightarrow 0, \; \beta \rightarrow 0 \quad -(4)$$

it is seen that the inverse distance dependence is recovered.

b) In the limit:

$$\beta \rightarrow 0, \; \kappa_0 \neq 0 \quad -(5)$$

eqn. (2) becomes an undamped resonator equation:

$$\frac{\partial^2 \phi}{\partial r^2} + \kappa_0^2 \phi = -\frac{\rho}{\epsilon_0} \quad -(6)$$

and $\phi \rightarrow \infty$ at resonance. In the limit (5)

$$\omega_r \rightarrow \kappa_0^2 r - \frac{4}{r} \quad -(7)$$

2)

From eq. (7) it is seen that there is a characteristic frequency ω_0, which can be expressed as:

$$\boxed{\kappa_0^2 = \frac{1}{r}\left(\omega_r + \frac{4}{r}\right)} \quad — (8)$$

This is a Hooke's law wavenumber ~~frequency~~ defined by the spin connection and the radial coordinate. For an undamped oscillator with a simple cosinal dependence

$$\rho = -\rho_0 \cos(\kappa r) \quad — (9)$$

resonance occurs at:

$$\kappa_0 = \kappa. \quad — (10)$$

<u>CONCLUSION</u>

The important deduction is that the potential ϕ can be amplified to ~~infinity~~ by eq. (10). The spin connection in this case is:

$$\omega_r = \kappa^2 r - \frac{4}{r} \quad — (11)$$