Appendix *A* – Tanh Solutions

```
<< "VectorAnalysis`"
SetCoordinates(Cartesian(x, y, z));
```

Calculate *A* from Equation 24

```
a1 = ax(x, y, z, t);
a2 = ay(x, y, z, t);
a3 = az(x, y, z, t);
```

- ### Equations 24

$$\text{eqn1} = \frac{\partial \text{a1}}{\partial z}\,\text{a2} - \frac{\partial \text{a1}}{\partial y}\,\text{a3};$$

$$\text{eqn2} = \frac{\partial \text{a2}}{\partial x}\,\text{a3} - \frac{\partial \text{a2}}{\partial z}\,\text{a1};$$

$$\text{eqn3} = \frac{\partial \text{a3}}{\partial y}\,\text{a1} - \frac{\partial \text{a3}}{\partial x}\,\text{a2};$$

eqn = {eqn1 == 0, eqn2 == 0, eqn3 == 0}

$$\{\text{ay}(x, y, z, t)\,\text{ax}^{(0,0,1,0)}(x, y, z, t) - \text{az}(x, y, z, t)\,\text{ax}^{(0,1,0,0)}(x, y, z, t) = 0,$$
$$\text{az}(x, y, z, t)\,\text{ay}^{(1,0,0,0)}(x, y, z, t) - \text{ax}(x, y, z, t)\,\text{ay}^{(0,0,1,0)}(x, y, z, t) = 0,$$
$$\text{ax}(x, y, z, t)\,\text{az}^{(0,1,0,0)}(x, y, z, t) - \text{ay}(x, y, z, t)\,\text{az}^{(1,0,0,0)}(x, y, z, t) = 0\}$$

■ Solution to Equations (note three solution sets) This is displaying triplets of {ax,ay,az}.

FullSimplify[DSolve[eqn, {a1, a2, a3}, {x, y, z, t}]];
α1 = a1 /. %
α2 = a2 /. %%
α3 = a3 /. %%%

$$\left\{\frac{c_1[t]\,(c_5[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,c_6[t])}{c_3[t]},\right.$$
$$\frac{c_1[t]\,(c_5[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,(c_6[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,c_7[t]))}{c_3[t]},$$
$$\frac{1}{c_3[t]} c_1[t]\,(c_5[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])$$
$$\left.(c_6[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,(c_7[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,c_8[t])))\right\}$$

$$\left\{\frac{c_2[t]\,(c_5[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,c_6[t])}{c_3[t]},\right.$$
$$\frac{c_2[t]\,(c_5[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,(c_6[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,c_7[t]))}{c_3[t]},$$
$$\frac{1}{c_3[t]} c_2[t]\,(c_5[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])$$
$$\left.(c_6[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,(c_7[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,c_8[t])))\right\}$$

$$\{c_5[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,c_6[t],$$
$$c_5[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,(c_6[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,c_7[t]),$$
$$c_5[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])$$
$$(c_6[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,(c_7[t] + \tanh(x\,c_1[t] + y\,c_2[t] + z\,c_3[t] + c_4[t])\,c_8[t]))\}$$

■ Directional Anisotropy in *A* (all solution sets have same degree on anisotropy)

$$\mathbf{FullSimplify}\left[\frac{\alpha 2}{\alpha 1}\right]$$
$$\mathbf{FullSimplify}\left[\frac{\alpha 3}{\alpha 1}\right]$$
$$\mathbf{FullSimplify}\left[\frac{\alpha 3}{\alpha 2}\right]$$

$$\left\{\frac{c_2[t]}{c_1[t]}, \frac{c_2[t]}{c_1[t]}, \frac{c_2[t]}{c_1[t]}\right\}$$

$$\left\{\frac{c_3[t]}{c_1[t]}, \frac{c_3[t]}{c_1[t]}, \frac{c_3[t]}{c_1[t]}\right\}$$

$$\left\{\frac{c_3[t]}{c_2[t]}, \frac{c_3[t]}{c_2[t]}, \frac{c_3[t]}{c_2[t]}\right\}$$

Calculating ω From Equations (21), (22), (23)

$$\omega 1 = \text{FullSimplify}\left[-\frac{\frac{\partial \alpha 1}{\partial y}}{\alpha 2}\right]$$

$$\omega 2 = \text{FullSimplify}\left[-\frac{\frac{\partial \alpha 2}{\partial z}}{\alpha 3}\right]$$

$$\omega 3 = \text{FullSimplify}\left[-\frac{\frac{\partial \alpha 3}{\partial x}}{\alpha 1}\right]$$

$$\left\{-\frac{\text{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_1[t]\,c_6[t]}{c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_6[t]},\right.$$
$$-\frac{\text{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_1[t]\,(c_6[t]+2\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t])}{c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t])},$$
$$-\left(\text{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_1[t]\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(2\,c_7[t]+3\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t]))\right)\Big/$$
$$\left.(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_7[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t])))\right\}$$

