

93(a): Riemann Tensor elements from Christoffel Symbols

The Riemann tensor is described by:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu} \Gamma^{\rho}{}_{\nu\sigma} - \partial_{\nu} \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda} \Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda} \Gamma^{\lambda}{}_{\mu\sigma} \quad - (1)$$

Thus:

$$R^{\circ}{}_{202} (\rho=0, \sigma=2, \mu=0, \nu=2) \quad - (2)$$

$$= \partial_0 \Gamma^{\circ}{}_{22} - \partial_2 \Gamma^{\circ}{}_{02} + \Gamma^{\circ}{}_{0\lambda} \Gamma^{\lambda}{}_{22} - \Gamma^{\circ}{}_{2\lambda} \Gamma^{\lambda}{}_{02}$$

with summation over repeated λ : - (3)

$$\Gamma^{\circ}{}_{0\lambda} \Gamma^{\lambda}{}_{22} = \Gamma^{\circ}{}_{00} \Gamma^{\circ}{}_{22} + \Gamma^{\circ}{}_{01} \Gamma^1{}_{22} + \dots$$

$$\Gamma^{\circ}{}_{2\lambda} \Gamma^{\lambda}{}_{02} = \Gamma^{\circ}{}_{20} \Gamma^{\circ}{}_{02} + \Gamma^{\circ}{}_{21} \Gamma^1{}_{02} + \dots \quad - (4)$$

So:

$$R^{\circ}{}_{202} = \Gamma^{\circ}{}_{01} \Gamma^1{}_{22} = -r e^{-2\beta} \partial_1 d \quad \checkmark \checkmark \quad - (5)$$

$$R^{\circ}{}_{303} (\rho=0, \sigma=3, \mu=0, \nu=3) \quad \checkmark \checkmark$$

$$R^{\circ}{}_{303} = \Gamma^{\circ}{}_{01} \Gamma^1{}_{33} = -r e^{-2\beta} \sin^2 \theta \partial_1 d \quad \checkmark \checkmark \quad - (6)$$

These two elements are calculated correctly by Carroll.

The correct calculation of $R^{\circ}{}_{101}$ is as follows.

$$\begin{aligned}
 2) \quad R^{\circ}_{101} &= d_0 \Gamma^{\circ}_{11} - d_1 \Gamma^{\circ}_{01} + \Gamma^{\circ}_{0\lambda} \Gamma^{\lambda}_{11} - \Gamma^{\circ}_{1\lambda} \Gamma^{\lambda}_{01} \\
 &= d_0 \left(e^{2(\beta-d)} d_0 \beta \right) - d_1 (d_1 d) \\
 &\quad + \Gamma^{\circ}_{11} (\Gamma^{\circ}_{00} - \Gamma^{\circ}_{01}) + \Gamma^{\circ}_{01} (\Gamma^{\circ}_{11} - \Gamma^{\circ}_{01}) \\
 &= d_0 \left(e^{2(\beta-d)} d_0 \beta \right) - d_1 (d_1 d) \\
 &\quad + e^{2(\beta-d)} d_0 \beta (d_0 d - d_0 \beta) + d_1 d (d_1 \beta - d_1 d)
 \end{aligned}$$

$$R^{\circ}_{101} = e^{2(\beta-d)} \left(d_0 (d_0 \beta) + d_0 \beta (d_0 d - d_0 \beta) \right) + d_1 d (d_1 \beta - d_1 d) - d_1 (d_1 d) \quad (7)$$

Unfortunately there is a sign error in Carroll's calculation, he uses:
 $- d_0 \beta (d_0 d - d_0 \beta)$.

All of Carroll's Christoffel symbols are correctly calculated and the sign error does not seem to affect the rest of chapter 7 of Carroll. However, I need a correct eq. (7) for page 93.