1) 95(4) : Checking Units in Carroll's Chapter 8.

Correcting Carroll for a missing c, the RW metric is:

$$ds^2 = -c^2 dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta + \sin^2\theta \, d\phi^2)\right) \quad -(1)$$

Therefore a must be unitless and $1-kr^2$ must be unitless because eq. (1) must reduce to the Minkowski metric:

$$ds^2 = -c^2 dt^2 + dr^2 + r^2(d\theta + \sin^2\theta \, d\phi^2) - (2)$$

The Friedmann equations used by Carroll are:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad -(3)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \quad -(4)$$

In SI: $G = m \, kgm^{-1}$, $\rho = kgm \, m^{-3}$, $-(5)$

so $\ddot{a}$ is in m^{-2} from eq. (3), because a is unitless. From eq. (4), $\dot{a}$ is in m^{-1} and k is in m^{-2}. This is consistent with the fact that r^2 is m^2, so $1-kr^2$ is unitless.

$-(6)$

Carroll also defines:

$$H = \frac{\dot{a}}{a} \quad \text{and} \quad \Omega = \frac{8\pi G}{3H^2}\rho = \frac{\rho}{\rho_{crit}}$$

So H is in m^{-1}, and Ω is unitless. The Friedmann eqn. is then:

$$\Omega - 1 = \frac{k}{H^2 a^2}$$

2) Up to this point the units are correct. However, Carroll starts to use $k = 0, \pm 1$. This is the source of the confusion. The SI units of k are m^{-2}.

Carroll also defines:

$$C := \frac{8\pi G}{3}\rho a^3 = \text{constant} \quad -(6)$$

It is unclear how this can be a constant when $a(t)$ is a function of time. The units of C should be m^{-2} (inverse metres).

However, Carroll later uses: $-(7)$

$$a = \frac{C}{2}(1-\cos\phi), \quad t = \frac{C}{2}(\phi - \sin\phi),$$

$$a = \frac{C}{2}(\cosh\phi - 1), \quad t = \frac{C}{2}(\sinh\phi - \phi), \quad -(8)$$

$$a = \left(\frac{9C}{4}\right)^{1/3} t^{2/3}. \quad -(9)$$

This is not self-consistent. In eq. (7a), a is m^{-2}, not unitless, in eq. (7b), C is sec not unitless. Similarly in eq. (8). In eq. (9), a is $m^{-2/3} t^{2/3}$.

In addition, Big Bang theory is incorrect as shown by Crothers.

My remedy to this confusion is to leave the results of paper 95 as:

3)

$$\rho_e = 4\pi \epsilon_0 \phi G \rho \quad \checkmark\checkmark \quad (C \; m^{-3}) \quad -(10)$$

$$J_r(\text{flat universe}) = \frac{A^{(0)}}{\mu_0}\left(\frac{4\pi G\rho}{a^2}\right) \quad \checkmark\checkmark \quad (C s^{-1} m^{-2}) \quad -(11)$$

$$J_r(\text{closed universe}) = \frac{A^{(0)}}{\mu_0}\left(4\pi G\rho\left(\frac{1-kr^2}{a^2}\right)\right) \quad -(12)$$

$$\boxed{k = 1.0 \; m^{-2}} \quad -(13)$$

$$r^2 J_\theta = r^2 \sin^2\theta \, J_\phi = \frac{A^{(0)}}{\mu_0}\left(\frac{4\pi G\rho}{a^2}\right) \quad -(14)$$

$$J_r(\text{open universe}) = \frac{A^{(0)}}{\mu_0}\left(4\pi G\rho\left(\frac{1-kr^2}{a^2}\right)\right) \quad -(15)$$

$$\boxed{k = -1.0 \; m^{-2}} \quad -(16)$$

$$r^2 J_\theta = r^2 \sin^2\theta \, J_\phi = \frac{A^{(0)}}{\mu_0}\left(\frac{4\pi G\rho}{a^2}\right) \quad -(17)$$

<u>Value of a^2</u>

My suggested remedy for plotting is to use in eq. (11):

$$a \propto t^{2/3} \quad -(18)$$

In eqs (12) and (14):

$$a \propto \left(\frac{1-\cos\phi}{\phi - \sin\phi}\right) t \quad -(19)$$

In eqns. (15) and (17):

$$a \propto \left(\frac{\cosh\phi - 1}{\sinh\phi - \phi}\right) t \quad -(20)$$