$$\left\{-\frac{\text{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_2[t]\,c_6[t]}{c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_6[t]},\right.$$
$$-\frac{\text{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_2[t]\,(c_6[t]+2\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t])}{c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t])},$$
$$-\left(\text{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_2[t]\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(2\,c_7[t]+3\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t]))\right)\Big/$$
$$\left.(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_7[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t])))\right\}$$

$$\left\{-\frac{\text{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_3[t]\,c_6[t]}{c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_6[t]},\right.$$
$$-\frac{\text{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_3[t]\,(c_6[t]+2\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t])}{c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t])},$$
$$-\left(\text{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_3[t]\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(2\,c_7[t]+3\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t]))\right)\Big/$$
$$\left.(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_7[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t])))\right\}$$

Solution Check on $\omega 1$

$$\omega 11 = \text{FullSimplify}\left[-\frac{\frac{\partial \alpha 1}{\partial z}}{\alpha 3}\right];$$

FullSimplify[ω11 − ω1]

{0, 0, 0}

Anisotropy in ω

$$\text{FullSimplify}\left[\frac{\omega 2}{\omega 1}\right]$$

$$\text{FullSimplify}\left[\frac{\omega 3}{\omega 1}\right]$$

$$\text{FullSimplify}\left[\frac{\omega 3}{\omega 2}\right]$$

$$\left\{\frac{c_2[t]}{c_1[t]}, \frac{c_2[t]}{c_1[t]}, \frac{c_2[t]}{c_1[t]}\right\}$$

$$\left\{\frac{c_3[t]}{c_1[t]}, \frac{c_3[t]}{c_1[t]}, \frac{c_3[t]}{c_1[t]}\right\}$$

$$\left\{\frac{c_3[t]}{c_2[t]}, \frac{c_3[t]}{c_2[t]}, \frac{c_3[t]}{c_2[t]}\right\}$$

Calculate phi using Equation 26

Null

$$\text{phi1} = \text{FullSimplify}\left[\text{DSolve}\left[\frac{\partial \text{phi}(x, y, z, t)}{\partial x} - \omega 1[[1]]\, \text{phi}(x, y, z, t) == 0, \text{phi}(x, y, z, t), \{x, y, z, t\}\right]\right][[1]][[1]]$$

$$\text{phi2} = \text{FullSimplify}\left[\text{DSolve}\left[\frac{\partial \text{phi}(x, y, z, t)}{\partial y} - \omega 2[[1]]\, \text{phi}(x, y, z, t) == 0, \text{phi}(x, y, z, t), \{x, y, z, t\}\right]\right][[1]][[1]]$$

$$\text{phi3} = \text{FullSimplify}\left[\text{DSolve}\left[\frac{\partial \text{phi}(x, y, z, t)}{\partial z} - \omega 3[[1]]\, \text{phi}(x, y, z, t) == 0, \text{phi}(x, y, z, t), \{x, y, z, t\}\right]\right][[1]][[1]]$$

$$\text{phif} = \text{phi3} /.\ c_7[x, y, t] \to \phi 0(t)$$

$$\text{phi}(x, y, z, t) \to \frac{\cosh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_7[y, z, t]}{\cosh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_5[t] + \sinh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_6[t]}$$

$$\text{phi}(x, y, z, t) \to \frac{\cosh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_7[x, z, t]}{\cosh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_5[t] + \sinh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_6[t]}$$

$$\text{phi}(x, y, z, t) \to \frac{\cosh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_7[x, y, t]}{\cosh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_5[t] + \sinh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_6[t]}$$

$$\text{phi}(x, y, z, t) \to \frac{\cosh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, \phi 0(t)}{\cosh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_5[t] + \sinh(x\, c_1[t] + y\, c_2[t] + z\, c_3[t] + c_4[t])\, c_6[t]}$$

Calculating $\omega 0$ using Equation 27

This calculation compares the differing solutions for $\omega 0$. It is interesting to note that the ratios $\frac{C[i]'[t]}{C[i][t]}$ are all equal for i=1,2,3

Two solutions are possible

(1) all the C[i][t] are equal or

(2) C[i][t] does not depend on time

$$\omega 01 = -\frac{\frac{\partial \alpha 1}{\partial t}}{\alpha 1}$$

$$\omega 02 = -\frac{\frac{\partial \alpha 2}{\partial t}}{\alpha 2};$$

$$\omega 03 = -\frac{\frac{\partial \alpha 3}{\partial t}}{\alpha 3};$$

Simplify[ω03 − ω02]
Simplify[ω02 − ω01]
Simplify[ω03 − ω01]

