RA-DIANT

Next Generation Renewable Energy Device

Energy Independence for Highest and Best Good for All

Revision History

Version No	Release Date	Reason for Change	Author
1.0	14 Jan 2024	N A	Anil Goel

The purpose of this work is to raise the understanding and vibration of current human collective.

Let it seep into your consciousness. Analyse it. Dissect it. Look at it carefully and then put it back together again.

Reread, contemplate, pray, discern and only accept those thought forms that resonate with your inner heart leaving behind those that do not resonate with it.

Freewill is your privilege and your responsibility. Use it in wisdom!

Dedicated to 6^{th} Density Service to Others (STO) Beings, who are my teachers and guides in realising free energy and make it available for highest and best good for all humanity on planet Earth.

My profound gratitude to Horst Eckardt, my mentor in this realm, for his support and commitment in realising free energy and make it available for highest and best good for all humanity on planet Earth.

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Executive Summary

There exists a fundamental law of duality between electricity and magnetism. The law reveals itself by the systematic study and analysis of already known quantities and equations. One of the key reasons that deprived humanity from discovering such a fundamental law is the heavy usage of usual Vector algebra, which is omnipresent in science and lacks many necessary geometrical concepts and perennially thwarts realisation of the true nature of reality.

The analysis discovers unknown quantities such as an electric potential and a voltage area density which are the dual of electric charge and current area density, respectively. In addition, the analysis unravels the mystery of the perennial myth that electric and magnetic fields can only be perpendicular i.e. there is only a perpendicular interaction between electric and magnetic fields. The analysis shows that there also exists a parallel interaction between electric and magnetic fields. The well known Gauss Law and Faraday Law, which have been attributed to perpendicular interaction, in actuality pertain to parallel interaction.

Furthermore, it is shown that the electric and magnetic fields can have velocities faster than light, bringing in aspects of general relativity. The fields rotate/spin with different cycles in various interactions and are primary quantities. The electric potential and charge are nothing more than spinning spacetime structures.

In addition, it is shown that the length measurement and hence the speed of light is affected by the electric and magnetic fields which makes it almost impossible to correctly measure the speed of light with currently used methods. The vector velocities of two interacting fields result in a ubiquitous scalar speed of light which is Lorentz invariant and the rotational/ spinning component.

Furthermore, it is shown that the power density and the energy density inherently represent the same concept, but the power density is due to the perpendicular interaction while the energy density is due to the parallel interaction. The energy density is not a scalar quantity or Lorentz invariant in the truest sense of the word.

In addition, it is shown that there is an unaccounted power density, firstly due to the newly discovered voltage area density and secondly due to the orthogonal complement or dual space.

Furthermore, it is shown that there exists a fourth space dimension for equations of transformation i.e. equation of unipolar induction and equation of convention, to be valid at the same time.

In addition, it is shown that there are additional unaccounted power terms, which are always present in reality irrespective of our view.

These are the key reasons that the prevalent theory of electromagnetism cannot corroborated with free energy devices and many experimental results and continues to plague human collective consciousness.

1. Law of Duality

Table 1.1

Duality					
Electricity			**Magnetism**		
Coulomb law	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$	$\frac{V}{m^2}$	Gauss law	$\nabla \cdot \mathbf{H} = 0$	$\frac{A}{m^2}$
Faraday law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$	$\frac{V}{m^2}$	Ampère-Maxwell law	$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$

The duality of the Faraday and Ampère-Maxwell law reveals itself when the dual entities, i.e. electric field and magnetic field strength or magnetic flux density and dielectric displacement, are being employed in the same equation.

Table 1.2

Duality					
Electricity			**Magnetism**		
Yang	Male		Yin	Female	
Positive	$+$		Negative	$-$	
North pole	$\mathbf{N}$		South pole	$\mathbf{S}$	
Direction of movement	$\mathbf{N} \longrightarrow \mathbf{S}$		Direction of movement	$\mathbf{N} \longleftarrow \mathbf{S}$	
Vacuum permittivity	ϵ_0	$\frac{As}{Vm}$	Vacuum permeability	μ_0	$\frac{Vs}{Am}$
Voltage	U	V	Current	I	A
Electric field strength	$\mathbf{E}$	$\frac{V}{m}$	Magnetic field strength	$\mathbf{H}$	$\frac{A}{m}$
Magnetic flux density or Magnetic induction	$\mathbf{B} = \mu_0 \mathbf{H}$	$\frac{Vs}{m^2}$	Dielectric displacement or Electric induction	$\mathbf{D} = \epsilon_0 \mathbf{E}$	$\frac{As}{m^2}$
Coulomb law	$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}$	$\frac{V}{m^2}$	Gauss law	$\nabla \cdot \mathbf{H} = 0$	$\frac{A}{m^2}$

Duality					
Electricity			**Magnetism**		
Gauss law	$\nabla \cdot \mathbf{B} = 0$	$\frac{Vs}{m^3}$	Coulomb law	$\nabla \cdot \mathbf{D} = \rho_e$	$\frac{As}{m^3}$
-					
Faraday law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$	$\frac{V}{m^2}$	Ampère-Maxwell law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} = 0$	$\frac{V}{m^2}$		$\nabla \times \mathbf{H} - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
Ampère-Maxwell law	$\nabla \times \mathbf{B} - \mu_0 \frac{\partial \mathbf{D}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$	Faraday law	$\nabla \times \mathbf{D} + \epsilon_0 \frac{\partial \mathbf{B}}{\partial t} = 0$	$\frac{As}{m^3}$
	$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$		$\nabla \times \mathbf{D} + \frac{1}{c^2} \frac{\partial \mathbf{H}}{\partial t} = 0$	$\frac{As}{m^3}$

The vacuum permittivity ϵ_0 converts the electricity into magnetism and vacuum permeability μ_0 converts the magnetism into electricity. Thus Coulomb law and Gauss law keep shuffling between two aspects depending on which aspect is represented in the equation.

Coulomb law:

(1) $$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0} \quad [1.1]$$

(2) $$\nabla \cdot \mathbf{D} = \rho_e \quad [1.2]$$

Gauss law:

(3) $$\nabla \cdot \mathbf{H} = 0 \quad [1.3]$$

(4) $$\nabla \cdot \mathbf{B} = 0 \quad [1.4]$$

Faraday law:

(5) $$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \quad [1.5]$$

(6) $$\nabla \times \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} = 0 \quad [1.6]$$

(7) $$\nabla \times \mathbf{D} + \epsilon_0 \frac{\partial \mathbf{B}}{\partial t} = 0$$ [1.7]

(8) $$\nabla \times \mathbf{D} + \frac{1}{c^2} \frac{\partial \mathbf{H}}{\partial t} = 0$$ [1.8]

Ampère-Maxwell law:

(9) $$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$ [1.9]

(10) $$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$ [1.10]

(11) $$\nabla \times \mathbf{H} - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}$$ [1.11]

(12) $$\nabla \times \mathbf{B} - \mu_0 \frac{\partial \mathbf{D}}{\partial t} = \mu_0 \mathbf{J}$$ [1.12]

Unipolar induction by Faraday is expressed by

(13) $$\mathbf{E} = \mathbf{v} \times \mathbf{B}$$ [1.13]

where $\mathbf{v}$ is a velocity of a moving frame or relative velocity between frames.

The dual of equation[1.13] is the equation of convection expressed by

(14) $$\mathbf{H} = -\mathbf{v} \times \mathbf{D}$$ [1.14]

The equations [1.13] and [1.14] together are called equations of transformation. The equations of transformation operate simultaneously.

Maxwell's equations can be derived from the equation of unipolar induction and the equation of convection.

Table 1.3

Duality					
Electricity			**Magnetism**		
Vacuum permittivity	ϵ_0	$\frac{As}{Vm}$	Vacuum permeability	μ_0	$\frac{Vs}{Am}$
Voltage	U	V	Current	I	A
Electric field strength	$\mathbf{E}$	$\frac{V}{m}$	Magnetic field strength	$\mathbf{H}$	$\frac{A}{m}$
Magnetic flux density or Magnetic induction	$\mathbf{B} = \mu_0 \mathbf{H}$	$\frac{Vs}{m^2}$	Dielectric displacement or Electric induction	$\mathbf{D} = \epsilon_0 \mathbf{E}$	$\frac{As}{m^2}$
Unipolar induction	$\mathbf{E} = \mathbf{v} \times \mathbf{B}$	$\frac{V}{m}$	Equation of convection	$\mathbf{H} = -\mathbf{v} \times \mathbf{D}$	$\frac{A}{m}$

Applying the curl on the equation of convection [1.14] gives

(15) $$\nabla \times \mathbf{H} = -\nabla \times (\mathbf{v} \times \mathbf{D})$$ [1.15]

According to Vector algebra

(16) $$\nabla \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\nabla \cdot \mathbf{C}) - \mathbf{C}(\nabla \cdot \mathbf{B}) + (\mathbf{C} \cdot \nabla)\mathbf{B} - (\mathbf{B} \cdot \nabla)\mathbf{C}$$ [1.16]

Therefore

(17) $$\nabla \times (\mathbf{v} \times \mathbf{D}) = \mathbf{v}(\nabla \cdot \mathbf{D}) - \mathbf{D}(\nabla \cdot \mathbf{v}) + (\mathbf{D} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{D}$$ [1.17]

(18) $$\nabla \times (\mathbf{v} \times \mathbf{D}) = \mathbf{v}(\nabla \cdot \mathbf{D}) - \mathbf{D}(\nabla \cdot \mathbf{v}) + \mathbf{D} \cdot (\nabla \mathbf{v}) - (\mathbf{v} \cdot \nabla)\mathbf{D}$$ [1.18]

(19) $$\nabla \times \mathbf{H} = -\left(\mathbf{v}(\nabla \cdot \mathbf{D}) - \mathbf{D}(\nabla \cdot \mathbf{v}) + \mathbf{D} \cdot (\nabla \mathbf{v}) - (\mathbf{v} \cdot \nabla)\mathbf{D}\right)$$ [1.19]

In Vector algebra, the gradient is usually introduced as a vector not as a tensor (1 form) as the components of space coordinates do not change when you convert a vector to a tensor in a Cartesian coordinate system. Hence, the dot product of a gradient vector with another vector still produces a scalar. However, this little maneuver of avoiding the concept of tensor in Vector algebra not only violates the fundamental property of dot product, i.e. makes the dot product being non commutative which unnecessarily creates confusion, but also has a few side effects, for example the gradient of the velocity $\nabla \mathbf{v}$ cannot be defined in the Vector algebra.

(20) $$\nabla \cdot \mathbf{v} \neq \mathbf{v} \cdot \nabla$$ [1.20]

(21) $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$ [1.21]

In accordance with special relativity, we have to assume that $\mathbf{v}$ is a constant. Therefore

(22) $$\frac{\partial v_x}{\partial x} = 0 \qquad \frac{\partial v_y}{\partial y} = 0 \qquad \frac{\partial v_z}{\partial z} = 0$$ [1.22]

On substituting equation [1.22] in equation [1.21] gives

(23) $$\nabla \cdot \mathbf{v} = 0$$ [1.23]

The dot product $\mathbf{v} \cdot \nabla$ is defined as a directional derivative in the direction of $\mathbf{v}$ and is a vector operator.

