On Certain Conceptual Anomalies in Einstein's Theory of Relativity

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There are a number of conceptual anomalies occurring in the Standard exposition of Einstein's Theory of Relativity. These anomalies relate to issues in both mathematics and in physics and penetrate to the very heart of Einstein's theory. This paper reveals and amplifies a few such anomalies, including the fact that Einstein's field equations for the so-called static vacuum configuration, $R_{\mu\nu} = 0$, violates his Principle of Equivalence, and is therefore erroneous. This has a direct bearing on the usual concept of conservation of energy for the gravitational field and the conventional formulation for localisation of energy using Einstein's pseudo-tensor. Misconceptions as to the relationship between Minkowski spacetime and Special Relativity are also discussed, along with their relationships to the pseudo-Riemannian metric manifold of Einstein's gravitational field, and their fundamental geometric structures pertaining to spherical symmetry.

Introduction

In a series of papers [1-17] I have previously provided mathematical demonstrations of the invalidity of the concept of the black hole and also of the expansion of the Universe with a Big Bang cosmology. In those papers I took on face value the fundamental line-elements from which these physical concepts have allegedly been derived by the Standard Model relativists, and demonstrated in purely mathematical terms that they are inconsistent with the geometrical structure of those line-elements, and are therefore false. I do not reiterate those demonstrations herein, referring the reader to the relevant papers for the details, and instead consider, in the main, various conceptual matters underlying the structure of Einstein's Theory of Relativity, and show that there are some very serious anomalies in the usual exposition, which render much of what has been claimed for General Relativity to be false.

Misconception: That Ricc $= 0$ fully describes the gravitational field

Setting $R_{\mu\nu} = 0$ imposes upon an observer in the alleged gravitational field, a consideration of the perceived source of the field in terms of its *centre of mass*, and so $g_{00} = 0$ is not a physically meaningful condition. In other words, the notion of gravitational collapse to a point-mass is not justified: it is ill-posed. A centre of mass is not a physical object, only a mathematical artifice. This same artifice occurs in Newton's theory as well, and in Newton's theory it is not a physical object either, and

nobody, quite rightly, considers it a physical object in Newton's universe. Oddly, the centre of mass is taken, by unconscious assumption or blind conviction, to be a real object in Einstein's theory. Gravitational collapse is a conceptual anomaly in General Relativity that has no basis in the physical world or in General Relativity. It is built upon a false idea as a result of not realising that $R_{\mu\nu} = 0$ imposes consideration of the perceived source of the alleged gravitational field in terms of its centre of mass only, and so can say *absolutely nothing* about the size or mass of the source of the field.

In view of the foregoing, a single line-element is insufficient for the full description of the gravitational field of an object such as a star. One needs two line-elements: one for the interior of the object and one for the region outside it. These line-elements, although different, are not disjoint, being coupled by quantities that are determined from the line-element for the interior of the star and by a common Gaussian curvature at the surface boundary of the object, as the study by Schwarzschild [18] (and my generalisation thereof [5]) for the ideal case of a homogeneous incompressible sphere of fluid teaches us. In this ideal case it is shown that there is an upper limit and a lower limit on the size of the sphere, beyond which it cannot exist. Newton's theory also requires a different equation to describe the field inside an object such as a star, to that equation describing the field outside it in terms of its centre of mass. No limitations are imposed on the size of an object according to Newton's theory because there is no limitation on the speed of an object in Newton's mechanics.

Misconception: That General Relativity permits point-masses

Point-masses are meaningless [11] - the notion is an oxymoron, a confounding of mathematical concepts with physical concepts. Furthermore, Special Relativity forbids the existence of infinite densities because infinite densities require infinite energies, which are forbidden by Special Relativity. Thus, if point-masses are permitted by General Relativity, it does so in violation of Special Relativity, and so it is not consistent. Thus, General Relativity also forbids point-masses and hence irresistible gravitational collapse to a point-mass. This is amplified further in the next section.

That point-masses are not permitted by General Relativity has also been demonstrated by Schwarzschild [18, 19], Brillouin [20], Abrams [21, 22, 23, 24], Stavroulakis [25, 26, 27, 28, 29].