$$\Bigg\{\Bigg(c_3[t]\Bigg(-\frac{(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_6[t])\,c_1'(t)}{c_3[t]}+\frac{c_1[t]\,(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_6[t])\,c_3'(t)}{c_3[t]^2}-\frac{1}{c_3[t]}c_1[t]\left(c_6[t]\,(x\,c_1'(t)+y\,c_2'(t)+z\,c_3'(t)+c_4'(t))\,\mathrm{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])+c_5'(t)+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_6'(t)\right)\Bigg)\Bigg)\Big/(c_1[t]\,(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_6[t])),$$

$$\Bigg(c_3[t]\Bigg(-\frac{(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t]))\,c_1'(t)}{c_3[t]}+\frac{c_1[t]\,(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t]))\,c_3'(t)}{c_3[t]^2}-\frac{1}{c_3[t]}c_1[t]\Big((c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t])\,(x\,c_1'(t)+y\,c_2'(t)+z\,c_3'(t)+c_4'(t))\,\mathrm{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])+c_5'(t)+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\left(c_7[t]\,(x\,c_1'(t)+y\,c_2'(t)+z\,c_3'(t)+c_4'(t))\,\mathrm{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])+c_6'(t)+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7'(t)\right)\Big)\Bigg)\Bigg)\Big/$$
$$(c_1[t]\,(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_7[t]))),$$

$$\Bigg(c_3[t]\Bigg(-\frac{1}{c_3[t]}(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_7[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t])))\,c_1'(t)+\frac{1}{c_3[t]^2}c_1[t]\,(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_7[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t])))\,c_3'(t)-\frac{1}{c_3[t]}c_1[t]\Big((c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_7[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t]))\,(x\,c_1'(t)+y\,c_2'(t)+z\,c_3'(t)+c_4'(t))\,\mathrm{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])+c_5'(t)+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\Big((c_7[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t])\,(x\,c_1'(t)+y\,c_2'(t)+z\,c_3'(t)+c_4'(t))\,\mathrm{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])+c_6'(t)+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\left(c_8[t]\,(x\,c_1'(t)+y\,c_2'(t)+z\,c_3'(t)+c_4'(t))\,\mathrm{sech}^2(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])+c_7'(t)+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8'(t)\right)\Big)\Big)\Bigg)\Bigg)\Big/$$
$$(c_1[t]\,(c_5[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_6[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,(c_7[t]+\tanh(x\,c_1[t]+y\,c_2[t]+z\,c_3[t]+c_4[t])\,c_8[t]))))\Bigg\}$$

$$\left\{\frac{c_2'(t)}{c_2[t]}-\frac{c_3'(t)}{c_3[t]},\ \frac{c_2'(t)}{c_2[t]}-\frac{c_3'(t)}{c_3[t]},\ \frac{c_2'(t)}{c_2[t]}-\frac{c_3'(t)}{c_3[t]}\right\}$$

$$\left\{\frac{c_1'(t)}{c_1[t]}-\frac{c_2'(t)}{c_2[t]},\ \frac{c_1'(t)}{c_1[t]}-\frac{c_2'(t)}{c_2[t]},\ \frac{c_1'(t)}{c_1[t]}-\frac{c_2'(t)}{c_2[t]}\right\}$$

$$\left\{\frac{c_1'(t)}{c_1[t]}-\frac{c_3'(t)}{c_3[t]},\ \frac{c_1'(t)}{c_1[t]}-\frac{c_3'(t)}{c_3[t]},\ \frac{c_1'(t)}{c_1[t]}-\frac{c_3'(t)}{c_3[t]}\right\}$$

Appendix *B* – Solutions with General Function Series

This model presents a full solution for the background em problem using an unspecifed series form. It appears that and power series in a general wave-like function would satisfy the equations in general. Consistency checks have been preformed on variables that can be calculated in more than one way.

```
<< "VectorAnalysis`"
SetCoordinates(Cartesian(x, y, z));
```

Assumed Form for *A*

Assume that the wave number vector and frequency are constants.

(1) $\mathsf{A}=\sum_{n=\mathrm{mo}}^{m} d_n \left(f\left[k_x\, x + k_y\, y + k_z\, z - v\, t\right]\right)^n \{\frac{k_x}{k_z},\frac{k_y}{k_z},1\}$

mo = 0;
m = 4;
$f(x, y, z, t) = g(\mathrm{kx}\, x + \mathrm{ky}\, y + \mathrm{kz}\, z + t\,(-\beta))$;

$$\mathrm{ax}(x, y, z, t) = \sum_{n=\mathrm{mo}}^{m} \frac{d(n)\,\mathrm{kx}\, f(x, y, z, t)^n}{\mathrm{kz}};$$

$$\mathrm{ay}(x, y, z, t) = \sum_{n=\mathrm{mo}}^{m} \frac{d(n)\,\mathrm{ky}\, f(x, y, z, t)^n}{\mathrm{kz}};$$

$$\mathrm{az}(x, y, z, t) = \sum_{n=\mathrm{mo}}^{m} d(n)\, f(x, y, z, t)^n;$$

a1 = ax(x, y, z, t);
a2 = ay(x, y, z, t);
a3 = az(x, y, z, t);
a = {a1, a2, a3};

