Checking Paper 94(3) Part 2

1) Eq. (10) is:

$$\underline{\nabla} \times \underline{\ddot{A}} - \underline{\ddot{\omega}} \times \underline{A} - 2\,\underline{\dot{\omega}} \times \underline{\dot{A}} - \underline{\omega} \times \underline{\ddot{A}} = \underline{\ddot{B}}_{\text{driving}} \quad -(1)$$

where:

$$\underline{\nabla} \times \underline{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(r A_\phi\right)\underline{k} \quad -(2)$$

$$\underline{\omega} \times \underline{A} = \left(\omega_r A_\phi - \omega_\phi A_r\right)\underline{k} \quad -(3)$$

So eq. (27) should be:

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r \ddot{A}_\phi\right) - \left(\ddot{\omega}_r A_\phi - \ddot{\omega}_\phi A_r\right) - 2\left(\dot{\omega}_r \dot{A}_\phi - \dot{\omega}_\phi \dot{A}_r\right) - \left(\omega_r \ddot{A}_\phi - \omega_\phi \ddot{A}_r\right) = \ddot{B}_{Z\,\text{driving}} \quad -(4)}$$

2) Eq. (15) is:

$$\underline{\nabla} \times \underline{\ddot{A}} + \underline{\nabla} \times \left(\dot{\omega}_0 \underline{A}\right) + \underline{\nabla} \times \left(\omega_0 \underline{\dot{A}}\right) = \underline{\ddot{B}}_{\text{driving}} \quad -(5)$$

so from eq. (2):

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r \ddot{A}_\phi + \dot{\omega}_0 r A_\phi + \omega_0 r \dot{A}_\phi\right) = \ddot{B}_{Z\,\text{driving}} \quad -(33)}$$

This is eq. (33) ✓

2) Eq. (4) is:

$$\underline{\nabla} \cdot \underline{E} = 0 \quad -(6)$$

i.e. $$\underline{\nabla} \cdot \left(\dot{\underline{A}} + \omega_0 \underline{A} \right) = 0 \quad -(7)$$

Now use:

$$\underline{\nabla} \cdot \underline{A} = \frac{1}{r} \frac{\partial}{\partial r} \left(r A_r \right) + \frac{1}{r} \frac{\partial}{\partial \phi} \left(A_\phi \right) \quad -(8)$$

so:

$$\underline{\nabla} \cdot \dot{\underline{A}} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \dot{A}_r \right) + \frac{1}{r} \frac{\partial}{\partial \phi} \left(\dot{A}_\phi \right) \quad -(9)$$

$$\underline{\nabla} \cdot \left(\omega_0 \underline{A} \right) = \left(\underline{\nabla} \omega_0 \right) \cdot \underline{A} + \omega_0 \underline{\nabla} \cdot \underline{A} \quad -(10)$$

The Gradient in Cylindrical Polar Coordinates

This is given in eq. (14), page 1061 of "Vector Analysis Problem Solver":

$$\boxed{\underline{\nabla} f = \frac{\partial f}{\partial r} \underline{e}_r + \frac{1}{r} \frac{\partial f}{\partial \phi} \underline{e}_\phi + \frac{\partial f}{\partial Z} \underline{e}_Z} \quad -(11)$$

So in eq. (10):

$$\underline{\nabla} \omega_0 = \frac{\partial \omega_0}{\partial r} \underline{e}_r + \frac{1}{r} \frac{\partial \omega_0}{\partial \phi} \underline{e}_\phi + \frac{\partial \omega_0}{\partial Z} \underline{e}_Z \quad -(12)$$

3) Therefore eq. (7) is:

$$\boxed{\begin{aligned}&\frac{1}{r}\frac{\partial}{\partial r}\left(r\dot{A}_r\right)+\frac{1}{r}\frac{\partial\dot{A}_\phi}{\partial\phi}\\&+\omega_0\left(\frac{1}{r}\frac{\partial}{\partial r}\left(rA_r\right)+\frac{1}{r}\frac{\partial A_\phi}{\partial\phi}\right)\\&+\frac{\partial\omega_0}{\partial r}A_r+\frac{1}{r}\frac{\partial\omega_0}{\partial\phi}A_\phi=0\end{aligned}} \quad -(13)$$

I think that eq. (34) of Paper 94(3) needs to be eq (13) of this note.

<u>Equation (12)</u>

This is

$$\boxed{\ddot{\underline{A}}+c\ddot{\omega}\underline{A}+c\omega^{\circ}\dot{\underline{A}}=-\dot{\underline{E}}_{\text{driving}}} \quad -(14)$$

Eqs. (35) and (36) need slight corrections:

$$\ddot{A}_r+c\ddot{\omega}A_r+c\omega^{\circ}\dot{A}_r=-\dot{E}_r \quad -(15)$$

$$\ddot{A}_\phi+c\ddot{\omega}A_\phi+c\omega^{\circ}\dot{A}_\phi=-\dot{E}_\phi \quad -(16)$$

Eq. (37) and (38) should be eq. (13) of this note.