1) 95(1): Background Details of the Friedmann Lemaître Robertson Walker metric (FLRW).

These notes are based on Carroll's chapter 8. It is known from paper 93 that FLRW is deeply flawed, but in this paper it is evaluated warts and all. The use of metrics in this way is of course completely new.

The FLRW metric is:

$$ds^2 = -c^2 dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right) \quad —(1)$$

The replacement:

$$k \rightarrow k/|k|, \quad r \rightarrow |k|^{1/2} r, \quad a \rightarrow a|k|^{-1/2} \quad -(2)$$

leaves (1) invariant. Here $k = -1$ is constant negative curvature and is called an open universe. The case $k = 0$ is called a flat universe, and $k = 1$ is positive curvature and a closed universe. The Ricci tensor of this metric is non-zero. Matter and energy are modelled by a perfect fluid:

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p\, g_{\mu\nu} \quad —(3)$$

where ρ and p are energy density and pressure respectively as measured in the rest frame, and U^μ is the four velocity of the fluid. If a fluid that is isotropic in some frame leads to a metric which is isotropic in some frame the two frames coincide. The fluid is at rest in comoving coordinates. In this case:

$$U^\mu = (1, 0, 0, 0), \quad —(4)$$

$$T_{\mu\nu} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & & & \\ 0 & & g_{ij}\, p & \\ 0 & & & \end{bmatrix} \quad —(5)$$

This means:

$$T^{\mu}{}_{\nu} = \mathrm{diag}(-\rho, p, p, p) \quad -(6)$$

and

$$T = T^{\mu}{}_{\mu} = -\rho + 3p. \quad -(7)$$

<u>The equation of state</u> is:

$$p = w\rho \quad -(8)$$

where w is a constant independent of time. <u>Conservat.</u> <u>of energy</u> is given by:

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \quad -(9)$$

i.e.:

$$\rho \propto a^{-3(1+w)} \quad -(10)$$

Cosmological fluids are usually modelled by dust and radiation. Dust is collisionless and non-relativistic with no pressure ($w = 0$). Examples are stars and galaxies. For dust energy density is dominated by the rest energy, which is proportional to the number density. Radiation is modelled by:

$$T^{\mu\nu} = \frac{1}{4\pi}\left(F^{\mu\lambda}F^{\nu}{}_{\lambda} - \frac{1}{4}g^{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma}\right) \quad -(11)$$

(Gaussian units, no $\underline{B}^{(3)}$ considered, MWE). The radiation pressure is:

$$p = \frac{1}{3}\rho, \quad \rho \propto a^{-4} \quad -(12)$$

For dust:

$$\rho \propto a^{-3}. \quad -(13)$$

3)

The EH equation is now used:

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) \quad -(14)$$

For $\mu\nu = 00$:

$$-3\frac{\ddot{a}}{a} = 4\pi G (\rho + 3p) \quad -(15)$$

where $\dot{a} := da/dt$. For $\mu\nu = ij$:

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{2k}{a^2} = 4\pi G (\rho - p) \quad -(16)$$

These give the Friedmann equations

$$\boxed{\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi}{3} G (\rho + 3p) \quad -(17) \\ \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad -(18) \end{aligned}}$$

The metric (1) obeys these equations. The Hubble parameter is:

$$H = \frac{\dot{a}}{a}. \quad -(19)$$

So:

$$\boxed{\frac{dH}{dt} = \frac{\ddot{a}a - \dot{a}\dot{a}}{a^2}} \quad -(20)$$

The deceleration parameter is:

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \quad -(21)$$

4) The density parameter is:

$$\Omega = \frac{8\pi G}{3H^2}\rho \qquad -(22)$$

Solutions

1) For dust in an open universe:

$$a = \frac{C}{2}\left(\cosh\phi - 1\right), \qquad -(23)$$

$$t = \frac{C}{2}\left(\sinh\phi - \phi\right), \qquad -(24)$$

$$k = -1. \qquad -(25)$$

2) For dust in a flat universe:

$$a = \left(\frac{9C}{4}\right)^{1/3} t^{2/3}, \qquad -(26)$$

$$k = 0. \qquad -(27)$$

3) For dust in a closed universe:

$$a = \frac{C}{2}\left(1 - \cos\phi\right), \qquad -(28)$$

$$t = \frac{C}{2}\left(1 - \sin\phi\right). \qquad -(29)$$

$$k = 1. \qquad -(30)$$

Here:

$$C = \frac{8\pi G}{3}\rho a^3 = \text{constant} \qquad -(31)$$

These are the background details of the FLRW metric based on Carroll, Chapter 8.