Equations 24 is satisfied

Null

Equation (24) is a three component equation given by

(2) $A_j \partial_k A_i = A_k \partial_i A_j$ where {i,j=1,2,3}in cyclic permutation.

$$\textbf{eqn1} = \text{Simplify}\left[\frac{\partial a1}{\partial z}\, a2 - \frac{\partial a1}{\partial y}\, a3 == 0\right]$$

$$\textbf{eqn2} = \text{Simplify}\left[\frac{\partial a2}{\partial x}\, a3 - \frac{\partial a2}{\partial z}\, a1 == 0\right]$$

$$\textbf{eqn3} = \text{Simplify}\left[\frac{\partial a3}{\partial y}\, a1 - \frac{\partial a3}{\partial x}\, a2 == 0\right]$$

True

True

True

▪ Calculating ω

ω is calculated using one part of each of the equations equations (21), (22), and (23)

$$\omega 1 = \text{FullSimplify}\left[-\frac{\frac{\partial a1}{\partial y}}{a2}\right];$$

$$\omega 2 = \text{FullSimplify}\left[-\frac{\frac{\partial a2}{\partial z}}{a3}\right];$$

$$\omega 3 = \text{FullSimplify}\left[-\frac{\frac{\partial a3}{\partial x}}{a1}\right];$$

$$\omega = \{\omega 1, \omega 2, \omega 3\}$$

$$\{-(\text{kx}\,(4\,d(4)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)^3 + 3\,d(3)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)^2 + 2\,d(2)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta) + d(1))\; g'(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)) / (d(0) + g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)\; (d(1) + g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)\,(d(2) + g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)\,(d(3) + d(4)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta))))),$$

$$-(\text{ky}\,(4\,d(4)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)^3 + 3\,d(3)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)^2 + 2\,d(2)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta) + d(1))\; g'(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)) / (d(0) + g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)\; (d(1) + g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)\,(d(2) + g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)\,(d(3) + d(4)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta))))),$$

$$-(\text{kz}\,(4\,d(4)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)^3 + 3\,d(3)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)^2 + 2\,d(2)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta) + d(1))\; g'(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)) / (d(0) + g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)\; (d(1) + g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)\,(d(2) + g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)\,(d(3) + d(4)\,g(\text{kx}\,x + \text{ky}\,y + \text{kz}\,z - t\,\beta)))))\}$$

▪ Solution Check on ω

ω is verified using the second part of each of the equations equations (21), (22), and (23)

$$\omega 11 = \text{Simplify}\left[-\frac{\frac{\partial a1}{\partial z}}{a3}\right];$$

$$\omega 22 = \text{Simplify}\left[-\frac{\frac{\partial a2}{\partial x}}{a1}\right];$$

$$\omega 33 = \text{Simplify}\left[-\frac{\frac{\partial a3}{\partial y}}{a2}\right];$$

Simplify[ω11 − ω1]
Simplify[ω22 − ω2]
Simplify[ω33 − ω3]

0

0

0

- **Calculating ωo from equation 27 with verification from three components of *A***

Null

ω_o is calculated for each component A_i and then checked for consistency. (3) $\omega_o = -\frac{\partial_t A_i}{A_i}$ for each i = 1, 2, 3

$$\omega 01 = \text{Simplify}\left[-\frac{\frac{\partial a1}{\partial t}}{a1}\right]$$

$$\omega 02 = \text{Simplify}\left[-\frac{\frac{\partial a2}{\partial t}}{a2}\right];$$

$$\omega 03 = \text{Simplify}\left[-\frac{\frac{\partial a3}{\partial t}}{a3}\right];$$

Simplify[ω03 − ω02]
Simplify[ω02 − ω01]

$$\Big(\beta \big(4\, d(4)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\,\beta)^3 + 3\, d(3)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\,\beta)^2 + 2\, d(2)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\,\beta) + d(1)\big)\, g'(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\,\beta)\Big) \Big/ \Big(d(4)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\,\beta)^4 + d(3)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\,\beta)^3 + d(2)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\,\beta)^2 + d(1)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\,\beta) + d(0)\Big)$$

0

0

Null

The calculation of the scalar potential ϕ is also redundant. It is given in general by equation (26) repeated here

(4) $\underline{\nabla}\phi$-$\omega\phi$=0

This produces three equations with a different constant of integration for each one. The solutions for ϕ are equated, resulting a a single constant of integration which may be a function of time. Once this is done, the solution is self-consistent.

$$\text{eqn4} = \text{Simplify}\left[\frac{\partial \text{phi}(x, y, z, t)}{\partial x} - \omega 1\, \text{phi}(x, y, z, t)\right];$$

