1)

93(8) : Change of Polarization of Electric Field Vector due to Gravitation

This is calculated from the inhomogeneous ECE field equation:

$$\partial_\mu F^{a\mu\nu} = -A^{(0)} R^{a}{}_{\mu}{}^{\mu\nu}, \quad -(1)$$

from which:

$$(\underline{\nabla} \cdot \underline{E})^0 = -\phi^{(0)} \left(R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30} \right) \quad -(2)$$

Here the superscript 0 on the LHS denotes the use of a scalar potential (time like). Here:

$$R^0{}_1{}^{\mu\nu} = g^{1\sigma} g^{\nu\rho} R^0{}_{1\sigma\rho} \quad -(3)$$

so

$$R^0{}_1{}^{10} = g^{11} g^{00} R^0{}_{110} = -g^{11} g^{00} R^0{}_{101} \quad -(4)$$

$$R^0{}_2{}^{20} = -g^{22} g^{00} R^0{}_{202} \quad -(5)$$

$$R^0{}_3{}^{30} = -g^{33} g^{00} R^0{}_{303} \quad -(6)$$

From Carroll's chapter 7:

$$R^0{}_{202} = -r e^{-2\beta} \partial_1 \alpha \quad -(7)$$

where:

$$e^{-2\beta} = 1 - \frac{2GM}{rc^2} \quad -(8)$$

$$e^{2\alpha} (2r \partial_1 \alpha + 1) = 1 \quad -(9)$$

2)

So:

$$\boxed{R^{0}{}_{2}{}^{02} = \frac{1}{r^2}\left(\frac{GM}{rc^2}\right)\left(1 - \frac{2GM}{rc^2}\right)^{-1}} \quad -(10)$$

because:

$$g^{22} = \frac{1}{r^2}, \quad g^{00} = -\left(1 - \frac{2mG}{rc^2}\right)^{-1} \quad -(11)$$

In ~~the~~ spherical polar coordinate system:

$$\underline{E} = E_r \underline{e}_r + E_\theta \underline{e}_\theta + E_\phi \underline{e}_\phi \quad -(12)$$

so:

$$\underline{\nabla} \cdot \underline{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 E_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(E_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial \phi} \quad -(13)$$

so:

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(E_\theta \sin\theta\right) = -\frac{\phi^{(0)}}{r^2}\left(\frac{GM}{rc^2}\right)\left(1 - \frac{2GM}{rc^2}\right)^{-1} \quad -(14)$$

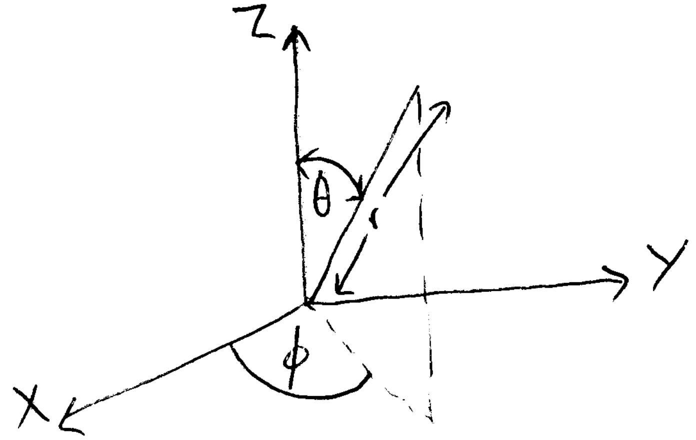
i.e

$$\frac{1}{r}\left(\frac{\partial E_\theta}{\partial\theta} + \cot\theta \, . \, E_\theta\right) = -\frac{\phi^{(0)}}{r^2}\left(\frac{GM}{rc^2}\right)\left(1 - \frac{2GM}{rc^2}\right)^{-1} \quad -(15)$$

3)

$$\frac{\partial E_\theta}{\partial \theta} + E_\theta \cot\theta = -\frac{\phi^{(0)}}{r}\left(\frac{6M}{rc^2}\right)\left(1-\frac{2GM}{rc^2}\right)^{-1}$$

— (16)



Interpretation

It can be seen that the E_θ component changes with θ, meaning that its plane of polarization is rotated. The deflection of light at the distance of closest approach is

$$\delta = \frac{4GM}{r_0 c^2} \quad — (17)$$

Conclusion

We expect that the polarization of light deflected by (17) will be rotated.