Misconception: That Ricc $=0$ is admissible

$R_{\mu\nu} = 0$ is inconsistent with the physical foundations of General Relativity as adduced by Einstein in that it violates Einstein's Principle of Equivalence, and so writing $R_{\mu\nu} = 0$ is erroneous in the first place. The motive to writing $R_{\mu\nu} = 0$ is due to conceptual anomaly. First, $R_{\mu\nu} = 0$ does not generalise Special Relativity but only Minkowski space. That is, $R_{\mu\nu} = 0$ generalises the pseudo-Efcleethean* geometry of Minkowski space into a pseudo-Riemannian geometry. Since $R_{\mu\nu} = 0$ imposes the centre of mass configuration on the perceived source of the field, the source of the field is not in the field (the line-element is undefined at the centre of mass). Since $R_{\mu\nu} = 0$ excludes by definition all masses and energy, the resulting curvature of spacetime has only kinematic properties. One cannot say that a material object follows a timelike geodesic in the field of $R_{\mu\nu} = 0$ because one cannot introduce any material object into that field. One cannot say that light follows a null geodesic in the field of $R_{\mu\nu} = 0$ because one cannot introduce energy into the field of $R_{\mu\nu} = 0$, and photons carry energy (if not also mass). One can only say that points travelling at the speed c of light in vacuo, in the spacetime of $R_{\mu\nu} = 0$, follow a null geodesic and one can only say that other points that move with a speed less than c follow timelike geodesics and that no points can move along a spacelike path. Time dilation and length contraction are kinematic effects of Minkowski space, which is a geometry in which points cannot move with a speed greater than c, by definition. The physical nature of light does not play a part in Minkowski geometry. The dynamics of Special Relativity are assumed to take place in Minkowski space, just as Newton's dynamics are assumed to take place in Efcleethean 3-Space. Thus, it is assumed that masses can simply be inserted into Minkowski space, just as masses are assumed to be able to be inserted into Efcleethean 3-Space for Newton's dynamics. (This is not the case in General Relativity, wherein mass, energy and spacetime interact, one upon the other.) Then with the assumption that masses can be inserted into Minkowski space, the dynamics of Special Relativity are developed, subject to the kinematic nature of Minkowski space with its limitation on the upper speed of a point therein, and with the assignation of a point moving with speed c to a photon. The dynamics of Special Relativity are the result of the kinematics of Minkowski space (i.e. the mere geometry thereof) imposed upon masses inserted into Minkowski space and attached to moving points so that the distinction between point and mass is lost by subsuming mass into a centre of mass (a mathematical point). On the Principle of Equivalence, according to Einstein [30],

> "Let now K be an inertial system. Masses which are sufficiently far from each other and from other bodies are then, with respect to K, free from acceleration. We shall also refer these masses to a system of co-ordinates K', uniformly accelerated with respect to K. Relatively to K' all the masses have equal and parallel accelerations; with respect to K' they behave just as if a gravitational field were present and K' were accelerated. Overlooking for the present the question as to the 'cause' of such a gravitational field, which will occupy us later, there is nothing to prevent our conceiving this gravitational field as real, that is, the conception that K' is 'at rest' and a gravitational field is present we may consider as equivalent to the conception that only K is an 'allowable' system of co-ordinates and no gravitational field is present. The assumption of the complete physical equivalence of the systems of co-ordinates, K and K', we call the 'principle of equivalence'; this principle is evidently intimately connected with the law of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relatively to each other. In fact, through this conception we arrive at the unity of the nature of inertia and gravitation."

Also, according to Einstein [30],

> "Stated more exactly, there are finite regions, where, with respect to a suitably chosen space

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*For the geometry due to Efcleethees, usually and abominably rendered as Euclid.

of reference, material particles move freely without acceleration, and in which the laws of special relativity, which have been developed above, hold with remarkable accuracy."