Simplify[DSolve[eqn4 == 0, phi(x, y, z, t), {x, y, z, t}]][[1]][[1]];
p1 = phi(x, y, z, t) /. %;
pp1 = p1 /. c_1[y, z, t] → ϕ0(t)

$$\text{eqn5} = \text{Simplify}\left[\frac{\partial \text{phi2}(x, y, z, t)}{\partial y} - \omega 2\, \text{phi2}(x, y, z, t)\right];$$

Simplify[DSolve[eqn5 == 0, phi2(x, y, z, t), {x, y, z, t}]][[1]][[1]];
p2 = phi2(x, y, z, t) /. %;
pp2 = Simplify[p2 /. c_1[x, z, t] → ϕ0(t)];

$$\text{eqn6} = \text{Simplify}\left[\frac{\partial \text{phi3}(x, y, z, t)}{\partial z} - \omega 3\, \text{phi3}(x, y, z, t)\right];$$

Simplify[DSolve[eqn6 == 0, phi3(x, y, z, t), {x, y, z, t}]][[1]][[1]];
p3 = phi3(x, y, z, t) /. %;
pp3 = p3 /. c_1[x, y, t] → ϕ0(t);
Simplify[pp2 − pp1]
Simplify[pp3 − pp2]

$$\phi 0(t) \Big/ \big(d(4)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta)^4 + d(3)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta)^3 + d(2)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta)^2 + d(1)\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta) + d(0)\big)$$

0

0

Appendix C - Energy Calculation

Abstract

This model presents an energy density calculaaaation for the background em problem using a travelling wave form for the magnetic vector potential A. Constant values for the wave number and frequency have been assumed.

The solution for the travelling wave assumption for the magnetic vector potential is given by the following equations.

$$\mathrm{A} = \left\{\frac{\mathrm{kx}\, g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}{\mathrm{kz}},\ \frac{\mathrm{ky}\, g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}{\mathrm{kz}},\ g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)\right\}$$

$$\phi = \frac{\phi_0}{g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}$$

$$\omega = \left\{-\frac{\mathrm{kx}\, g'(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}{g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)},\ -\frac{\mathrm{ky}\, g'(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}{g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)},\ -\frac{\mathrm{kz}\, g'(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}{g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}\right\}$$

$$\omega_0 = -\frac{\beta\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}{g[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}$$

$$u = \frac{c^2\left(\left(\mathrm{kx}^4+\left(\mathrm{ky}^2+\mathrm{kz}^2\right)\mathrm{kx}^2+\mathrm{ky}^4+\mathrm{kz}^4+\mathrm{ky}^2\,\mathrm{kz}^2\right)c^2+\left(\mathrm{kx}^2+\mathrm{ky}^2+\mathrm{kz}^2\right)\beta^2\right)g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)^4\, g'(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)^2-\mathrm{kz}^2\left(\left(\mathrm{kx}^2+\mathrm{ky}^2+\mathrm{kz}^2\right)c^2+\beta^2\right)\phi_0^2\, g'(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)^2}{c^4\,\mathrm{kz}^2\, g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)^4\,\mu_0}$$

$$s = \left\{-\frac{\mathrm{kx}\left(\mathrm{kx}^2+\mathrm{ky}^2+\mathrm{kz}^2\right)\beta\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]^2}{\mathrm{kz}^2\,\mu_0}+\frac{\mathrm{kx}\,\beta\,\phi_0^2\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]^2}{c^2\, g[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]^4\,\mu_0},\right.$$
$$-\frac{\mathrm{ky}\left(\mathrm{kx}^2+\mathrm{ky}^2+\mathrm{kz}^2\right)\beta\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]^2}{\mathrm{kz}^2\,\mu_0}+\frac{\mathrm{ky}\,\beta\,\phi_0^2\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]^2}{c^2\, g[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]^4\,\mu_0},\ \left.-\frac{\left(\mathrm{kx}^2+\mathrm{ky}^2+\mathrm{kz}^2\right)\beta\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]^2}{\mathrm{kz}\,\mu_0}+\frac{\mathrm{kz}\,\beta\,\phi_0^2\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]^2}{c^2\, g[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]^4\,\mu_0}\right\}$$

In[1]:=

```
PageBreakBelow;
```

Calculation of Potentials

In[2]:=

```
<< VectorAnalysis`
SetCoordinates[Cartesian[x, y, z]];
```

■ Assumed Form for A

In[4]:=

```
ax[x, y, z, t] = kx / kz * g[kx * x + ky * y + kz * z - β * t];
ay[x, y, z, t] = ky / kz * g[kx * x + ky * y + kz * z - β * t];
az[x, y, z, t] = g[kx * x + ky * y + kz * z - β * t];

a1 = ax[x, y, z, t];
a2 = ay[x, y, z, t];
a3 = az[x, y, z, t];
a = {a1, a2, a3}
j = {jx[z, y, z, t], jy[x, y, z, t], jz[x, y, z, t]};

eqn1 = D[a1, z] * a2 - D[a1, y] * a3;
eqn2 = D[a2, x] * a3 - D[a2, z] * a1;
eqn3 = D[a3, y] * a1 - D[a3, x] * a2;
eqn = {eqn1 == 0, eqn2 == 0, eqn3 == 0}
```