According to Vector algebra, written in matrix form:

(24) $$\mathbf{A} \cdot \mathbf{B} = \mathbf{A}^T \mathbf{B}$$ [1.24]

Therefore

(25) $$\mathbf{v} \cdot \nabla = \mathbf{v}^T \nabla$$ [1.25]

(26) $$\mathbf{v}^T = \left[\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t} \right]$$ [1.26]

(27) $$\nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix}$$ [1.27]

Therefore

(28) $$\mathbf{v} \cdot \nabla = \left[\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}\right] \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix}$$ [1.28]

(29) $$\begin{aligned} \mathbf{v} \cdot \nabla &= \frac{\partial x}{\partial t}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial}{\partial y} + \frac{\partial z}{\partial t}\frac{\partial}{\partial z} \\ &= \frac{\partial}{\partial t} + \frac{\partial}{\partial t} + \frac{\partial}{\partial t} \\ &= \sum_{i=1}^{3} \frac{\partial}{\partial t} \end{aligned}$$ [1.29]

To define the gradient of the velocity $\nabla \mathbf{v}$, we must look elsewhere. The Vector algebra is a stripped down version of Clifford algebra in three dimensions. I will expound on what has been stripped in Vector algebra in the next chapter.

According to Clifford Algebra,

(30) $$\nabla \mathbf{v} = \nabla \cdot \mathbf{v} + \nabla \wedge \mathbf{v}$$ [1.30]

(31) $$\nabla \mathbf{v} = \nabla \cdot \mathbf{v} + I \nabla \times \mathbf{v}$$ [1.31]

In accordance with special relativity, we have to assume that $\mathbf{v}$ is a constant. Therefore the curl of the velocity $\nabla \times \mathbf{v}$ will be zero.

(32) $$\nabla \mathbf{v} = 0 + I0 = 0$$ [1.32]

Therefore, the gradient of the velocity $\nabla \mathbf{v}$ always vanishes.

We need to import the Clifford algebra identity equation [1.32] into the Vector algebra.

On substituting equation [1.23] and [1.32] in equation [1.19] gives

(33) $$\nabla \times \mathbf{H} = -\left(\mathbf{v}(\nabla \cdot \mathbf{D}) - \mathbf{D}(0) + \mathbf{D} \cdot (0) - \frac{\partial \mathbf{D}}{\partial t}\right)$$ [1.33]

(34) $$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} - \mathbf{v}(\nabla \cdot \mathbf{D})$$ [1.34]

We define the current area density by

(35) $$\mathbf{J} = -\mathbf{v}(\nabla \cdot \mathbf{D})$$ [1.35]

On substituting equation [1.35] in equation [1.34] gives

(36) $$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$ [1.36]

Thus, the equation of convection and the Ampère-Maxwell law equation are two expressions of same law.

It is evident from equation [1.34] (the second term on RHS) that the Coulomb law is inherent in the equation of convection.

Applying the curl on Faraday's unipolar induction [1.13] gives

(37) $$\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B})$$ [1.37]

According to Vector algebra [1.16]

$$\nabla \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\nabla \cdot \mathbf{C}) - \mathbf{C}(\nabla \cdot \mathbf{B}) + (\mathbf{C} \cdot \nabla)\mathbf{B} - (\mathbf{B} \cdot \nabla)\mathbf{C}$$ [1.16]

Therefore

(38) $$\nabla \times (\mathbf{v} \times \mathbf{B}) = \mathbf{v}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{v}) + (\mathbf{B} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{B}$$ [1.38]

(39) $$\nabla \times (\mathbf{v} \times \mathbf{B}) = \mathbf{v}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{v}) + \mathbf{B} \cdot (\nabla \mathbf{v}) - (\mathbf{v} \cdot \nabla)\mathbf{B}$$ [1.39]

(40) $$\nabla \times \mathbf{E} = \mathbf{v}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{v}) + \mathbf{B} \cdot (\nabla \mathbf{v}) - (\mathbf{v} \cdot \nabla)\mathbf{B}$$ [1.40]

On substituting equation [1.23] and [1.32] in equation [1.40] gives

(41) $$\nabla \times \mathbf{E} = \mathbf{v}(\nabla \cdot \mathbf{B}) - \frac{\partial \mathbf{B}}{\partial t}$$ [1.41]

For special case [1.4]

$$\nabla \cdot \mathbf{B} = 0$$ [1.4]

On substituting equation [1.4] in equation [1.41] gives

(42) $$\nabla \times \mathbf{E} = 0 - \frac{\partial \mathbf{B}}{\partial t}$$ [1.42]

(43) $$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$ [1.43]

Thus the Faraday law is a special case of Faraday's unipolar induction unlike the Ampère-Maxwell law.

Table 1.4

Duality					
Electricity			**Magnetism**		
Unipolar induction	$\mathbf{E} = \mathbf{v} \times \mathbf{B}$	$\frac{V}{m}$	Equation of convection	$\mathbf{H} = -\mathbf{v} \times \mathbf{D}$	$\frac{A}{m}$
Coulomb law	$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}$	$\frac{V}{m^2}$	Gauss law	$\nabla \cdot \mathbf{H} = 0$	$\frac{A}{m^2}$
Gauss law	$\nabla \cdot \mathbf{B} = 0$	$\frac{Vs}{m^3}$	Coulomb law	$\nabla \cdot \mathbf{D} = \rho_e$	$\frac{As}{m^3}$
Faraday law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$	$\frac{V}{m^2}$	Ampère-Maxwell law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} = 0$	$\frac{V}{m^2}$		$\nabla \times \mathbf{H} - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
Ampère-Maxwell law	$\nabla \times \mathbf{B} - \mu_0 \frac{\partial \mathbf{D}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$	Faraday law	$\nabla \times \mathbf{D} + \epsilon_0 \frac{\partial \mathbf{B}}{\partial t} = 0$	$\frac{As}{m^3}$

Duality					
Electricity			Magnetism		
	$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$		$\nabla \times \mathbf{D} + \frac{1}{c^2}\frac{\partial \mathbf{H}}{\partial t} = 0$	$\frac{As}{m^3}$

We know now that the magnetic monopoles do exist in the nature as observed by Helmholtz institute and London centre of nanotechnology. The measurement of the magnetic monopole is the real problem, not its existence.

By applying the rules of duality, we should be able to theoretically correct the Maxwell equations as we have already seen that the dual equation of convection results in the Ampère-Maxwell law.

The dual of an elementary electric charge e would be an **ELEMENTARY MAGNETIC POTENTIAL** h.

(44) $e\ (Ampere\ second) \longleftarrow dual \longrightarrow p\ (Volt\ second)$ [1.44]

The dual of an electric charge q would be a **MAGNETIC POTENTIAL** ϕ_h.

(45) $q = ne\ (Ampere\ second) \longleftarrow dual \longrightarrow \phi_p = np\ (Volt\ second)$ [1.45]

where n is the number of elementary electric charge or elementary magnetic potential.

The dual of an electric charge density ρ_e would be a **MAGNETIC POTENTIAL DENSITY** ρ_p.

(46) $\rho_e \left(\frac{As}{m^3}\right) \longleftarrow dual \longrightarrow \rho_p \left(\frac{Vs}{m^3}\right)$ [1.46]

The dual of an electric current area density $\mathbf{J}$ would be a **MAGNETIC VOLTAGE AREA DENSITY** $\mathbf{V}$.

(47) $\mathbf{J} = -\,\mathbf{v}(\nabla \cdot \mathbf{D})\left(\frac{A}{m^2}\right) \longleftarrow dual \longrightarrow \mathbf{V} = \mathbf{v}(\nabla \cdot \mathbf{B})\left(\frac{V}{m^2}\right)$ [1.47]

The dual of the Coulomb law would be a **DUAL COULOMB LAW.**

(48) $$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0} \longleftarrow dual \longrightarrow \nabla \cdot \mathbf{H} = \frac{\rho_p}{\mu_0}$$ [1.48]

(49) $$\nabla \cdot \mathbf{D} = \rho_e \longleftarrow dual \longrightarrow \nabla \cdot \mathbf{B} = \rho_p$$ [1.49]

The dual of the Ampère-Maxwell law would be a **DUAL AMPÈRE-MAXWELL LAW.**

(50) $$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \longleftarrow dual \longrightarrow \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{V}$$ [1.50]

Table 1.5

Duality					
Electricity			**Magnetism**		
Yang	Male		Yin	Female	
Positive	$+$		Negative	$-$	
North pole	$\mathbf{N}$		South pole	$\mathbf{S}$	
Direction of movement	$\mathbf{N} \longrightarrow \mathbf{S}$		Direction of movement	$\mathbf{N} \longleftarrow \mathbf{S}$	
Vacuum permittivity	ϵ_0	$\frac{As}{Vm}$	Vacuum permeability	μ_0	$\frac{Vs}{Am}$
Voltage	U	V	Current	I	A
Electric field strength	$\mathbf{E}$	$\frac{V}{m}$	Magnetic field strength	$\mathbf{H}$	$\frac{A}{m}$
Magnetic flux density or Magnetic induction	$\mathbf{B} = \mu_0 \mathbf{H}$	$\frac{Vs}{m^2}$	Dielectric displacement or Electric induction	$\mathbf{D} = \epsilon_0 \mathbf{E}$	$\frac{As}{m^2}$
Elementary electric charge	e	$C = As$	Elementary magnetic potential	p	$Wb = Vs$
Electric charge	q	$C = As$	Magnetic potential	ϕ_p	$Wb = Vs$

Duality					
Electricity			Magnetism		
Electric charge density	ρ_e	$\frac{A\,s}{m^3}$	Magnetic potential density	ρ_p	$\frac{Vs}{m^3}$
Unipolar induction	$\mathbf{E} = \mathbf{v} \times \mathbf{B}$	$\frac{V}{m}$	Equation of convection	$\mathbf{H} = -\mathbf{v} \times \mathbf{D}$	$\frac{A}{m}$
Coulomb law	$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}$	$\frac{V}{m^2}$	Dual Coulomb law	$\nabla \cdot \mathbf{H} = \frac{\rho_p}{\mu_0}$	$\frac{A}{m^2}$
Dual Coulomb law	$\nabla \cdot \mathbf{B} = \rho_p$	$\frac{Vs}{m^3}$	Coulomb law	$\nabla \cdot \mathbf{D} = \rho_e$	$\frac{A\,s}{m^3}$
Electric current area density	$\mathbf{J} = -\mathbf{v}(\nabla \cdot \mathbf{D})$	$\frac{A}{m^2}$	Magnetic voltage area density	$\mathbf{V} = \mathbf{v}(\nabla \cdot \mathbf{B})$	$\frac{V}{m^2}$
Dual Ampère-Maxwell law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{V}$	$\frac{V}{m^2}$	Ampère-Maxwell law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} = \mathbf{V}$	$\frac{V}{m^2}$		$\nabla \times \mathbf{H} - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
Ampère-Maxwell law	$\nabla \times \mathbf{B} - \mu_0 \frac{\partial \mathbf{D}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$	Dual Ampère-Maxwell law	$\nabla \times \mathbf{D} + \epsilon_0 \frac{\partial \mathbf{B}}{\partial t} = \epsilon_0 \mathbf{V}$	$\frac{A\,s}{m^3}$
	$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$		$\nabla \times \mathbf{D} + \frac{1}{c^2} \frac{\partial \mathbf{H}}{\partial t} = \epsilon_0 \mathbf{V}$	$\frac{A\,s}{m^3}$
Special Case					
Dual Gauss law	$\nabla \cdot \mathbf{E} = 0$	$\frac{V}{m^2}$	Gauss law	$\nabla \cdot \mathbf{H} = 0$	$\frac{A}{m^2}$
Gauss law	$\nabla \cdot \mathbf{B} = 0$	$\frac{Vs}{m^3}$	Dual Gauss law	$\nabla \cdot \mathbf{D} = 0$	$\frac{A\,s}{m^3}$
Electric current area density	$\mathbf{J} = 0$	$\frac{A}{m^2}$	Magnetic voltage area density	$\mathbf{V} = 0$	$\frac{V}{m^2}$

Duality					
Electricity			Magnetism		
Faraday law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$	$\frac{V}{m^2}$	Dual Faraday law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0$	$\frac{A}{m^2}$

As pointed out earlier, the vacuum permittivity ϵ_0 converts the electricity into magnetism and the vacuum permeability μ_0 converts the magnetism into electricity. Thus Ampère-Maxwell law and dual Ampère-Maxwell law keep shuffling between two aspects depending on which aspect is represented in the equation.