5)

Charge Density of the Coulomb Law (ρ_e)

In paper 93 this was found to be:

$$\boxed{J^0 = -3\phi \frac{\ddot{a}}{a}} \quad -(32)$$

Using the first Friedmann equation:

$$J^0 = 4\pi \phi G (\rho + 3p) \quad -(33)$$

For dust:

$$p = 0 \quad -(34)$$

so:

$$J^0 = 4\pi \phi G \rho . \quad -(35)$$

This means that the Coulomb law in SI units is

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho_e}{\epsilon_0} \quad -(36)$$

where:

$$\boxed{\rho_e = 4\pi \epsilon_0 \phi G \rho} \quad -(37)$$

Units of Eq. (37)

$\rho_e = C m^{-3}$, $\epsilon_0 = J^{-1} C^2 m^{-1}$, $\phi = \text{volt} = J C^{-1}$

$G = m \, kgm^{-1}$, $\rho = kgm \, m^{-3}$.

In eq. (33): $J^0 = \text{volt} \, m \, kgm^{-1} \, kgm \, m^{-3} = \text{volt}$ ✓

So:

$$\boxed{\underline{\nabla} \cdot \underline{E} = J^0 = \rho_e / \epsilon_0} \quad -(38)$$

where $\rho_e / \epsilon_0 = C m^{-3} J C^{-2} m = J C^{-1} m^{-2}$ ✓✓

6)

Discussion of Eq. (37)

The exact self-consistency of ECE theory is shown by the fact that eq. (37) is the same structure as eq. (5.102) of GCUFT vol. 1:

$$\rho_e = \epsilon_0 G \left(\frac{\phi}{c^2}\right) \rho \quad -(39)$$

In eq. (39), ϕ / c^2 has the units of volts. The only difference between eqs. (37) and (39) is the factor 4π, which is a convention. Both equations indicate that charge density cannot exist without mass density. For example the electron carries charge and mass.

Current Densities of the FLRW Metric

The radial component is, from paper 93:

$$J_r = -\frac{A^{(0)}}{\mu_0} \frac{(kr^2 - 1)}{a^3} \left(\frac{2}{a} (k + \dot{a}^2) + \ddot{a} \right) \quad -(40)$$

and the angular components are:

$$J_\theta = J_\phi \sin^2\theta = \frac{A^{(0)}}{\mu_0 a^3 r^2} \left(\frac{2}{a} (k + \dot{a}^2) + \ddot{a} \right) \quad -(41)$$

Eq. (40) can be written as:

$$J_r = A^{(0)} \left(1 - kr^2\right) \left(\frac{2}{a} \left(k + \dot{a}^2\right) + \ddot{a} \right)$$

7) It is seen that the radial factor of eq. (1):

$$f = \frac{a^2}{1-kr^2} \quad \text{— (43)}$$

appears as an inverse in eq. (42).

For a flat universe $(k=0)$, eq. (42) reduces to:

$$J_r = \frac{A^{(0)}}{\mu_0} \cdot \frac{1}{a^2} \left(2\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a} \right) \quad \text{— (44)}$$

For dust $(p=0)$:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho \quad \text{— (45)}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad \text{— (46)}$$

So for $k=0$, $p=0$:

$$\boxed{J_r = \frac{A^{(0)}}{\mu_0} \cdot \frac{1}{a^2} \cdot 4\pi G\rho} \quad \text{— (47)}$$

From eq. (26):

$$a = \left(\frac{9C}{4}\right)^{1/3} t^{2/3} \quad \text{— (48)}$$

where C is a constant.

The usual interpretation of eq. (47) is that at $t=0$ $(a=0)$ there is a singularity. This is where the FLRW metric becomes unphysical