However, $R_{\mu\nu} = 0$ does not generalise Special Relativity, only the geometry of Minkowski space. The source of the field, as a centre of mass, is not in the field of $R_{\mu\nu} = 0$. No masses or energy can be arbitrarily inserted into the spacetime of $R_{\mu\nu} = 0$. Thus, $R_{\mu\nu} = 0$ violates Einstein's Principle of Equivalence. Furthermore, one cannot assign the value of the constant appearing in the Schwarzschild line-element to the Newtonian potential in the infinitely far field because Schwarzschild space is asymptotically Minkowski space, not asymptotically Special Relativity and not asymptotically Newtonian dynamics. And in Newton's theory, the potential is defined as the work per unit mass, on a mass that can, in principle, be inserted into the gravitational field of another mass. One cannot insert any masses, by definition, into the field of $R_{\mu\nu} = 0$. The infinitely far field of $R_{\mu\nu} = 0$ does not become Newtonian - it becomes Minkowski space only. Newton's law of gravitation is based *a priori* on the interaction of two masses; Einstein's theory of gravitation is not. The claim that the constant in the Schwarzschild solution can be associated with the infinitely far field Newtonian potential was never made by Schwarzschild, because he clearly knew this cannot be done. He only stated in his 1st paper on the subject [19] that the constant was to be physically interpreted as some function of the mass. That function cannot be ascertained from the line-element for $R_{\mu\nu} = 0$. The value of the constant was determined by Schwarzschild in his 2nd paper [18], on the sphere of homogeneous incompressible fluid. In that paper it is obtained that the constant is determined from the interior line-element, where the energy-momentum tensor is not zero, not from the alleged field for $R_{\mu\nu} = 0$, and with it the fact that there are two non-Newtonian masses, the active and the passive mass respectively, both from the interior line-element.

With a line-element for $R_{\mu\nu} = 0$ alone, one can only rightly say that the geometry is modified from that of Minkowski space, by the presence of a non-zero constant. When that constant is zero, Minkowski space is recovered, and with that recovery of Minkowski space, one can again arbitrarily insert masses and energies and develop the dynamics of Special Relativity. It does not follow, that with the setting of the constant to zero, that the pseudo-Riemannian metric manifold of $R_{\mu\nu} = 0$ collapses into Special Relativity. Special Relativity is merely an augmentation to Minkowski space by the arbitrary insertion of mass and energy into Minkowski space with the constrained kinematic features of Minkowski space

applied to those masses and energies. The collapse of $R_{\mu\nu} = 0$ into Minkowski space takes with it only a geometry and hence only a system of kinematics, not a system of dynamics.

Still, the writing of $R_{\mu\nu} = 0$ outside the source is erroneous, even though in the footsteps of Einstein, who claimed $R_{\mu\nu} = 0$ for a mass island. Schwarzschild only did as I have done - taken Einstein at his word. However, in writing $R_{\mu\nu} = 0^*$, Einstein has violated his own theory, by violating his Principle of Equivalence .

This does not invalidate the detailed analysis by Schwarzschild [18, 19], Brillouin [20], Abrams [21, 22, 23, 24], and myself [1-17], since those works are based upon the implication, *if $R_{\mu\nu} = 0$ outside the source of the field then certain things follow* (but no black holes are possible). The validity of $R_{\mu\nu} = 0$ is entirely another question. Now, since $R_{\mu\nu} = 0$ violates Einstein's Principle of Equivalence, it is erroneous. This invalidates the black hole from an even deeper level, and much more besides.

Misconception: That the quantity r in the Schwarzschild metric is not the radius of curvature

Recall that the so-called "Schwarzschild" line-element (which is in fact not Schwarzschild's line-element [19]), is

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$
(1)

wherein it is alleged that r is the radius, and that r can go down to zero. The Standard Model relativists erroneously claim that $\alpha = 2m$, by means of a far field comparison with the Newtonian potential. When pressed for an explanation of what they mean by r being the radius, the Standard Model relativists give (depending on which Standard Model relativist one asks) various vague definitions. Their vague definitions all repose in mere jargon, either in attempts to mask conceptual confusion or in ignorance. That the Standard Model relativists call $r = 2m$ in the foregoing line-element the "Schwarzschild radius" testifies to what they think r signifies, particularly given the fact that they also claim that the "Schwarzschild radius" is able to be deduced from Newton's theory [11]. Yet not a single proponent of the Standard Model has correctly identified the quantity r appearing in expression (1). According to Taylor and Wheeler [32], r is the "reduced circumference", since the great circumference C associated with (1) is given by

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*Coincidently, that $R_{\mu\nu} = 0$ is inadmissible was realised independently and at about the same time as the Author, by Dr. M. W. Evans [31], via a different line of thought - by using ECE theory.