On a closer inspection of the Table 1.5, it seems that the so called electric charge or electric monopole is actually a magnetic charge or magnetic monopole.

i) This can be easily ascertained by observing the properties of the so called electric charge.

ii) The basic units of so called electric charge e is Ampere Second (As). The ampere is the unit associated with magnetism and the volt is the unit associated with electricity.

iii) The so called electric charge repels another electric charge of same polarity and attracts the positive charge (positron) with opposite polarity, which is the fundamental characteristic of magnetic North and South poles.

The electric charge $e(As)$ produces the so called electric current area density $\mathbf{J}\left(\frac{A}{m^2}\right)$.

Similarly, the so called magnetic potential or magnetic monopole is actually an electric potential or an electric monopole. Thus, magnetic charge $e(As)$ produces magnetic current area density $\mathbf{J}\left(\frac{A}{m^2}\right)$ and electric potential $p(Vs)$ produces electric voltage area density $\mathbf{V}\left(\frac{V}{m^2}\right)$.

As such it doesn't matter what an entity is called as long as we know what they really are, but it creates unnecessary confusion and is prone to errors for example in the literature, usually area is dropped from magnetic current area density $\mathbf{J}\left(\frac{A}{m^2}\right)$ which causes confusion as density is commonly related to per unit volume not to per unit area.

In this work, we will use the correct and consistent terminology to avoid any confusion and errors which may arise due to wrong terminologies.

Table 1.6

Duality					
Electricity			**Magnetism**		
Yang	Male		Yin	Female	
Positive	$+$		Negative	$-$	
North pole	$\mathbf{N}$		South pole	$\mathbf{S}$	
Direction of movement	$\mathbf{N} \longrightarrow \mathbf{S}$		Direction of movement	$\mathbf{N} \longleftarrow \mathbf{S}$	
Vacuum permittivity	ϵ_0	$\frac{As}{Vm}$	Vacuum permeability	μ_0	$\frac{Vs}{Am}$
Voltage	U	V	Current	I	A
Electric field strength	$\mathbf{E}$	$\frac{V}{m}$	Magnetic field strength	$\mathbf{H}$	$\frac{A}{m}$
Magnetic flux density or Magnetic induction	$\mathbf{B} = \mu_0\mathbf{H}$	$\frac{Vs}{m^2}$	Dielectric displacement or Electric induction	$\mathbf{D} = \epsilon_0\mathbf{E}$	$\frac{As}{m^2}$
Elementary electric potential	p	$Wb = Vs$	Elementary magnetic charge	e	$C = As$
Electric potential	ϕ_p	$Wb = Vs$	Magnetic charge	q	$C = As$
Electric potential density	ρ_p	$\frac{Vs}{m^3}$	Magnetic charge density	ρ_e	$\frac{As}{m^3}$
Unipolar induction	$\mathbf{E} = \mathbf{v} \times \mathbf{B}$	$\frac{V}{m}$	Equation of convection	$\mathbf{H} = -\mathbf{v} \times \mathbf{D}$	$\frac{A}{m}$

Duality					
Electricity			**Magnetism**		
Coulomb law	$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}$	$\frac{V}{m^2}$	Dual Coulomb law	$\nabla \cdot \mathbf{H} = \frac{\rho_p}{\mu_0}$	$\frac{A}{m^2}$
Dual Coulomb law	$\nabla \cdot \mathbf{B} = \rho_p$	$\frac{Vs}{m^3}$	Coulomb law	$\nabla \cdot \mathbf{D} = \rho_e$	$\frac{As}{m^3}$
Electric voltage area density	$\mathbf{V} = \mathbf{v}(\nabla \cdot \mathbf{B})$	$\frac{V}{m^2}$	Magnetic current area density	$\mathbf{J} = -\mathbf{v}(\nabla \cdot \mathbf{D})$	$\frac{A}{m^2}$
Dual Ampère-Maxwell law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{V}$	$\frac{V}{m^2}$	Ampère-Maxwell law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} = \mathbf{V}$	$\frac{V}{m^2}$		$\nabla \times \mathbf{H} - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
Ampère-Maxwell law	$\nabla \times \mathbf{B} - \mu_0 \frac{\partial \mathbf{D}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$	Dual Ampère-Maxwell law	$\nabla \times \mathbf{D} + \epsilon_0 \frac{\partial \mathbf{B}}{\partial t} = \epsilon_0 \mathbf{V}$	$\frac{As}{m^3}$
	$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$		$\nabla \times \mathbf{D} + \frac{1}{c^2} \frac{\partial \mathbf{H}}{\partial t} = \epsilon_0 \mathbf{V}$	$\frac{As}{m^3}$
Special Case					
Dual Gauss law	$\nabla \cdot \mathbf{E} = 0$	$\frac{V}{m^2}$	Gauss law	$\nabla \cdot \mathbf{H} = 0$	$\frac{A}{m^2}$
Gauss law	$\nabla \cdot \mathbf{B} = 0$	$\frac{Vs}{m^3}$	Dual Gauss law	$\nabla \cdot \mathbf{D} = 0$	$\frac{As}{m^3}$
Electric voltage area density	$\mathbf{V} = 0$	$\frac{V}{m^2}$	Magnetic current area density	$\mathbf{J} = 0$	$\frac{A}{m^2}$
Faraday law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$	$\frac{V}{m^2}$	Dual Faraday law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0$	$\frac{A}{m^2}$

The vector form of Ohm's law was generalised by Gustav Kirchhoff and is

(51) $$\mathbf{J} = \sigma \mathbf{E}$$ [1.51]

where σ (sigma) is a material-dependent parameter called the conductivity which is generally a tensor. In most cases, σ is assumed to be a scalar quantity.

For vacuum

(52) $$\mathbf{D} = \epsilon_0 \mathbf{E}$$ [1.52]

(53) $$\mathbf{J} = \sigma_0 \mathbf{E}$$ [1.53]

where σ_0 is vacuum conductivity.

On substituting equation [1.52] with equation [1.53] gives

(54) $$\mathbf{J} = \frac{\sigma_0}{\epsilon_0}\mathbf{D}$$ [1.54]

(55) $$\mathbf{J} = \omega_p \mathbf{D}$$ [1.55]

where $\omega_p = \dfrac{\sigma_0}{\epsilon_0}$ is the **ANGULAR FREQUENCY OF OUTWARD SPIRALLING ELECTRIC INDUCTION** in free space as observed in the skin effect in the copper cable.

The dual of the skin effect would be a contraction, a **CORE EFFECT**.

(56) $$Skin\ effect \longleftarrow dual \longrightarrow Core\ effect$$ [1.56]

The dual of a vacuum conductivity would be a vacuum **POTENTIATIVITY**.

(57) $$\sigma_0 \left(\frac{A}{Vm}\right) \longleftarrow dual \longrightarrow \eta_0 \left(\frac{V}{Am}\right)$$ [1.57]

The dual of Ohm's law [1.53] would be the **DUAL OHM'S LAW** i.e. **ELECTRIC VOLTAGE AREA DENSITY IS PROPORTIONAL TO THE MAGNETIC FIELD STRENGTH.**

(58) $$\mathbf{J} = \sigma_0 \mathbf{E} \left(\frac{A}{m^2}\right) \longleftarrow dual \longrightarrow \mathbf{V} = \eta_0 \mathbf{H} \left(\frac{V}{m^2}\right)$$ [1.58]

(59) $\mathbf{V} = \eta_0 \mathbf{H}$ [1.59]

(60) $\mathbf{B} = \mu_0 \mathbf{H}$ [1.60]

On substituting equation [1.60] in equation [1.59] gives

(61) $\mathbf{V} = \frac{\eta_0}{\mu_0} \mathbf{B}$ [1.61]

(62) $\mathbf{V} = \omega_e \mathbf{B}$ [1.62]

where $\omega_e = \frac{\eta_0}{\mu_0}$, is the **ANGULAR FREQUENCY OF AN INWARD SPIRALLING MAGNETIC INDUCTION** in free space, as observed in the core effect in the fibre optic cable.

Thus, the dual of the angular frequency of an outward spiralling electric induction is the angular frequency of an inward spiralling magnetic potential.

(63) $\omega_p = \frac{\sigma_0}{\epsilon_0} \left(\frac{1}{s} \right) \longleftarrow dual \longrightarrow \omega_e = \frac{\eta_0}{\mu_0} \left(\frac{1}{s} \right)$ [1.63]

(64) $\omega_p = \frac{\sigma_0}{\epsilon_0}$ [1.64]

(65) $\omega_e = \frac{\eta_0}{\mu_0}$ [1.65]

On multiplying equation [1.64] with [1.65] gives

(66) $\omega_p \omega_e = \frac{\sigma_0 \eta_0}{\epsilon_0 \mu_0}$ [1.66]

(67) $\omega_p \omega_e = c^2 \ \sigma_0 \eta_0$ [1.67]

(68) $\omega_p \omega_e = c^2 \ \kappa_e \kappa_h$ [1.68]

where

(69) $\kappa_p \propto \sigma_p$, is the **ELECTRIC WAVE NUMBER** [1.69]

(70) $\kappa_e \propto \eta_e$, is the **MAGNETIC WAVE NUMBER** [1.70]

Therefore

(71) $$\sigma_p \propto \frac{1}{\lambda_p}$$ [1.71]

(72) $$\eta_e \propto \frac{1}{\lambda_e}$$ [1.72]

where

(73) $$\kappa_p \kappa_e = \kappa_0^2$$ [1.73]

(74) $$\omega_p \omega_e = \omega_0^2$$ [1.74]

where $\kappa_0 \left(\frac{1}{m}\right)$ and $\omega_0 \left(\frac{1}{s}\right)$ are constants.

Substituting equation [1.55] in Ampère-Maxwell law gives

(75) $$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \omega_p \mathbf{D}$$ [1.75]

(76) $$\nabla \times \mathbf{H} = \epsilon_0 \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E}\right)$$ [1.76]

(77) $$\nabla \times \mathbf{B} = \epsilon_0 \mu_0 \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E}\right)$$ [1.77]

(78) $$\nabla \times \mathbf{B} = \frac{1}{c^2} \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E}\right)$$ [1.78]

Substituting equation [1.62] in Faraday law gives

(79) $$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \omega_e \mathbf{B}$$ [1.79]

(80) $$\nabla \times \mathbf{E} = -\mu_0 \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H} \right)$$ [1.80]

(81) $$\nabla \times \mathbf{D} = -\epsilon_0 \mu_0 \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H} \right)$$ [1.81]

(82) $$\nabla \times \mathbf{D} = -\frac{1}{c^2} \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H} \right)$$ [1.82]

Table 1.7

Duality					
Electricity			**Magnetism**		
Yang	Male		Yin	Female	
Positive	$+$		Negative	$-$	
North pole	$\mathbf{N}$		South pole	$\mathbf{S}$	
Direction of movement	$\mathbf{N} \longrightarrow \mathbf{S}$		Direction of movement	$\mathbf{N} \longleftarrow \mathbf{S}$	
Vacuum permittivity	ϵ_0	$\frac{As}{Vm}$	Vacuum permeability	μ_0	$\frac{Vs}{Am}$
Vacuum conductivity	σ_0	$\frac{A}{Vm}$	Vacuum potentivity	η_0	$\frac{V}{Am}$
Vacuum angular frequency for Electric induction	$\omega_p = \frac{\sigma_0}{\epsilon_0}$	$\frac{1}{s}$	Vacuum angular frequency for Magnetic induction	$\omega_e = \frac{\eta_0}{\mu_0}$	$\frac{1}{s}$
Voltage	U	V	Current	I	A
Electric field strength	$\mathbf{E}$	$\frac{V}{m}$	Magnetic field strength	$\mathbf{H}$	$\frac{A}{m}$

