

1) Calculation of $d_1(\theta, d)$

Use :

$$d_1 d = \frac{1}{2r} (e^{-2d} - 1) \quad (1)$$

so:
$$d_1(d_1 d) = -\frac{1}{2r^2} (e^{-2d} - 1)$$

----- \rightarrow
$$-\frac{1}{r} (d_1 d) e^{-2d} \quad (2)$$

$$= -\frac{1}{2r^2} \left(\frac{1}{1-x} - 1 \right) - \frac{1}{2r^2} \left(\frac{x}{1-x} \right) (1-x)$$

$$= -\frac{1}{2r^2} \left(\frac{x}{1-x} + \frac{x}{(1-x)^2} \right)$$

So :

$$R^{\circ}_{101} = -\frac{1}{2r^2} \left(\frac{x^2}{(1-x)^2} - \frac{x}{1-x} - \frac{x}{(1-x)^2} \right)$$

$$= -\frac{x}{1-x} \cdot \frac{1}{2r^2} \left(\frac{x}{1-x} + 1 + \frac{1}{1-x} \right)$$

$$= -\frac{x}{1-x} \cdot \frac{1}{2r^2} \left(\frac{1+x}{1-x} + 1 \right)$$

$$R^{\circ}_{101} = -\frac{x}{r^2(1-x)^2} = R^{\circ}_{110}$$

2)

1. So:

$$R^{\circ}_{101} = -\frac{x}{r^2(1-x)^2} = R^{\circ}_{1^{10}}$$

$$R^{\circ}_{2^{20}} = R^{\circ}_{3^{30}} = -\frac{1}{2r^2} \frac{x}{1-x}$$

$$\underline{\nabla} \cdot \underline{E} = \phi \left(R^{\circ}_{1^{10}} + R^{\circ}_{2^{20}} + R^{\circ}_{3^{30}} \right)$$

$$\underline{\nabla} \cdot \underline{E} = \frac{x}{r^2(1-x)} \left(\frac{1}{1-x} + 1 \right)$$

$$x = \frac{2mG}{rc^2}$$