Out[10]= $\left\{\frac{\text{kx}\, g[\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta]}{\text{kz}}, \frac{\text{ky}\, g[\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta]}{\text{kz}}, g[\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta]\right\}$

Out[15]= {True, True, True}

The magnetic vector potential is given by the following; this expression satisfies equations 24

$$\text{A} = \left\{\frac{\text{kx}\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta)}{\text{kz}}, \frac{\text{ky}\, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta)}{\text{kz}}, g(\text{kx}\, x + \text{ky}\, y + \text{kz}\, z - t\, \beta)\right\}$$

{True, True, True}

■ Calculation of ϕ

By combining equations (21) and (25) we get the following equations

$$\frac{\phi(x,y,z,t)\,\frac{\partial \text{a1}}{\partial z}}{\text{a3}} + \frac{\partial\phi(x,y,z,t)}{\partial x}=0 \qquad \text{or}$$

$$\frac{\partial\phi(x,y,z,t)}{\partial x} + \frac{\frac{\partial \text{a1}}{\partial y}\,\phi(x,y,z,t)}{\text{a2}}=0$$

which easily solves to

$$\phi(x,\ y,\ z,\ t) = \frac{\phi_0}{g(\text{kx}\ x+\text{ky}\ y+\text{kz}\ z-t\ \beta)}$$

In[16]:=
```
eqnphi = D[p[x, y, z, t], x] + (D[a1, z] * p[x, y, z, t]) / a3;
FullSimplify[DSolve[eqnphi == 0, p[x, y, z, t], {x, y, z, t}]] /.
 C[1][y, z, t] -> Subscript[ϕ, 0]
eqnphia = D[pa[x, y, z, t], x] + (D[a1, y] * pa[x, y, z, t]) / a2;
FullSimplify[DSolve[eqnphia == 0, pa[x, y, z, t], {x, y, z, t}]] /.
 C[1][y, z, t] -> Subscript[ϕ, 0]
```

Out[17]= $\left\{\left\{\text{p}[x,\ y,\ z,\ t] \to \frac{\phi_0}{g[\text{kx}\ x + \text{ky}\ y + \text{kz}\ z - t\ \beta]}\right\}\right\}$

Out[19]= $\left\{\left\{\text{pa}[x,\ y,\ z,\ t] \to \frac{\phi_0}{g[\text{kx}\ x + \text{ky}\ y + \text{kz}\ z - t\ \beta]}\right\}\right\}$

ϕ can be generated from two similar equations by using equations (22) and (23) instead of (21). This verifies the consistency of the calculaion.

In[20]:=
```
eqnphi2 = p1[x, y, z, t] * D[a2, x] / a1 + D[p1[x, y, z, t], y];
FullSimplify[DSolve[eqnphi2 == 0, p1[x, y, z, t], {x, y, z, t}]] /.
 C[1][x, z, t] -> Subscript[ϕ, 0]
eqnphi3 = p2[x, y, z, t] * D[a3, x] / a1 + D[p2[x, y, z, t], z];
FullSimplify[DSolve[eqnphi3 == 0, p2[x, y, z, t], {x, y, z, t}]] /.
 C[1][x, y,  t] -> Subscript[ϕ, 0]
```

Out[21]= $\left\{\left\{\text{p1}[x,\ y,\ z,\ t] \to \frac{\phi_0}{g[\text{kx}\ x + \text{ky}\ y + \text{kz}\ z - t\ \beta]}\right\}\right\}$

Out[23]= $\left\{\left\{\text{p2}[x,\ y,\ z,\ t] \to \frac{\phi_0}{g[\text{kx}\ x + \text{ky}\ y + \text{kz}\ z - t\ \beta]}\right\}\right\}$

In[24]:=

$$\phi[x,\ y,\ z,\ t] = \frac{\phi_0}{g(\text{kx}\ x + \text{ky}\ y + \text{kz}\ z - t\ \beta)};$$

Calculating ω from equations (21),(22), (23)

In[25]:=
```
ω1 = -kx * g'[kx * x + ky * y + kz * z - β * t] / g[kx x + ky y + kz z - t β];
ω2 = -ky * g'[kx * x + ky * y + kz * z - β * t] / g[kx x + ky y + kz z - t β];
ω3 = -kz * g'[kx * x + ky * y + kz * z - β * t] / g[kx x + ky y + kz z - t β];
ω = {ω1, ω2, ω3}
ω1 - FullSimplify[-D[a1, y] / a2];
ω2 - FullSimplify[-D[a2, z] / a3];
ω3 - FullSimplify[-D[a3, x] / a1];
{%%%, %%, %}
```

