APPENDIX 4 : THE MEANING OF THE a INDEX

It is first noted that the ECE field equations originate in the Bianchi identity:

$$D \wedge F^a := R^a{}_b \wedge A^b \quad - (D1)$$

where the a and b indices denote those of a tangent space-time at point P in a base manifold in differential geometry. Thus:

$$D \wedge F^a = D_\mu F^a{}_{\nu\sigma} + D_\sigma F^a{}_{\mu\nu} + D_\nu F^a{}_{\sigma\mu} \quad - (D2)$$

where the Greek indices of the base manifold have been restored. In generally covariant unified field theory the electromagnetic field tensor is therefore a vector-valued two-form, i.e. an anti-symmetric tensor for each a. The field two-form is defined as:

$$F^a{}_{\mu\nu} = q^a{}_\kappa F^\kappa{}_{\mu\nu} \quad - (D3)$$

where $F^\kappa{}_{\mu\nu}$ is a tensor in the base manifold with three indices. It is seen that:

$$D \wedge F^\kappa := R^\kappa{}_b \wedge A^b \quad - (D4)$$

using the tetrad postulate:

$$D_\mu q^a{}_\kappa = 0. \quad - (D5)$$

Therefore Eq. (D1) can be written in the base manifold:

$$D_\mu F^\kappa{}_{\nu\sigma} + D_\sigma F^\kappa{}_{\mu\nu} + D_\nu F^\kappa{}_{\sigma\mu} := A^{(0)} \left(R^\kappa{}_{\mu\nu\sigma} + R^\kappa{}_{\sigma\mu\nu} + R^\kappa{}_{\nu\sigma\mu} \right). \quad - (D6)$$

In general relativity and unified field theory the base manifold is four dimensional space-time in which curvature and torsion are both present in general. So the electromagnetic field in this base manifold is a rank three tensor, not a rank two tensor as in special relativity and

Minkowski space-time. In the latter type of space-time there is no curvature and no torsion, so Minkowski space-time is flat space-time.

For example, consider the electric field components:

$$F^{\kappa}{}_{\mu\sigma} = F^{\kappa}{}_{10}\,,\ F^{\kappa}{}_{20}\,,\ F^{\kappa}{}_{30}\,. \qquad -(D7)$$

In the complex circular basis:

$$\kappa = (1),\ (2),\ (3) \qquad -(D8)$$

and we recover the three vector components $E_x^{(1)}$, $E_y^{(1)}$, and $E_z^{(1)}$. The first two denote complex conjugate plane waves:

$$\underline{E}^{(1)} = \underline{E}^{(2)*} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i(\omega t - \kappa z)}. \qquad -(D9)$$

So the meaning of κ Superimposed on $\mu\sigma$ is that one coordinate system is superimposed on another. When one coordinate system is imposed on the same coordinate system the only possibilities are:

$$F^{\kappa}{}_{\mu\sigma} = F^{1}{}_{10}\,,\ F^{2}{}_{20}\,,\ F^{3}{}_{30} = E^{1}{}_{1}\,,\ E^{2}{}_{2}\,,\ E^{3}{}_{3} \qquad -(D10)$$

as used in Appendix 4.