Duality					
Electricity			**Magnetism**		
Magnetic flux density or Magnetic induction	$\mathbf{B} = \mu_0 \mathbf{H}$	$\frac{Vs}{m^2}$	Dielectric displacement or Electric induction	$\mathbf{D} = \epsilon_0 \mathbf{E}$	$\frac{As}{m^2}$
Elementary electric potential	p	$Wb = Vs$	Elementary magnetic charge	e	$C = As$
Electric potential	ϕ_p	$Wb = Vs$	Magnetic charge	q	$C = As$
Electric potential density	ρ_p	$\frac{Vs}{m^3}$	Magnetic charge density	ρ_e	$\frac{As}{m^3}$
Unipolar induction	$\mathbf{E} = \mathbf{v} \times \mathbf{B}$	$\frac{V}{m}$	Equation of convection	$\mathbf{H} = -\mathbf{v} \times \mathbf{D}$	$\frac{A}{m}$
Coulomb law	$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}$	$\frac{V}{m^2}$	Dual Coulomb law	$\nabla \cdot \mathbf{H} = \frac{\rho_p}{\mu_0}$	$\frac{A}{m^2}$
Dual Coulomb law	$\nabla \cdot \mathbf{B} = \rho_p$	$\frac{Vs}{m^3}$	Coulomb law	$\nabla \cdot \mathbf{D} = \rho_e$	$\frac{As}{m^3}$
Electric voltage area density	$\mathbf{V} = \mathbf{v}(\nabla \cdot \mathbf{B})$	$\frac{V}{m^2}$	Magnetic current area density	$\mathbf{J} = -\mathbf{v}(\nabla \cdot \mathbf{D})$	$\frac{A}{m^2}$
Dual ohms law	$\mathbf{V} = \eta_0 \mathbf{H}$		Ohms law	$\mathbf{J} = \sigma_0 \mathbf{E}$	
	$\mathbf{V} = \omega_e \mathbf{B}$			$\mathbf{J} = \omega_p \mathbf{D}$	
Dual Ampère-Maxwell law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{V}$	$\frac{V}{m^2}$	Ampère-Maxwell law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$		$\nabla \times \mathbf{D} + \frac{1}{c^2}\frac{\partial \mathbf{H}}{\partial t} = -\epsilon_0 \mathbf{V}$	$\frac{As}{m^3}$
	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \omega_e \mathbf{B}$	$\frac{V}{m^2}$		$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \omega_p \mathbf{D}$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{E} = -\mu_0 \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H}\right)$	$\frac{V}{m^2}$		$\nabla \times \mathbf{H} = \epsilon_0 \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E}\right)$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{B} = \frac{1}{c^2} \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E}\right)$	$\frac{Vs}{m^3}$		$\nabla \times \mathbf{D} = -\frac{1}{c^2} \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H}\right)$	$\frac{As}{m^3}$

Duality					
Electricity			Magnetism		
Special Case					
Dual Gauss law	$\nabla \cdot \mathbf{E} = 0$	$\frac{V}{m^2}$	Gauss law	$\nabla \cdot \mathbf{H} = 0$	$\frac{A}{m^2}$
Gauss law	$\nabla \cdot \mathbf{B} = 0$	$\frac{Vs}{m^3}$	Dual Gauss law	$\nabla \cdot \mathbf{D} = 0$	$\frac{As}{m^3}$
Electric voltage area density	$\mathbf{V} = 0$	$\frac{V}{m^2}$	Magnetic current area density	$\mathbf{J} = 0$	$\frac{A}{m^2}$
Faraday law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$	$\frac{V}{m^2}$	Dual Faraday law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0$	$\frac{A}{m^2}$

The comprehensive table of duality is fully symmetric in all respect wherein

i) The Ampere changes into Volt or vice versa.

$$Ampere \longleftrightarrow Volt$$

ii) The charge changes into potential or vice versa.

$$Charge \longleftrightarrow Potential$$

iii) The current changes into potential or vice versa.

$$Current \longleftrightarrow Voltage$$

iv) The sign change in the equations are due to the fact that the electric potential propagates from positive potential to negative or less positive electric potential whereas the magnetic current propagates from negative magnetic charge to positive or less negative magnetic charge i.e. south magnetic pole to north magnetic pole.

$$+ \longrightarrow \quad Electric\ Potential \quad \longrightarrow -$$

$$N \longleftarrow \quad Magnetic\ Current \quad \longleftarrow S$$

Before further analysis, we need to get the right tool for the right job or at least to know which tools are missing from our popular mathematical tool box i.e. Vector algebra. We all know or at least we can imagine if we haven't experienced, what it is like to unscrew a star headed screw with slotted screwdriver or draw a circle without a compass.

2. Mathematical Tool Box

The original Maxwell equations were originally formulated in Quaternions algebra. The four well known Maxwell equations in Vector algebra are actually the **Maxwell-Heaviside equations** which lost lots of details and important aspects entailed in the original Maxwell equations at the expense of slight easiness of calculation and understanding.

It is important to understand and acknowledge the limitation of our mathematical tool box which lacks a few absolutely necessary tools i.e. Vector algebra lacks certain necessary geometrical concepts. For example, in Vector algebra, electromagnetic waves can only be transverse, which means that the electric field vector is always perpendicular to the magnetic field vector, and the curl of the fields is perpendicular to the fields themselves.

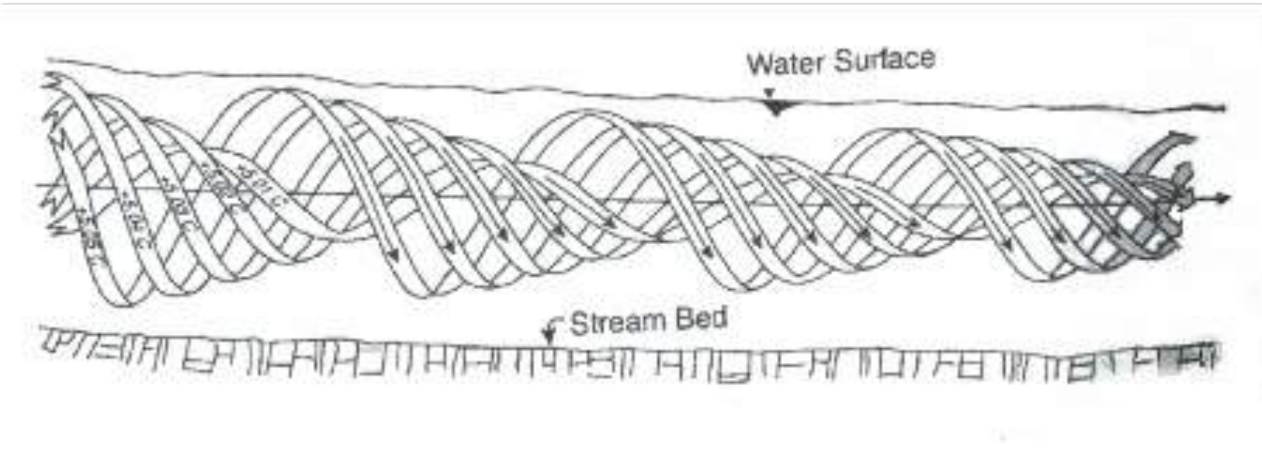
(1) $$\nabla \times \mathbf{E} \perp \mathbf{E}$$ [2.1]

(2) $$\nabla \times \mathbf{B} \perp \mathbf{B}$$ [2.2]

It has been known for more than a hundred years that the curl of a vector field and the field itself need not be perpendicular to one another. However, they can also be parallel.

In fact, towards the end of the nineteenth century, the Italian mathematician Eugenio Beltrami successfully developed a system of equations for the description of hydrodynamic flow, in which the curl of a vector is proportional to the vector itself.

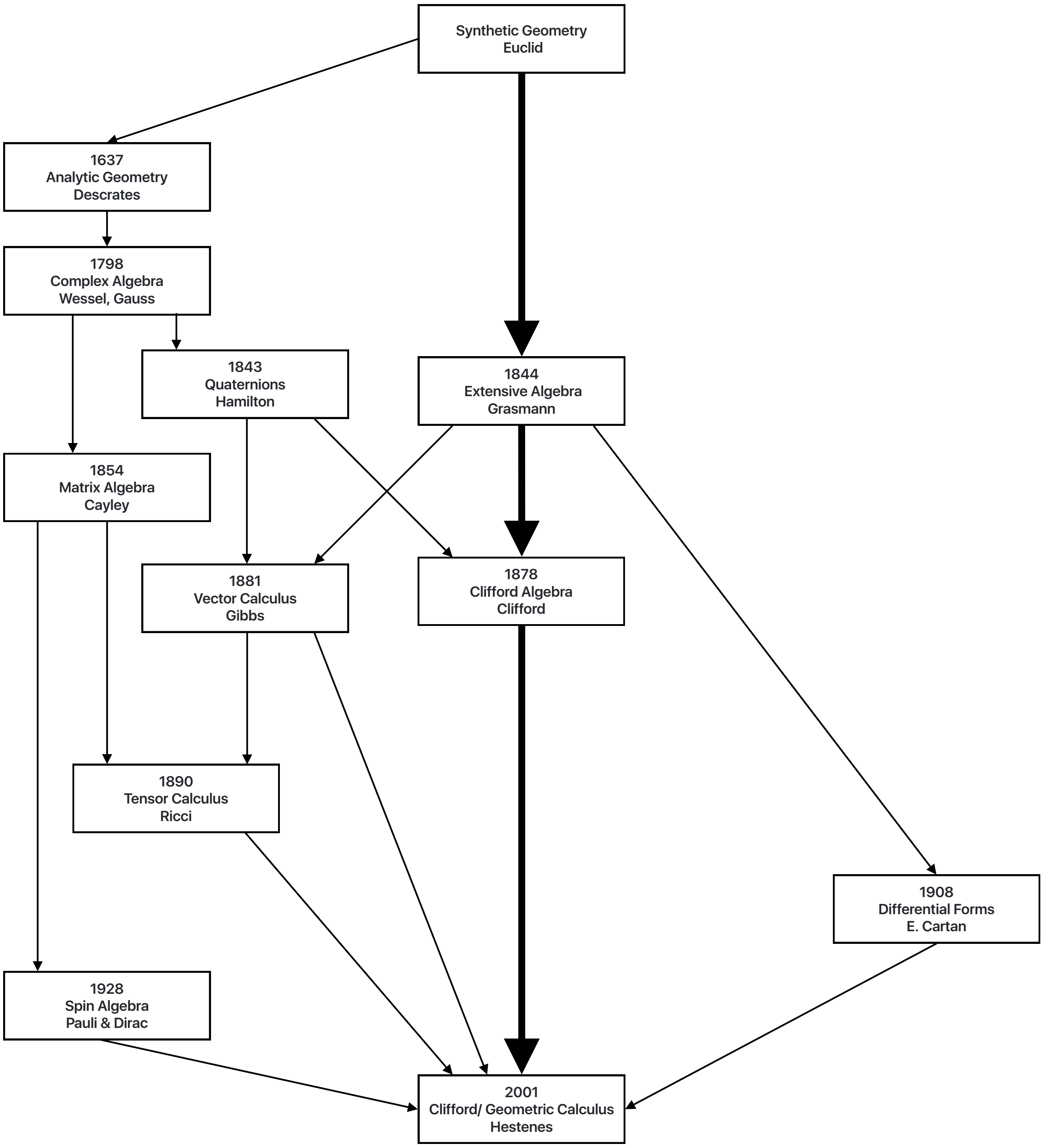
Figure 2.1



Beltrami Flow

This geometric aspect can better be understood and easily visualised in Clifford/ Geometric algebra, which is an algebrification of geometry and a geometrification of algebra at the same time. The Clifford algebra is the algebra of the physical space and subspaces on the real field. It provides a unified view of things. The popular Vector algebra is a stripped down version of Clifford algebra in three dimensions. The Clifford algebra is not complicated, it is only a few small steps beyond the Vector algebra. In addition, it extends easily into any number of dimensions with the same formalism, and elegantly unifies all Vector algebra systems under one roof.

