1) 93(14): Calculation of the θ Component of J

This is:

$$J_\theta = J^2_2 = -\frac{A^{(0)}}{\mu_0}\left(R^{2}{}_{0}{}^{20} + R^{2}{}_{1}{}^{21} + R^{2}{}_{3}{}^{23}\right) \quad -(1)$$

where

$$R^{2}{}_{3}{}^{23} = g^{22} g^{33} R^{2}{}_{323}$$

$$= g^{22} g^{33}\left(1 - e^{-2\beta}\right)\sin^2\theta$$

$$= \frac{1}{r^4 \sin^2\theta}\left(1 - e^{2\alpha}\right)\sin^2\theta$$

where:

$$e^{2\alpha} = -(1-x),$$

so

$$\boxed{R^{2}{}_{3}{}^{23} = \frac{1}{r^4}(2-x)} \quad -(2)$$

Also:

$$R^{2}{}_{1}{}^{21} = g^{11} g^{22} R_{2121} = g^{11} g^{22} R_{1212}$$

where:

$$R_{1212} = g_{11} R^{1}{}_{212},$$

so:

$$R^{2}{}_{1}{}^{21} = g^{11} g^{22} g_{11} R^{1}{}_{212}$$

where

$$g^{11} g_{11} = 1,$$

so

$$\boxed{R^{2}{}_{1}{}^{21} = g^{22} R^{1}{}_{212} = \frac{1}{2r^2}(x-2)} \quad -(3)$$

From previous calculations:

$$\boxed{R^{2}{}_{0}{}^{20} = -\frac{2}{r^4}(1+x)} \quad -(4)$$

Therefore:

$$R^{2}{}_{0}{}^{20} + R^{2}{}_{1}{}^{21} + R^{2}{}_{3}{}^{23} = \frac{1}{r^4}\left(2 - x - 2 - 2x\right) + \frac{1}{2r^2}(x-2)$$

$$= -\frac{3x}{r^4} - \frac{(2-x)}{2r^2}$$

So:

$$\boxed{J_\theta = J^2_2 = \frac{A^{(0)}}{\mu_0}\left(\frac{1}{r^2} - \frac{x}{r^2} + \frac{3x}{r^4}\right)} \quad -(5)$$

where:

$$x = \frac{2MG}{rc^2}. \quad -(6)$$

As $x \to 0$: $\quad -(7)$

$$J_\theta \to \frac{A^{(0)}}{\mu_0}\frac{1}{r^2}.$$

Notes

The J_θ component is expressed in the spherical polar coordinate system (r, θ, ϕ). So the occurrence of $1/r^2$ and $1/r^4$ terms in J_θ is due to the use of these coordinates and metric components. The current components J_r, J_θ and J_ϕ are due to $\tilde{R} \wedge q$ caused by curved spacetime, and thus due to mass. In these calculations, gravitational torsion has not yet been considered, only gravitational curvature.