Out[28]= $\left\{-\frac{\mathrm{kx}\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}{g[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}, -\frac{\mathrm{ky}\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}{g[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}, -\frac{\mathrm{kz}\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}{g[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}\right\}$

Out[32]= {0, 0, 0}

The vector portion of the spin connection is given by the following, and shown to satisfy equations 21, 22, 23

$$\omega = \left\{-\frac{\mathrm{kx}\, g'(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}{g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}, -\frac{\mathrm{ky}\, g'(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}{g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}, -\frac{\mathrm{kz}\, g'(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}{g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta)}\right\}$$

{0, 0, 0}

Calculating ω_0 from equation (27) with verification from three components of A note alternate for for ω_0

In[33]:=
```
ω01 = Simplify[-β * g'[kx * x + ky * y + kz * z - β * t] / g[kx x + ky y + kz z - t β]];
ω02 = D[Log[g[kx x + ky y + kz z - t β]], t];
ω01 == ω02

ω0 = ω01
```

Out[35]= True

Out[36]= $-\frac{\beta\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}{g[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}$

The scalar component of the spin connection is given by the following expression.

$$\omega_0 = -\frac{\beta\, g'[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}{g[\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta]}$$

An alternate expression for this is

$$\omega_0 = \frac{\partial \log(g(\mathrm{kx}\, x+\mathrm{ky}\, y+\mathrm{kz}\, z-t\,\beta))}{\partial t}$$

An Observed Relationship between ω and ω_0

```
In[37]:= Simplify[{ω1 / ω01, ω2 / ω01, ω3 / ω01}]
```

Out[37]= $\left\{\frac{kx}{\beta}, \frac{ky}{\beta}, \frac{kz}{\beta}\right\}$

Note that the ratio of the vector spin connection to the scalar spin connection is the wave velocity

$$\frac{\omega_i}{\omega_0} = \left\{\frac{kx}{\beta}, \frac{ky}{\beta}, \frac{kz}{\beta}\right\}$$

Energy Density Calculation

Puthoff Energy Term

```
In[38]:=
gradaisquared =
  Simplify[Dot[Grad[a1], Grad[a1]] + Dot[Grad[a2], Grad[a2]] + Dot[Grad[a3], Grad[a3]]];
daidtsquared = Simplify[Dot[D[a, t], D[a, t]]];
ua = Simplify[(1 / (2 μ0)) (gradaisquared + daidtsquared / c^2)];
```

$$u_a = \frac{\left(kx^2 + ky^2 + kz^2\right) \left(kx^2 + ky^2 + kz^2 + \frac{\beta^2}{c^2}\right) g'[kx\, x + ky\, y + kz\, z - t\, \beta]^2}{2\, kz^2\, \mu_0}$$

```
In[41]:=
ub1 = Simplify[ (D[ϕ[x, y, z, t], t])^2];
ub2 = Simplify[Dot[Grad[ϕ[x, y, z, t]], Grad[ϕ[x, y, z, t]]]];
uϕ = Simplify[(1 / (2 * μ0 * c^2) * (ub1 / c^2 + ub2))];
```

$$u_\phi = \frac{\left(c^2 \left(kx^2 + ky^2 + kz^2\right) + \beta^2\right) \phi_0^2\, g'[kx\, x + ky\, y + kz\, z - t\, \beta]^2}{2\, c^4\, g[kx\, x + ky\, y + kz\, z - t\, \beta]^4\, \mu_0}$$

The energy density from the Puthoff article is given by

```
In[44]:= energydensity = Simplify[ua - uϕ + ρ * ϕ[x, y, z, t]];
```

$$u_{\text{Puthoff}} = \frac{\rho\, \phi_0}{g[kx\, x + ky\, y + kz\, z - t\, \beta]} + \frac{\left(kx^2 + ky^2 + kz^2\right) \left(kx^2 + ky^2 + kz^2 + \frac{\beta^2}{c^2}\right) g'[kx\, x + ky\, y + kz\, z - t\, \beta]^2}{2\, kz^2\, \mu_0} - \frac{\left(c^2 \left(kx^2 + ky^2 + kz^2\right) + \beta^2\right) \phi_0^2\, g'[kx\, x + ky\, y + kz\, z - t\, \beta]^2}{2\, c^4\, g[kx\, x + ky\, y + kz\, z - t\, \beta]^4\, \mu_0}$$

Spin Energy

In[45]:=
```
t1 = Dot[ω0 a, ω0 a];
t2 = (ω1 * a1)^2 + (ω2 * a2)^2 + (ω3 * a3)^2;
uaece = Simplify[(1 / (2 μ0)) * (t1 / c^2 + t2)];
```

$$u_{A\ ECE} = \frac{\left(c^2\left(kx^4+ky^4+kz^4\right)+\left(kx^2+ky^2+kz^2\right)\beta^2\right) g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{2\,c^2\,kz^2\,\mu_0}$$