$C = 2\pi r$. In fact, this quantity is calculated from (1) by

$$C = \int_0^{2\pi} r \sin \frac{\pi}{2} d\varphi = 2\pi r.$$

Other relativists call r in (1) the "areal radius", apparently because the area A of a spherical surface according to (1) is $A = 4\pi r^2$. This quantity is actually calculated from (1) by

$$A = \int_0^{2\pi} \left(\int_0^{\pi} r^2 \sin\theta d\theta \right) d\varphi = 4\pi r^2.$$

In my previous papers [1-17] I correctly referred to the quantity r in (1) as the *radius of curvature*, and demonstrated that in (1), $\alpha < r < \infty$. This is because the quantity r is in actual fact related directly to the Gaussian curvature of the spherical surface for some fixed value of r. The quantity r does not determine the geodesic radial distance (the proper radius) from the centre of spherical symmetry to the surface. The proper radius does not determine the great circumference or the surface area of a spherical surface, but it plays a rôle in the determination of the volume of the non-Efcleethean sphere defined on (1), by means of a straightforward triple integral. The proper radius R_p associated with (1) is given by

$$R_p = \int_\alpha^r \sqrt{\frac{r}{r-\alpha}} \; dr$$

$$= \sqrt{r(r-\alpha)} + \alpha \ln \left(\frac{\sqrt{r} + \sqrt{r-\alpha}}{\sqrt{\alpha}} \right).$$

Clearly the proper radius and the radius of curvature (Gaussian) are not the same. They approach each other asymptotically as $r \to \infty$, and are equal when $r \approx 1.467\alpha$. When $r > 1.467\alpha$, $R_p > r$, and when $r < 1.467\alpha$, $R_p < r$, so that as $r \to \alpha^+$, $r/R_p \to \infty$ [15].

In all my previous papers, except [17], I did not provide any mathematical proof that r in (1) is the radius of curvature (Gaussian), because I incorrectly assumed that the Standard Model relativists knew sufficient differential geometry to know what I was talking about. That assumption has proved quite erroneous, as I have received various derisive emails or other derisive criticisms from disgruntled Standard Model physicists, telling me, amongst other unseemly things, that r in (1) is an "areal radius", not a radius of curvature, and that the proper radius is "some kind of distance" from the surface described by $r = \alpha$ and that, according to R. P. Kerr, my arguments are "rubbish" [33]. Concerning r in (1), G. 't Hooft [34] says it is, *"... a gauge choice: it defines the coordinate r"*, and *"In the community of real physicists, the number $r = 2M$ (if $G = 1$)*

is conventionally called the Schwarzschild radius associated to the mass M (or the energy Mc^2), nothing deeper than that". Now "some kind of distance" is hardly a meaningful definition, that an arbitrary choice ("gauge" or otherwise) defines the variable r in (1) is not valid geometry, and "rubbish" is not a demonstration of anything. Of course, I speak from the perspective of an unreal physicist, it seems. But the fact is, unfortunately for the disgruntled, that there are no geometrically undefined quantities in the line-element (1), and so there are no quantities therein that can be arbitrarily interpreted or re-defined at the vagarious whim of "real" physicists (otherwise we could just as well claim that r in (1) is a unicorn, or a poached egg, should we feel so disposed). Owing to the response of the Standard Model relativists, and more so for the lack of response thereof, I gave from first principles a full mathematical description of a spherically symmetric metric manifold in a dedicated paper [17]. Evidently that paper was far too difficult for the Standard Model relativists to understand (a long list of Standard Model relativists, many prominent, who could not understand that paper, is given in reference [35]). Consequently, I give here another proof that r in (1) is the radius of curvature by virtue of its formal geometric relationship to the Gaussian curvature, so that the conceptual error of the Standard Model relativists is amplified once again, from a different perspective.

