13(6): The Carlton Line Element

This is:

$$g_{00} = A(C(r))^{1/2}, \quad g_{11} = -B(C(r))^{1/2},$$
$$g_{22} = -C(r), \quad g_{33} = -C(r)\sin^2\theta \quad -(1)$$

The radius of curvature is:

$$R_c = (C(r))^{1/2} \quad -(2)$$

It is assumed in the standard model that:

$$C(r) = r^2. \quad -(3)$$

In the infinitely far field:

$$\boxed{R_p \to R_c \to r} \quad -(4)$$

The general form of $C(r)$ is:

$$C(r) = R_c^2(r) = (|r - r_0|^n + \alpha^n)^{2/n} \quad -(5)$$

Here:

$$\boxed{R_c(r) \geq \alpha} \quad -(6)$$

The most general line element is given by Levi-Civita:

$$\boxed{dl'^2 = B(R_c)\,dR_c^2 + R_c^2(d\theta^2 + \sin^2\theta\,d\phi^2)}$$
(7)

2) This is valid for $M^3$, where Euclidean space is $E^3$. Crothers suggests:

$$\left.\begin{aligned} R_c(r) &= |r - r_0|, \\ dR_c(r) &= \frac{(r - r_0)}{|r - r_0|}\, dr, \\ dR_c^2(r) &= dr^2 \end{aligned}\right\} \quad - (8)$$

For the standard model metric, eqn. (7) becomes:

$$dl'^2 = \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad - (9)$$

for its space part. Thus:

$$\boxed{R_c = r, \quad R_p \neq r.} \quad - (10)$$

If the centre of a sphere can be located anywhere in Minkowski space relative to the origin of the coordinate system at $r = 0$:

$$ds^2 = -dt^2 + d(|r - r_0|)^2 + |r - r_0|^2 d\Omega^2 \quad - (11)$$

$$ds^2 = -dt^2 + dr^2 + |r - r_0|^2 d\Omega^2,$$

$$R_c = |r - r_0|$$

Therefore the most general line element is:

$$dl'^2 = B(R_c)\, dR_c^2 + (|r - r_0|)^2 d\Omega^2 \quad - (12)$$

3)

This compares with Carroll's:

$$d\ell'^2 = n(t, r)\,dr^2 + r^2\,d\Omega^2 \quad -(13)$$

So:

$$\boxed{\begin{aligned} n(t, r) &= B(R_c) \quad -(14) \\ r^2 &= (|r - r_0|)^2. \quad -(15) \end{aligned}}$$

I accept that $R_{\mu\nu} = 0$ identically is not valid, but:

$$G_{\mu\nu} = k\,T_{\mu\nu} \rightarrow 0 \quad -(16)$$

is valid.

In this way we obtain the NASA Cassini results, the r parameter used by Carroll is replaced by

$$r = |r - r_0| \quad -(17)$$

At the same time there can be no black holes because of (6).