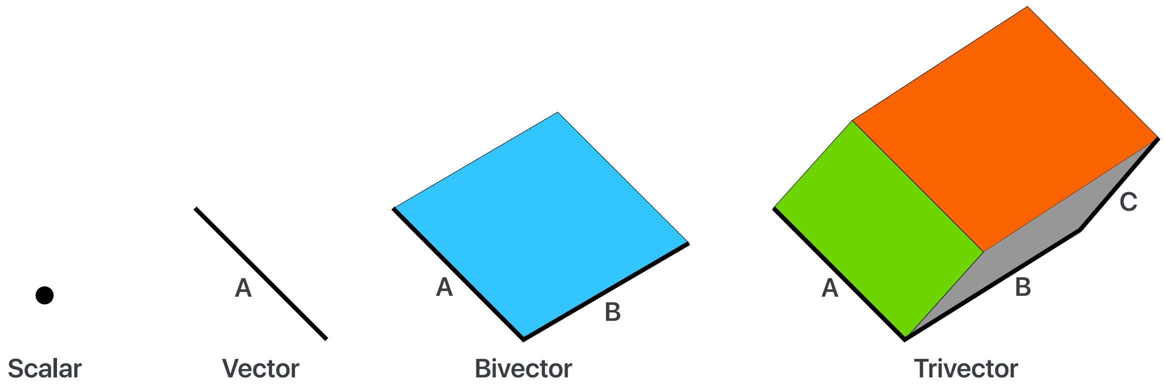
Figure 2.2



Unification

In Clifford algebra, a scalar can be visualised as an ideal point in space, which has no geometric extent. A vector can be visualised as a line segment, which has length and orientation. A bivector can be visualised as a patch of flat surface, which has area and orientation. A trivector can be visualised as a piece of three-dimensional space, which has a volume and an orientation.

Figure 2.3



Visualisation

Each object has a grade, according to how many dimensions are involved in its geometric extent. Therefore, Clifford algebra is a graded algebra. The situation is summarised in the following Table 2.1.

Table 2.1

Object	Visualisation	Geometric Extent	Grade
Scalar	Point	No Geometric Extent	0
Vector	Line Segment	Extent in 1 Direction	1
Bivector	Patch of Surface	Extent in 2 Directions	2
Trivector	Piece of Volume/ Space	Extent in 3 Directions	3

Two vectors $\mathbf{A}$ and $\mathbf{B}$ that are not collinear span a plane can be combined as $\mathbf{A} \wedge \mathbf{B}$. The operation $\wedge$ is called the outer (or wedge) product, and two vectors combined by $\wedge$ are called a bivector. The side on which the rotation of $\mathbf{A}$ toward $\mathbf{B}$ is anticlockwise, i.e., a positive rotation, is defined to be the front side, and the magnitude of bivector $\mathbf{A} \wedge \mathbf{B}$ is defined to be the area of the parallelogram defined by $\mathbf{A}$ and $\mathbf{B}$.

Non-coplanar vectors $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ span a 3D volume, which we denote by $\mathbf{A} \wedge \mathbf{B} \wedge \mathbf{C}$. Three vectors combined by $\wedge$ are called a trivector. As a geometric figure, it represents the entire space, but its sign is defined to be positive if $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are a right-handed system and negative if they are left-handed. The magnitude of trivector $\mathbf{A} \wedge \mathbf{B} \wedge \mathbf{C}$ is defined by the volume of the parallelepiped defined by $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$.

The orthogonal complement or dual of the plane specified by bivector $\mathbf{A} \wedge \mathbf{B}$ is a line orthogonal to it. The dual $(\mathbf{A} \wedge \mathbf{B})^*$ is defined to be a vector $\mathbf{C}$ orthogonal to plane $\mathbf{A} \wedge \mathbf{B}$ and by virtue of this to both $\mathbf{A}$ and $\mathbf{B}$ which lies in the plane $\mathbf{A} \wedge \mathbf{B}$, having the same magnitude as $\mathbf{A} \wedge \mathbf{B}$.

The orthogonal complement or dual of the space specified by trivector $\mathbf{A} \wedge \mathbf{B} \wedge \mathbf{C}$ is the origin (a scalar). The dual $(\mathbf{A} \wedge \mathbf{B} \wedge \mathbf{C})^*$ is defined to be a scalar α that has the same magnitude as $\mathbf{A} \wedge \mathbf{B} \wedge \mathbf{C}$.

Table 2.2

Dimension n=2					
	Blade			**Blade**	
Scalar	**Grade = 0**			**Grade = 0**	
Vector	**Grade = 1**			**Grade = 1**	
Bivector	**Grade = 2**			**Grade = 2**	**Pseudo scalar I_2**

Table 2.3

Dimension n=3					
	Blade			**Blade**	
Scalar	**Grade = 0**			**Grade = 0**	
Vector	**Grade = 1**			**Grade = 1**	
Bivector	**Grade = 2**			**Grade = 2**	**Pseudo vector**
Trivector	**Grade = 3**			**Grade = 3**	**Pseudo scalar I_3**

Table 2.4

Dimension n=4					
	Blade			**Blade**	
Scalar	**Grade = 0**			**Grade = 0**	
Vector	**Grade = 1**			**Grade = 1**	
Bivector	**Grade = 2**			**Grade = 2**	
Trivector	**Grade = 3**			**Grade = 3**	**Pseudo vector**
Quadvector	**Grade = 4**			**Grade = 4**	**Pseudo scalar I_4**

In n dimensions, the dual of a blade (a general term for a geometric object in Clifford algebra) of grade g is a blade of $(n-g)$ and vice versa as shown in Tables 2.2, 2.3 and 2.4.

The dual of a scalar is always a pseudo scalar. In three dimensions, the familiar "triple scalar product" $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C}$ produces a pseudo scalar. It's called pseudo because if you look at it in a mirror, it changes sign. This stands in contrast to an ordinary non-pseudo scalar, such as the number of marbles, which is unaffected by a reflection.

In three and four dimensions, it converts a vector to a certain "corresponding" pseudo vector aka axial vector. The familiar cross product $\mathbf{A} \times \mathbf{B}$ produces a pseudo vector aka axial vector.

Table 2.5

Pseudo Scalar Comparison					
$n=2$	I_2	**Bivector**	**Grade = 2**	$I_2^2 = -1$	$I_2\mathbf{A} = -\mathbf{A}I_2$
$n=3$	I_3	**Trivector**	**Grade = 3**	$I_3^2 = -1$	$I_3\mathbf{A} = \mathbf{A}I_3$
$n=4$	I_4	**Quadvector**	**Grade = 4**	$I_4^2 = -1$	$I_4\mathbf{A} = -\mathbf{A}I_4$

The dual $(\ldots)^*$ specifies the orthogonal complement of the direct representation, it represents the vector product or scalar triple product of the Vector algebra.

(3) $\quad (\mathbf{A} \wedge \mathbf{B})^* = \mathbf{A} \times \mathbf{B}$ [2.3]

(4) $(\mathbf{A} \wedge \mathbf{B} \wedge \mathbf{C})* = \mathbf{A} \cdot \mathbf{B} \times \mathbf{C}$ [2.4]

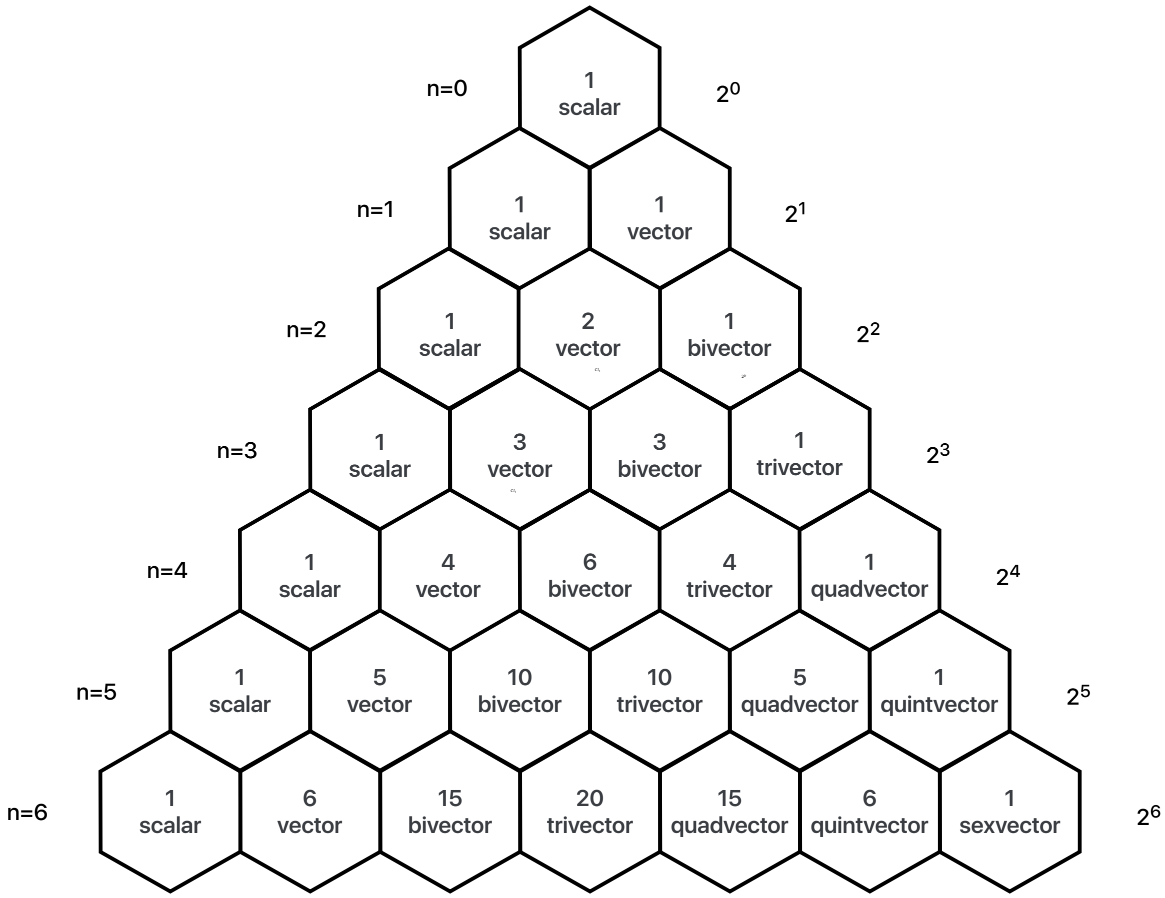
The dual representation of a field, i.e., the orthogonal complement of the direct representation, corresponds to the vector product or curl of a field of the Vector algebra.

(5) $(\nabla \wedge \mathbf{E})* = \nabla \times \mathbf{E} \longrightarrow \nabla \wedge \mathbf{E} = I\nabla \times \mathbf{E}$ [2.5]

(6) $(\nabla \wedge \mathbf{B})* = \nabla \times \mathbf{B} \longrightarrow \nabla \wedge \mathbf{B} = I\nabla \times \mathbf{B}$ [2.6]

The number of linear subspaces contained in a space depends on the number of dimensions involved as shown in the Figure 2.4which takes the form of Pascal's triangle. On each row, the total number of components is 2^n and contains the coefficients of the expanded binomial $(1 + x)^n$, and the power of x represents the grade of the subspace.

Figure 2.4



Pascal's Triangle

Like a complex number, which is the sum of the real part and so called the imaginary part which actually represents a rotation, a Clif is the sum of blades of various grades as shown in Table 2.6. In the literature, the Clif is called a multivector, but this term is misleading and may cause confusion and errors.