In[48]:=
```
t3 = (ω0 ϕ[x, y, z, t])^2;
t4 = (ω1 * ϕ[x, y, z, t])^2 + (ω2 * ϕ[x, y, z, t])^2 + (ω3 * ϕ[x, y, z, t])^2;
upece = (1 / (2 * μ0 * c^2)) * Simplify[t3 / c^2 + t4];
```

$$u_{\phi\ ECE} = \frac{\left(c^2\left(kx^2+ky^2+kz^2\right)+\beta^2\right)\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{2\,c^4\,g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0}$$

Total Energy Density with ρ being Zero

In[51]:=
```
u = FullSimplify[energydensity + uaece - upece] /. ρ → 0;
```

In[52]:=

$$u = \left(c^2\left(\left(kx^4+\left(ky^2+kz^2\right)kx^2+ky^4+kz^4+ky^2\,kz^2\right)c^2+\left(kx^2+ky^2+kz^2\right)\beta^2\right) g(kx\,x+ky\,y+kz\,z-t\,\beta)^4\, g'(kx\,x+ky\,y+kz\,z-t\,\beta)^2 - kz^2\left(\left(\left(kx^2+ky^2+kz^2\right)c^2+\beta^2\right)\phi_0^2\, g'(kx\,x+ky\,y+kz\,z-t\,\beta)^2\right)\Big/\left(c^4\,kz^2\,g(kx\,x+ky\,y+kz\,z-t\,\beta)^4\,\mu_0\right)$$

Power Flux

In[53]:=
```
sA = Simplify[(-1 / μ0) (D[a1, t] Grad[a1] + D[a2, t]  Grad[a2] + D[a3, t]  Grad[a3])]
```

Out[53]=

$$\left\{\frac{kx\left(kx^2+ky^2+kz^2\right)\beta\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{kz^2\,\mu_0},\ \frac{ky\left(kx^2+ky^2+kz^2\right)\beta\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{kz^2\,\mu_0},\ \frac{\left(kx^2+ky^2+kz^2\right)\beta\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{kz\,\mu_0}\right\}$$

In[54]:=
```
sϕ = Simplify[-(1 / (μ0 * c^2)) D[ϕ[x, y, z, t], t] Grad[ϕ[x, y, z, t]]]
```

Out[54]=

$$\left\{\frac{kx\,\beta\,\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{c^2\,g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0},\ \frac{ky\,\beta\,\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{c^2\,g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0},\ \frac{kz\,\beta\,\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{c^2\,g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0}\right\}$$

In[55]:=

```
S_A ECE = Simplify[-(1 / μ0) ω0 ( a1^2 + a2^2 + a3^2) ω]
```

Out[55]=

$$\left\{-\frac{kx\left(kx^2+ky^2+kz^2\right)\beta\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{kz^2\,\mu_0},\right.$$
$$\left.-\frac{ky\left(kx^2+ky^2+kz^2\right)\beta\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{kz^2\,\mu_0},\ -\frac{\left(kx^2+ky^2+kz^2\right)\beta\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{kz\,\mu_0}\right\}$$

In[56]:=

```
S_ϕ ECE = Simplify[-(1 / (μ0 * c^2)) ( ω0 ω ϕ[x, y, z, t]^2 )]
```

Out[56]=

$$\left\{-\frac{kx\,\beta\,\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{c^2\, g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0},\right.$$
$$\left.-\frac{ky\,\beta\,\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{c^2\, g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0},\ -\frac{kz\,\beta\,\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{c^2\, g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0}\right\}$$

In[57]:=

```
S_TOTAL = Simplify[%%% + %% - %%% - % + ϕ[x, y, z, t] j] /. jx[z, y, z, t] → 0 /. jy[x, y, z, t] → 0 /.
   jz[x, y, z, t] → 0
```

Out[57]=

$$\left\{-\frac{kx\left(kx^2+ky^2+kz^2\right)\beta\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{kz^2\,\mu_0}+\frac{kx\,\beta\,\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{c^2\, g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0},\right.$$
$$-\frac{ky\left(kx^2+ky^2+kz^2\right)\beta\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{kz^2\,\mu_0}+\frac{ky\,\beta\,\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{c^2\, g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0},$$
$$\left.-\frac{\left(kx^2+ky^2+kz^2\right)\beta\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{kz\,\mu_0}+\frac{kz\,\beta\,\phi_0^2\, g'[kx\,x+ky\,y+kz\,z-t\,\beta]^2}{c^2\, g[kx\,x+ky\,y+kz\,z-t\,\beta]^4\,\mu_0}\right\}$$

In[58]:=

```
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```