Consider Efcleethean 3-Space. A hypersphere in Efcleethean 3-Space is a 2-sphere, described by

$$ds^2 = r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2. \tag{2}$$

The generalisation of (2) to a 2-D Riemannian manifold, is given by [36],

$$ds^2 = R_c^2 d\theta^2 + R_c^2 \sin^2\theta d\varphi^2, \tag{3}$$

wherein R_c is a function of the variable r appearing in (2). Expression (3) describes a geometric surface, i. e. a 2-D Riemannian manifold [37]. Now for a 2-D Riemannian manifold, the Riemannian curvature reduces to the Gaussian curvature G, and depends only upon the components of the metric tensor and their derivatives. It is given by [36, 38, 39, 40, 41],

$$G = \frac{R_{1212}}{g}, \tag{4}$$

where $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor of the first kind and g is the determinant of the metric tensor. In the case of (3), and hence similarly for (2), since (2) and (3) have precisely the same geometric form, $g = g_{11}g_{22}$. Also,

$$R_{1212} = g_{11} R_{212}^1,$$

$$R_{212}^1 = \frac{\partial \Gamma_{22}^1}{\partial x^1} - \frac{\partial \Gamma_{21}^1}{\partial x^2} + \Gamma_{22}^k \Gamma_{k1}^1 - \Gamma_{21}^k \Gamma_{k2}^1,$$

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$$\Gamma^{\alpha}_{\alpha\beta} = \Gamma^{\alpha}_{\beta\alpha} = \frac{\partial}{\partial x^{\beta}} \left(\frac{1}{2} \ln |g_{\alpha\alpha}| \right),$$

$$\Gamma^{\alpha}_{\beta\beta} = -\frac{1}{2g_{\alpha\alpha}} \frac{\partial g_{\beta\beta}}{\partial x^{\alpha}}, \quad (\alpha \neq \beta),$$

and all other $\Gamma^{\alpha}_{\beta\gamma}$ vanish. In the above, $k, \alpha, \beta = 1, 2$, $x^1 = \theta$ and $x^2 = \phi$, of course. Simple calculations then show that for (3),

$$G = \frac{1}{R_c^2}$$

and so R_c is the inverse square root of the Gaussian curvature, i.e. the radius of curvature (which is not "rubbish" by any stretch of a rational imagination, despite what the "illustrious" Mr. Kerr, and other Standard Model relativists and "real" physicists such as Mr. 't Hooft, might say).

The geometer N. Stavroulakis [29] has also noted that r in (1) is the radius of curvature.

Misconception: That Einstein's pseudo-tensor is meaningful

Einstein's pseudo-tensor is claimed to represent the energy and momentum of the gravitational field. That it is not a tensor, and therefore not in keeping with the basic principles of General Relativity, is problematic in itself. However, that issue has been ignored by the Standard Model relativists (perhaps blissfully so), who routinely apply the pseudo-tensor in relation to the localisation of gravitational energy, the conservation of energy and the flow of energy and momentum (e.g. [38, 41, 42, 43]).

Einstein's pseudo-tensor, $\sqrt{-g}t^{\mu}_{\nu}$, is defined by (e.g. [38, 41, 42, 43]),

$$\sqrt{-g}t^{\mu}_{\nu} = \frac{1}{2} \left(\delta^{\mu}_{\nu} L - \frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}} g^{\sigma\rho}_{,\nu} \right), \tag{5}$$

wherein L is given by

$$L = -g^{\alpha\beta} \left(\Gamma^{\gamma}_{\alpha\kappa} \Gamma^{\kappa}_{\beta\gamma} - \Gamma^{\gamma}_{\alpha\beta} \Gamma^{\kappa}_{\gamma\kappa} \right). \tag{6}$$

In a remarkable paper published in 1917, T. Levi-Civita [44] provided a clear and rigorous proof that Einstein's pseudo-tensor is a meaningless concoction of mathematical symbols, and therefore any argument relying upon it is fallacious. I repeat Levi-Civita's proof.