Table 2.6

Dimensions	Clif
$n = 2$	$C = scalar + vector + bivector$
$n = 3$	$C = scalar + vector + bivector + trivector$
$n = 4$	$C = scalar + vector + bivector + trivector + quadvector$

Similar to multiplication of complex numbers, any geometric object in Clifford algebra can be multiplied with any other object. The geometric product of two vectors (grade=1 only) gives only two terms, a scalar term and a bivector term (assuming the bivector term is nonzero). The geometric product of a vector and a bivector has two terms, a vector term and a trivector term. The geometric product of two bivectors (grade=2 only) has three terms, a scalar term, a bivector term and a quadvector term. The geometric product is not commutative.

The geometric product of two vectors is

(7) $$\mathbf{AB} = \langle \mathbf{A} \cdot \mathbf{B} \rangle_0 + \langle \mathbf{A} \wedge \mathbf{B} \rangle_2$$ [2.7]

(8) $$\langle \mathbf{A} \cdot \mathbf{B} \rangle_0 = scalar$$ [2.8]

(9) $$\langle \mathbf{A} \wedge \mathbf{B} \rangle_2 = bivector$$ [2.9]

The equation [2.8] is the dot product, inner product, or scalar product from the Vector algebra where where $\mathbf{A}$ and $\mathbf{B}$ are ordinary grade =1 vectors. The grade of the dot product is zero i.e. the grade of the dot product is $|\,grade\ \mathbf{A} - grade\ \mathbf{B}\,|$.

(10) $$\langle \mathbf{A} \cdot \mathbf{B} \rangle_0 = \frac{\mathbf{AB} + \mathbf{BA}}{2}$$ [2.10]

where $\mathbf{A}$ and $\mathbf{B}$ have $grade = 1$

(11) $$\langle \mathbf{A} \wedge \mathbf{B} \rangle_2 = \frac{\mathbf{AB} - \mathbf{BA}}{2}$$ [2.11]

where $\mathbf{A}$ and $\mathbf{B}$ have $grade \leq 1$

The geometric product of a vector and a bivector is

(12) $\mathbf{A}\langle\mathbf{B}\rangle_2 = \langle\mathbf{A}\cdot\mathbf{B}\rangle_1 + \langle\mathbf{A}\wedge\mathbf{B}\rangle_3$ [2.12]

(13) $\langle\mathbf{A}\cdot\mathbf{B}\rangle_1 = vector$ [2.13]

(14) $\langle\mathbf{A}\wedge\mathbf{B}\rangle_3 = trivector$ [2.14]

The equation [2.13] is called the contraction. It generalises the dot product. There are multiple contractions possible between two higher grade objects and the highest contraction is the lowest-grade piece of the geometric product.

(15) $\langle\mathbf{A}\cdot\langle\mathbf{B}\rangle_{\mathbf{2}}\rangle_1 = \dfrac{\mathbf{AB}-\mathbf{BA}}{2}$ [2.15]

(16) $\langle\mathbf{A}\wedge\langle\mathbf{B}\rangle_{\mathbf{2}}\rangle_3 = \dfrac{\mathbf{AB}+\mathbf{BA}}{2}$ [2.16]

The signs in equation [2.15] and [2.16] are the opposite way round to the case of two vectors in equation [2.10] and [2.11] respectively.

The geometric product of a vector with a blade of grade r is

(17) $\mathbf{A}\langle\mathbf{B}\rangle_r = \langle\mathbf{A}\cdot\langle\mathbf{B}\rangle_r\rangle_{(r-1)} + \langle\mathbf{A}\wedge\langle\mathbf{B}\rangle_r\rangle_{(r+1)}$ [2.17]

(18) $\langle\mathbf{A}\cdot\langle\mathbf{B}\rangle_{\mathbf{r}}\rangle_{(r-1)} = \dfrac{\mathbf{AB}+(-1)^{r-1}\mathbf{BA}}{2}$ [2.18]

(19) $\langle\mathbf{A}\wedge\langle\mathbf{B}\rangle_{\mathbf{r}}\rangle_{(r+1)} = \dfrac{\mathbf{AB}-(-1)^{r+1}\mathbf{BA}}{2}$ [2.19]

The symmetries of inner/ dot products and outer/ wedge product alternate for the odd or even grade blade $\langle\mathbf{B}\rangle_r$.

The geometric product of two bivectors is

(20) $\langle \mathbf{A} \rangle_2 \langle \mathbf{B} \rangle_2 = \langle \mathbf{A} \cdot \cdot \ \mathbf{B} \rangle_0 + \langle \mathbf{A} \cdot \mathbf{B} \rangle_2 + \langle \mathbf{A} \wedge \mathbf{B} \rangle_4$ [2.20]

(21) $\langle \mathbf{A} \cdot \cdot \ \mathbf{B} \rangle_0 = scalar$ [2.21]

(22) $\langle \mathbf{A} \cdot \mathbf{B} \rangle_2 = bivector$ [2.22]

(23) $\langle \mathbf{A} \wedge \mathbf{B} \rangle_4 = quadvector$ [2.23]

The equation [2.21] represents the contraction. The quadvector vanishes in three dimensions and represents a pseudo scalar in four dimensions.

The geometric product of two trivectors is

(24) $\langle \mathbf{A} \rangle_3 \langle \mathbf{B} \rangle_3 = \langle \mathbf{A} \cdot \cdot \cdot \mathbf{B} \rangle_0 + \langle \mathbf{A} \cdot \cdot \ \mathbf{B} \rangle_2 + \langle \mathbf{A} \cdot \mathbf{B} \rangle_4 + \langle \mathbf{A} \wedge \mathbf{B} \rangle_6$ [2.24]

(25) $\langle \mathbf{A} \cdot \cdot \cdot \mathbf{B} \rangle_0 = scalar$ [2.25]

(26) $\langle \mathbf{A} \cdot \cdot \ \mathbf{B} \rangle_2 = bivector$ [2.26]

(27) $\langle \mathbf{A} \cdot \mathbf{B} \rangle_4 = quadvector$ [2.27]

(28) $\langle \mathbf{A} \wedge \mathbf{B} \rangle_6 = sextvector$ [2.28]

The equation [2.25] represents the contraction. The quadvector vanishes in three dimensions and represents a pseudo scalar in four dimensions. The sextvector vanishes in both three and four dimensions.

On multiplying an object of grade r by an object of grade s $(s \geq r)$, the geometric product contains $(r + 1)$ terms of grades starting from $|s - r|$ with an increment by 2 until $|s + r|$. There are r terms of inner/ dot products and one term of outer/ wedge product.

(29) $$\langle \mathbf{A} \rangle_r \langle \mathbf{B} \rangle_s = \langle \mathbf{A} \ (rdot) \ \mathbf{B} \rangle_{|s-r|} + \ldots + \langle \mathbf{A} \cdot \cdot \ \mathbf{B} \rangle_{|s+r-4|} + \langle \mathbf{A} \cdot \mathbf{B} \rangle_{|s+r-2|} + \langle \mathbf{A} \wedge \mathbf{B} \rangle_{|s+r|}$$ [2.29]

The definite symmetry (or antisymmetry) of each grade$\langle \ldots \rangle_{|s+r-2t|}$, where $t = 0,\ 1,\ \ldots,\ r$, is represented by $(-1)^{\frac{r(r-1)}{2}}(-1)^{\frac{s(s-1)}{2}}\ (-1)^{\frac{(r+s-2t)(r+s-2t-1)}{2}}$.

The Clifford algebra is amazingly elegant in handling rotations. It emphasises the plane of rotation rather than the axis of rotation and works equally well in any number of dimensions. The plane of rotation seems more natural than an axis of rotation. For example, if you are sitting in an aircraft, you can see the plane of rotation for yaw-wise rotations spread out in front of you, running left/right. It is somewhat less natural to visualise the vertical axis (even though the two representations are technically equivalent in three dimensions).

In Clifford algebra a rotation is represented by the geometric product of two vectors which is Lorentz-invariant. The exact choice of vectors doesn't matter; there are many different pairs of vectors that specify the same rotation. The plane containing the two vectors is the plane of rotation, and the angle between the two vectors is half the angle of rotation. The half angle has a deep physical significance and avoids any singularities. The rotation is expressed by

(30) $$\mathbf{x}' = R^{\sim}\mathbf{x}R$$ [2.30]

where $\mathbf{x}$ is the unrotated vector, $\mathbf{x}'$ is a vector that is rotated relative to $\mathbf{x}$ by an angle θ, R is a rotor with rotor angle $\dfrac{\theta}{2}$ and $R^{\sim}$ is the reverse of R. The reverse is a generalisation of the notion of the complex conjugate.

Compound rotations are represented by a product of rotors in the obvious way. For example, if you carry out a rotation described by rotor R_1 and then follow it by another rotation described by rotor R_2, the overall rotation composed of the two rotations is specified by the rotor R_{12} by multiplying the two rotors, in order

(31) $$R_{12} = R_1 R_2$$ [2.31]

The rotor can also be expressed in exponential form

(32) $$R(\frac{\theta}{2}) = cos(\frac{\theta}{2}) + e_1 e_2 sin(\frac{\theta}{2})$$ [2.32]

(33) $R(\frac{\theta}{2}) = e^{\frac{\theta}{2}e_1e_2}$ [2.33]

where e_1 and e_2 are orthonormal basis vectors.

There is a two-to-one relationship between rotors and rotations. Any given rotation can be represented by two inequivalent rotors i.e. R and $-R$. If something rotates by 2π radians in any plane, it gets back the same attitude, but the rotor picks up a minus sign. It must rotate 4π radians to get back the original rotor. This is known as spinors. This is can be experienced first hand by the Dirac string trick or/ and the Philippine wine-glass trick.

A Clif consisting of terms of odd grades is called an odd Clif, and a Clif consisting of terms of even grades including grade zero i.e. scalar, is called an even Clif. It is easily seen that the product of two even Clifs and the product of two odd Clifs are even Clifs, and the product of even and odd Clifs is an odd Clif. The sums and scalar multiples of even Clifs are even Clifs and their products are also even Clifs. This means that the set of even Clifs forms by itself a closed algebra i.e. an even sub algebra of the Clifford algebra.

In fact, the Complex numbers are even a sub algebra of Clifford algebra in two dimensions, containing just scalars and bivectors. Similarly, the Quaternions, which are a higher dimensional generalisation of the complex numbers, are an even sub algebra of Clifford algebra in three dimensions, containing just scalars and bivectors. The reason, we are not able to immediately recognise it, is because the real even subspaces i.e. bivectors have also been complexified. This means that the two dimensions of the bivectors have combined into one single dimension. Whenever we complexity things, we loose degree of transparency and understanding. This is the reason why most of the people are still unaware that the imaginary part of complex number represents rotation and made the Quaternions algebra very difficult to understand. The Dirac algebra is nothing but the Clifford algebra with complexification in spacetime. The Pauli algebra is the even sub algebra of the Dirac algebra. The quaternion algebra can also be seen as the even subalgebra of the Pauli algebra. The complex numbers can also be seen as the even subalgebra of the quaternions.

In addition, the Clifford algebra resolves the apparent chirality (handedness) in electromagnetism i.e. the right-hand rule for electric generators, and the left-hand rule for electric motors. The cross product is not preserved under reflection, and thus it introduces a fake chirality in electromagnetism. It is an artifact of the mathematics used to describe the world, not a property of the world itself.

Thus, the Vector algebra lacks few important concepts which are essential for our analysis, visualisation and correct geometrical interpretation, namely

i) Bivector, Trivector and so on
ii) Orthogonal complement or Dual space
iii) Clif
iv) Geometric product
v) Rotations

It can be easily visualised from Figure 2.3 that a vector can be parallel to a bivector of which it is a part.

