1) 94(4): Time Dependent Resonance Structures from the Ampère Maxwell Law.

In general this law in ECE theory is:

$$\underline{\nabla} \times \underline{B} = \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} + \mu_0 \underline{J} \quad -(1)$$

where:

$$\underline{E} = - \underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} - c\omega^0 \underline{A} + c \phi \underline{\omega}. \quad -(2)$$

If we consider the law (1) to be:

$$\frac{\partial \underline{E}}{\partial t} = c^2 \left(\underline{\nabla} \times \underline{B} - \mu_0 \underline{J} \right)_{\text{driving}} \quad -(3)$$

$$= - \underline{\nabla} \frac{\partial \phi}{\partial t} - \frac{\partial^2 \underline{A}}{\partial t^2} - \frac{\partial}{\partial t} \left(c\omega^0 \underline{A} \right) + \frac{\partial}{\partial t} \left(c \phi \underline{\omega} \right)$$

then the most general resonance equation of time-dependent type is:

$$\frac{\partial^2 \underline{A}}{\partial t^2} + c \left(\frac{\partial \omega^0}{\partial t} \right) \underline{A} + c\omega^0 \frac{\partial \underline{A}}{\partial t}$$

$$= c \left(\frac{\partial \phi}{\partial t} \right) \underline{\omega} + c \phi \left(\frac{\partial \underline{\omega}}{\partial t} \right) + c^2 \mu_0 \underline{J} - \underline{\nabla} \frac{\partial \phi}{\partial t} - c^2 \underline{\nabla} \times \underline{B} \quad -(4)$$

If there is no current density and no charge density this reduces to:

$$\boxed{\frac{\partial^2 \underline{A}}{\partial t^2} + c\omega^0 \frac{\partial \underline{A}}{\partial t} + c\left(\frac{\partial \omega^0}{\partial t}\right)\underline{A} = -c^2\, \underline{\nabla} \times \underline{B} \qquad (5)}$$

There is resonance in $\underline{A}$ if and only if:

1) The scalar part, ω^0, of the spin connection is non-zero.
2) The time derivative $\partial \omega^0 / \partial t$ is non-zero.
3) $\underline{\nabla} \times \underline{B}$ is non-zero and <u>also</u> time dependent.

When investigating various claims, resonance structures such as this must be systematically developed, both time dependent and space dependent, and the conditions needed for resonance identified. An example consists of conditions 1) to 3) above. These conditions show already that the magnetic field in the design must be <u>both</u> space and time dependent

3)

In addition to eqn. (1) there exists the Coulomb law:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(6)$$

i.e.:

$$\underline{\nabla} \cdot \left(c\phi \underline{\omega} - \underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} - c\omega^\circ \underline{A} \right) = \rho / \epsilon_0 \quad -(7)$$

In the absence of charge:

$$\underline{\nabla} \cdot \left(\frac{\partial \underline{A}}{\partial t} + c\omega^\circ \underline{A} \right) = 0 \quad -(8)$$

So ω° may be eliminated between eqns. (5) and (8). Eq. (8) is:

$$\underline{\nabla} \cdot \frac{\partial \underline{A}}{\partial t} = -c\, \underline{\nabla} \cdot \left(\omega^\circ \underline{A} \right) \quad -(9)$$

$$\boxed{\underline{\nabla} \cdot \frac{\partial \underline{A}}{\partial t} = -c \left(\underline{A} \cdot \underline{\nabla} \omega^\circ + \omega^\circ \underline{\nabla} \cdot \underline{A} \right)} \quad -(10)$$

Therefore ω° is governed by eqs. (5) and (10). These two equations must be solved simultaneously.

From eq. (10):

$$\underline{\nabla} \cdot \left(\frac{\partial \underline{A}}{\partial t} + c\omega^\circ \underline{A} \right) = -c\, \underline{A} \cdot \underline{\nabla} \omega^\circ \quad -(11)$$

4) The integration of eq. (11) must now be carried out. For this we use the divergence theorem. For any well behaved vector field $\underline{V}(\underline{r})$ defined within a volume surrounded by a closed surface S:

$$\oint_S \underline{V} \cdot \underline{n} \, da = \int_V \underline{\nabla} \cdot \underline{V} \, d^3 r \quad - (12)$$

Thus for the Gauss law:

$$\int_V \left(\underline{\nabla} \cdot \underline{E} - \frac{\rho}{\epsilon_0} \right) d^3 r = 0 \quad - (13)$$

i.e.

$$\oint_S \underline{E} \cdot \underline{n} \, da = \frac{1}{\epsilon_0} \int_V \rho(r) \, d^3 r \quad - (14)$$

So the integration of eq. (11) is — (15)

$$\boxed{\oint_S \left(\frac{\partial \underline{A}}{\partial t} + c\omega^0 \underline{A} \right) \cdot \underline{n} \, da = - c \int_V \underline{A} \cdot \underline{\nabla} \omega^0 \, d^3 r}$$

i.e.:

$$\oint_S \frac{\partial \underline{A}}{\partial t} \cdot \underline{n} \, da = - \oint_S c\omega^0 \underline{A} \cdot \underline{n} \, da - c \int_V \underline{A} \cdot \underline{\nabla} \omega^0 d^3 r \quad - (16)$$

Eqn. (16) is:

5)

$$\oint_S \frac{\partial \underline{A}}{\partial t} \cdot \underline{n}\, da = -c\omega^{\circ} \oint_S \underline{A} \cdot \underline{n}\, da - c \int_V \underline{A} \cdot \underline{\nabla}\, \omega_{\circ}\, d^3 r \quad -(17)$$

and is a relation between $\omega_{\circ}$ and $\underline{A}$.
Furthermore it expresses $\frac{\partial \underline{A}}{\partial t}$ in terms of $\underline{A}$.

- Therefore the correct way to solve eq. (5) is <u>simultaneously</u> <u>with eq. (15)</u>.

If possible this should be done by computer for <u>the various</u> modes of $\underline{\nabla} \times \underline{B}$.

It can be seen that ω° is eliminated <u>and that eq. (5)</u> <u>reduces to an undamped oscillator</u>

- because $\partial \underline{A} / \partial t$ is eliminated in favour of $\underline{A}$.

<u>Conclusion.</u>

For various modes of $\underline{\nabla} \times \underline{B}$, $\underline{A}$ can go to infinity. There is no need to model ω°, because it can be expressed in terms of $\underline{A}$.