Note 94(3) : Determining Equations

It is assumed in this note that the electric and magnetic fields can act as driving fields for the potential. So the complete set of equations is:

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(1)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(2)$$

$$\underline{\nabla} \cdot \underline{E} = 0 \quad -(3)$$

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(4)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(5)$$

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} - c\omega^0 \underline{A} \quad -(6)$$

In this set of equations there is no charge / current density present, and no scalar potential.

From eq. (6):

$$\frac{\partial^2 \underline{A}}{\partial t^2} + c\omega^0 \frac{\partial \underline{A}}{\partial t} + c\left(\frac{\partial \omega^0}{\partial t}\right)\underline{A} = -\left(\frac{\partial \underline{E}}{\partial t}\right)_{\text{driving}} \quad -(7)$$

where from eq. (2):

$$\left(\frac{\partial \underline{E}}{\partial t}\right)_{\text{driving}} = c^2\left(\underline{\nabla} \times \underline{B}\right)_{\text{driving}} \quad -(8)$$

From eq. (3):

2)

$$\underline{\nabla} \cdot \left(\frac{\partial \underline{A}}{\partial t} + c\omega^{\circ} \underline{A} \right) = 0 \quad - (12)$$

So:

$$\frac{\partial \underline{A}}{\partial t} + c\omega^{\circ} \underline{A} = 0 \quad - (13)$$

is a possible solution.

From eq. (13) & eq. (7):

$$\boxed{\frac{\partial^2 \underline{A}}{\partial t^2} + \left(c\frac{\partial \omega^{\circ}}{\partial t} - (c\omega^{\circ})^2 \right) \underline{A} = -\left(\frac{\partial \underline{E}}{\partial t}\right)_{\text{driving}}}$$

$$- (14)$$

If:

$$\frac{\partial \omega^{\circ}}{\partial t} > c\omega^{\circ 2} \quad - (15)$$

eq. (14) is an undamped oscillator. At resonance:

$$|\underline{A}| \rightarrow \infty \quad - (16)$$

for a given $\left(-\partial \underline{E} / \partial t = c^2 \underline{\nabla} \times \underline{B}\right)_{\text{driving}}$.

From eqs. (4) and (5):

$$\underline{\nabla} \times \underline{A} = \underline{\omega} \times \underline{A} \quad - (17)$$

Eqs (13) and (17) are simultaneous equations for ω° and $\underline{\omega}$.