(34) $$\mathbf{A} \wedge \mathbf{B} \parallel \mathbf{A}$$ [2.34]

(35) $$\mathbf{A} \wedge \mathbf{B} \parallel \mathbf{B}$$ [2.35]

Therefore it is possible that

(36) $$\nabla \wedge \mathbf{E} \parallel \mathbf{E}$$ [2.36]

(37) $$\nabla \wedge \mathbf{B} \parallel \mathbf{B}$$ [2.37]

The Beltrami condition uses the original interpretation of direct representation of a curl of a field from the Clifford algebra and imports it into the Vector algebra i.e. applies it to the indirect representation of a curl of a field. Such solutions of the Maxwell-Heaviside equations are called the **Beltrami solutions**.

(38) $$\nabla \wedge \mathbf{E} \parallel \mathbf{E} \longrightarrow \nabla \wedge \mathbf{E} = \omega_p \mathbf{E} \longrightarrow \nabla \times \mathbf{E} = \kappa_p \mathbf{E}$$ [2.38]

Therefore

(39) $$\nabla \cdot \mathbf{E} = 0$$ [2.39]

(40) $$\nabla \wedge \mathbf{B} \parallel \mathbf{B} \longrightarrow \nabla \wedge \mathbf{B} = -\omega_e \mathbf{B} \longrightarrow \nabla \times \mathbf{B} = -\kappa_e \mathbf{B}$$ [2.40]

Therefore

(41) $\nabla \cdot \mathbf{B} = 0$ [2.41]

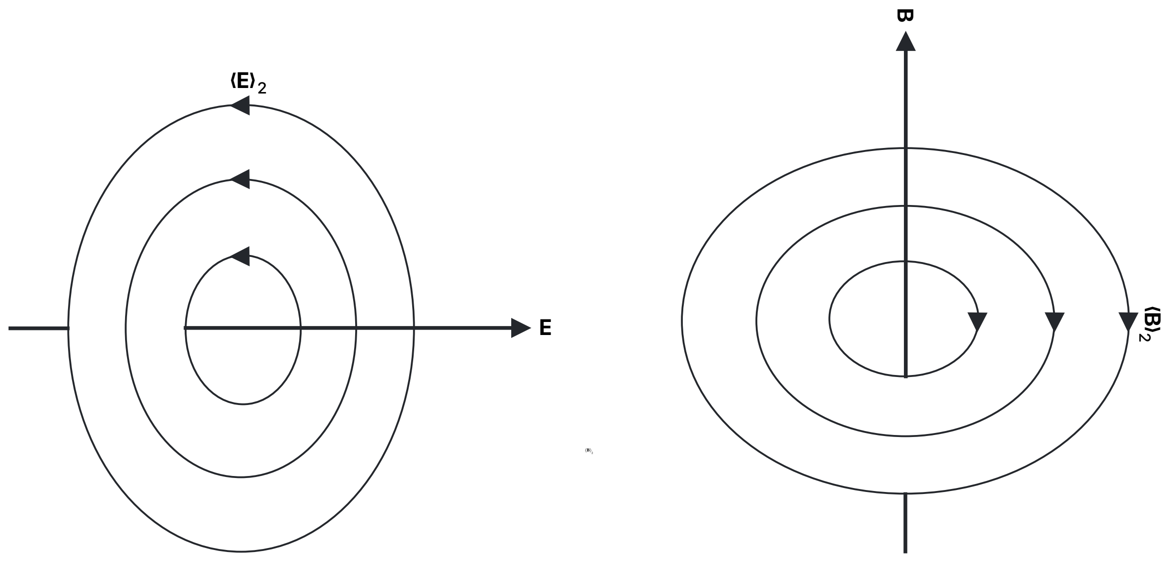
The ω_p is the rotation of vector $\mathbf{E}$ and ω_e is the rotation of vector $\mathbf{B}$. In the process of applying the direct representation curl of the field from the Clifford algebra into the Vector algebra, ω was appropriately changed to κ.

In the Clifford algebra literature, the electric field is represented by a vector $\mathbf{E}$, whereas the magnetic flux density is represented by a bivector $\langle \mathbf{B} \rangle_2$ to derive the Maxwell equations using the Lorentz force, which encompass Faraday's unipolar induction.

Thus, the electric field vector $\mathbf{E}$ has an orthogonal complement or dual, being a bivector $\langle \mathbf{E} \rangle_2$, and the magnetic flux density $\langle \mathbf{B} \rangle_2$ has an orthogonal complement or dual, being a vector $\mathbf{B}$. Thus, both the electric field $\mathbf{E}$ and the magnetic flux density $\mathbf{B}$ are either vectors in their true/direct representation and their orthogonal complement, or are dual bivectors $\langle \mathbf{E} \rangle_2$ and $\langle \mathbf{B} \rangle_2$, respectively, or vice versa.

Figure 2.5 shows that the two vectors $\mathbf{E}$, $\mathbf{B}$ and the two bivectors $\langle \mathbf{E} \rangle_2$, $\langle \mathbf{B} \rangle_2$ can be perpendicular and parallel at same time due to cross interaction between orthogonal complements or dual spaces.

Figure 2.5



Orthogonal Complement or Dual Space

Table 2.7-1

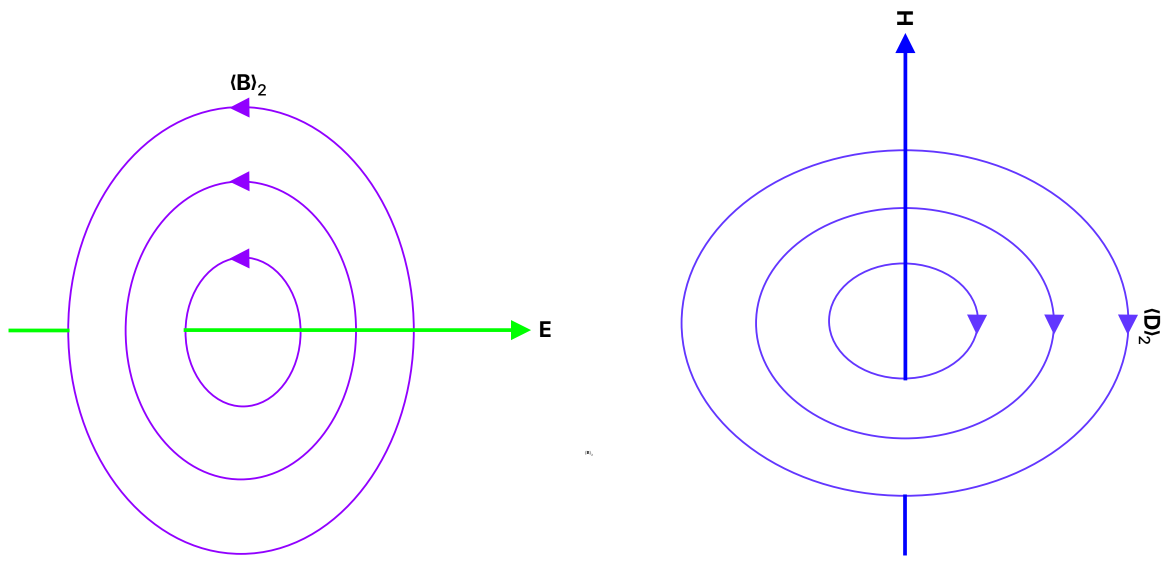
Penpendicular Interaction	Dual Parallel Interaction
$\mathbf{E} \perp \langle\mathbf{E}\rangle_2$	$\mathbf{E} \parallel \langle\mathbf{B}\rangle_2$
$\mathbf{B} \perp \langle\mathbf{B}\rangle_2$	$\mathbf{B} \parallel \langle\mathbf{E}\rangle_2$

In standard notations, the electric field vector $\mathbf{E}$ is represented by the electric field strength $\mathbf{E}$ and the magnetic field vector $\mathbf{B}$ is represented by the magnetic field strength $\mathbf{H}$. The electric field bivector $\langle\mathbf{E}\rangle_2$ is represented by the magnetic flux density $\mathbf{B}$ and the magnetic field bivector is $\langle\mathbf{B}\rangle_2$ is represented by the dielectric displacement $\mathbf{D}$.

(42) $$\begin{aligned}\mathbf{E} &\equiv \mathbf{E}\\ \langle\mathbf{E}\rangle_2 &\equiv \langle\mathbf{B}\rangle_2\end{aligned}$$ [2.42]

(43) $$\begin{aligned}\mathbf{B} &\equiv \mathbf{H}\\ \langle\mathbf{B}\rangle_2 &\equiv \langle\mathbf{D}\rangle_2\end{aligned}$$ [2.43]

Figure 2.6

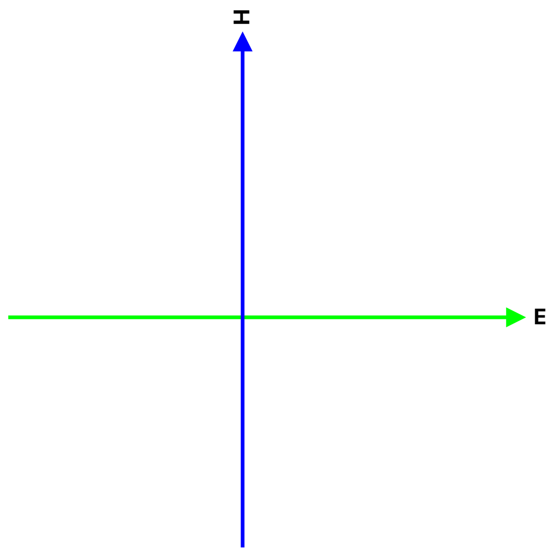


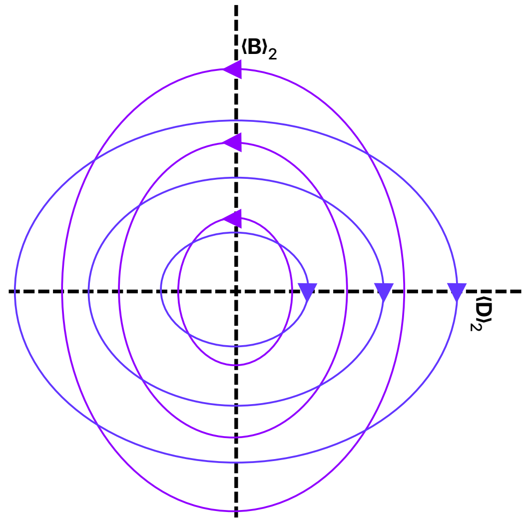
Orthogonal Complement or Dual Space

Table 2.8

Standard Notations	
Penpendicular Interaction	**Dual Parallel Interaction**
$\mathbf{E} \perp \mathbf{H}$	$\mathbf{E} \parallel \langle \mathbf{D} \rangle_2$
$\langle \mathbf{B} \rangle_2 \perp \langle \mathbf{D} \rangle_2$	$\mathbf{H} \parallel \langle \mathbf{B} \rangle_2$