Contracting (5) produces a linear invariant, thus

$$\sqrt{-g}t^{\mu}_{\mu} = \frac{1}{2} \left(4L - \frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}} g^{\sigma\rho}_{,\mu} \right). \tag{7}$$

Since L is, according to (6), quadratic and homogeneous with respect to the Riemann-Christoffel symbols, and

therefore also with respect to $g^{\sigma\rho}_{,\mu}$, one can apply Euler's theorem to obtain,

$$\frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}} g^{\sigma\rho}_{,\mu} = 2L. \tag{8}$$

Substituting (8) into (7) yields the linear invariant at L. This is a first-order, intrinsic differential invariant that depends only on the components of the metric tensor and their first derivatives. However, the mathematicians Ricci and Levi-Civita [45] proved, in 1900, that such invariants *do not exist*. This is sufficient to render Einstein's pseudo-tensor meaningless, and hence all arguments relying on it, false. In particular, Einstein's conception of the conservation of energy in the gravitational field is erroneous.

This obviously has immediate implications for the localisation of gravitational energy and gravitational radiation. Since $R_{\mu\nu} = 0$ violates Einstein's Principle of Equivalence and is thereby inadmissible, one can write the field equations in the form proposed by H. A. Lorentz [38] and independently by Levi-Civita [38, 44], thus

$$T_{\mu\nu} + \frac{1}{\kappa} G_{\mu\nu} = 0, \tag{9}$$

where $G_{\mu\nu} = \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right)$ is Einstein's tensor and $G_{\mu\nu}/\kappa$ are the components of a gravitational energy tensor. Thus, Einstein's tensor and the energy-momentum tensor vanish identically. The total energy is always zero. And there is no localisation of gravitational energy. Consequently, projects such as LIGO, and its counterparts around the world, such as the Australian International Gravitational Observatory (AIGO), are misguided, and have already squandered huge amounts of taxpayers' money on a fantasy (despite evidence being presented to the major participants in these projects [35]).

That Einstein (and Pauli) [38] both knew of Levi-Civita's 1917 paper [44], but did not take stock of all the contents thereof, leaves one wondering why.

Misconception: That "Schwarzschild's" solution is Schwarzschild's solution

It has been reported by a number of other authors besides me (e.g. [21, 46, 47]) that what is referred to almost ubiquitously in the literature as "Schwarzschild's" solution is not Schwarzschild's solution, but a corruption thereof. Here is Schwarzschild's solution:

$$ds^2 = \left(1 - \frac{\alpha}{R} \right) dt^2 - \left(1 - \frac{\alpha}{R} \right)^{-1} dR^2 - R^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2 \right),$$

$$R = R(r) = \left(r^3 + \alpha^3 \right)^{\frac{1}{3}}, \quad 0 < r < \infty.$$

Schwarzschild did not claim that $\alpha = 2m$. Schwarzschild did not breath a single word about black holes. Clearly,

Schwarzschild's solution precludes the possibility of the black hole. One only needs to actually read the original papers of Schwarzschild [19, 18] to verify these facts. In any event, the issue is moot, since $R_{\mu\nu} = 0$ is invalid in General Relativity.

Epilogue

In view of the foregoing, the concept of the black hole is entirely fallacious. Since the Big Bang cosmology has also been shown to be inconsistent with the geometric structure of General Relativity [10, 12, 15], much of what has been the focus of research by the Standard Model relativists, for many years, is invalid.

It is clear that Einstein's formulation for the gravitational field does not achieve what he had thought, or what contemporary Standard Model relativists claim. If the programme of reduction of physics to geometry is to be realised, as envisioned by Einstein, it must come from some reformulation of General Relativity in terms of a unified field theory possibly couched in Riemannian geometry, or from a deeper geometrical structure than currently entertained; if indeed it can be done at all.

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