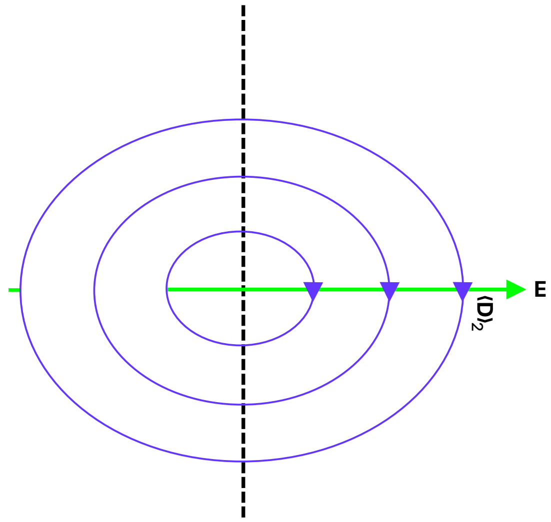
Figure 2.7

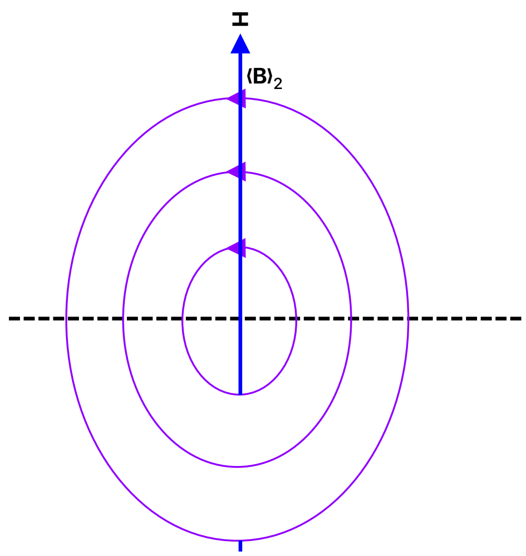




Perpendicular Interaction

Figure 2.8





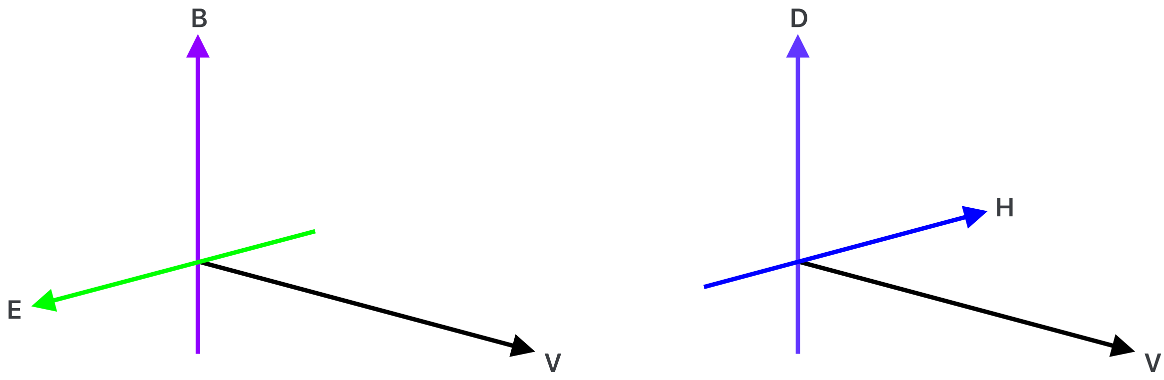
Parallel Interaction

As we are aware that in the Vector algebra, both the magnetic flux density $\mathbf{B}$ and the dielectric displacement $\mathbf{D}$ are also vectors and the equations of transformation, i.e., the unipolar induction by Faraday/ the Faraday law and the equation of convection/ the Ampère-Maxwell law, are valid at the same time.

$$\mathbf{E} = \mathbf{v} \times \mathbf{B} \longrightarrow \mathbf{v} \perp \mathbf{E} \perp \mathbf{B} \quad [1.13]$$

$$\mathbf{H} = -\,\mathbf{v} \times \mathbf{D} \longrightarrow \mathbf{v} \perp \mathbf{H} \perp \mathbf{D} \quad [1.14]$$

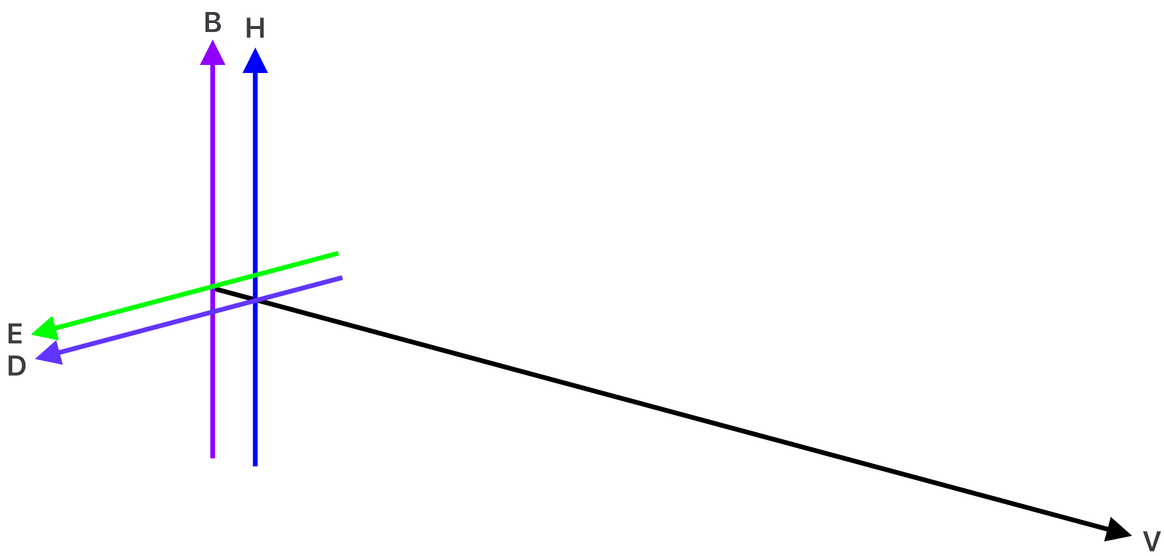
Figure 2.9



Fragmented Equations of Transformation

The equations {1.13] and [1.14] can only be possible if $\mathbf{E} \parallel \mathbf{D}$ and $\mathbf{H} \parallel \mathbf{B}$ as shown in Figure 2.10.

Figure 2.9



Unified Equations of Transformation

It is evident from Figure 2.10 that parallel interaction is inherent in the Vector algebra, even though the existence of parallel interaction has always been denied. As we will see later, the well known energy density formulation in the Vector algebra unquestionably confirms the existence of parallel interaction.

(44) $$\begin{aligned} \mathbf{E} \cdot \mathbf{D} \neq 0 \\ \mathbf{E} \times \mathbf{D} = 0 \end{aligned}$$ [2.44]

(45) $$\begin{aligned} \mathbf{H} \cdot \mathbf{B} \neq 0 \\ \mathbf{H} \times \mathbf{B} = 0 \end{aligned}$$ [2.45]

As the equations of transformation, i.e., the unipolar induction by Faraday and the equation of convection, are applicable to both perpendicular and parallel interaction, this implies that even the parallel interaction is a transverse wave like perpendicular interaction. The parallel interaction is not a longitudinal wave.

The magnetic flux density $\mathbf{B}$ and the dielectric displacement $\mathbf{D}$ are usually treated as bivectors both in spacetime and in three dimensions with separate time dimension, which is an even sub algebra of spacetime algebra. This results in zero energy density as we will discover this in our analysis. An easy and correct method to decipher the grade of the geometric object is to count the number of dimensions including the time dimension contained in the unit of the object. We always considered velocity $\mathbf{v}$ as vector, but it is a bivector which has a rotation. Like the curvature of the earth, we do not feel or observe the rotation when we drive our cars but it is always present in reality irrespective of our feeling or observation. This is particularly important in understanding various field velocities in our analysis. The grade of the electric and magnetic objects are summarised in the following Table 2.10.

Table 2.10

Duality							
Electricity				**Magnetism**			
Electric field strength	$\mathbf{E}$	$\frac{V}{m}$	**Vector**	Magnetic field strength	$\mathbf{H}$	$\frac{A}{m}$	**Vector**

Duality							
Electricity				Magnetism			
Magnetic flux density or Magnetic induction	$\mathbf{B}$	$\frac{Vs}{m^2}$	**Bivector** in 3 dimensions and **Trivector** in 4 dimensions	Dielectric displaceme nt or Electric induction	$\mathbf{D}$	$\frac{As}{m^2}$	**Bivector** in 3 dimensions and **Trivector** in 4 dimensions
Elementary electric potential	p	$Wb = Vs$	**Scalar** in 3 dimensions and **Vector** in 4 dimensions	Elementary magnetic charge	e	$C = As$	**Scalar** in 3 dimensions and **Vector** in 4 dimensions
Electric potential	ϕ_p	$Wb = Vs$	**Scalar** in 3 dimensions and **Vector** in 4 dimensions	Magnetic charge	q	$C = As$	**Scalar** in 3 dimensions and **Vector** in 4 dimensions
Electric voltage area density	$\mathbf{V}$	$\frac{V}{m^2}$	**Bivector**	Magnetic current area density	$\mathbf{J}$	$\frac{A}{m^2}$	**Bivector**
Velocity of electric field	$\mathbf{v}_E$	$\frac{m}{s}$	**Vector** in 3 dimensions and **Bivector** in 4 dimensions	Velocity of magnetic field	$\mathbf{v}_H$	$\frac{m}{s}$	**Vector** in 3 dimensions and **Bivector** in 4 dimensions

The standard electromagnetism in Vector algebra with separate time dimension neither has a concept of dual space nor distinguishes between the vector and the bivector. It completely ignores 50% of the reality, i.e. dual parallel interaction. It is akin to working with one eye which on top is colour blind. It has been distorting reality and perennially denying the existence of dual parallel interaction, i.e. $\mathbf{E} \parallel \langle \mathbf{D} \rangle_2$ and $\mathbf{H} \parallel \langle \mathbf{B} \rangle_2$.

3. Dual Parallel Interaction

The Helmholtz style equations can be derived by applying the Beltrami conditions of direct representation of a field, i.e dual space in Vector algebra. The Beltrami conditions [2.38] to [2.41] for vacuum are

$$\nabla \times \mathbf{E} = \kappa_p \mathbf{E} \quad [2.38]$$

$$\nabla \cdot \mathbf{E} = 0 \quad [2.39]$$

$$\nabla \times \mathbf{B} = -\kappa_e \mathbf{B} \quad [2.40]$$

$$\nabla \cdot \mathbf{B} = 0 \quad [2.41]$$

The equation [2.41] is known as **Gauss law.** This is not a special case as postulated before but inherent in the dual parallel interaction. The equation [2.39] can be seen as the **DUAL GAUSS LAW.**

On applying the curl operator twice to equation [2.38] and equation [2.40] gives

(1) $$\nabla \times \nabla \times \mathbf{E} = \kappa_p \nabla \times \mathbf{E} = \kappa_p^2 \mathbf{E} \quad [3.1]$$

(2) $$\nabla \times \nabla \times \mathbf{B} = -\kappa_e \nabla \times \mathbf{B} = \kappa_e^2 \mathbf{B} \quad [3.2]$$

These are similar to Trkalian equations, except that the wave number is corresponding to the wave length of the field.

On using vector calculus Identity gives

(3) $$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \kappa_p^2 \mathbf{E} \quad [3.3]$$

(4) $$\nabla \times \nabla \times \mathbf{B} = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \kappa_e^2 \mathbf{B} \quad [3.4]$$

On substituting equation [2.38] in equation [3.3] gives

(5) $$\nabla^2 \mathbf{E} + \kappa_p^2 \mathbf{E} = 0 \quad [3.5]$$

On substituting equation [2.40] in equation [3.4] gives

(6) $$\nabla^2 \mathbf{B} + \kappa_e^2 \mathbf{B} = 0$$ [3.6]

The equations [3.5] and [3.6] are similar to Helmholtz equations, except the wave number is corresponding to the field.

According to Beltrami condition/ Gauss law [2.39] and [2.41], the divergences of the $\mathbf{E}$ and $\mathbf{B}$ fields are always zero in the dual parallel interaction.

Therefore

(7) $$\mathbf{J} = -\,\mathbf{v}(\nabla \cdot \mathbf{D}) = 0$$ [3.7]

(8) $$\mathbf{V} = \mathbf{v}(\nabla \cdot \mathbf{B}) = 0$$ [3.8]

The dual Ampere Maxwell law [1.50] becomes

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{V}$$ [1.50]

(9) $$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$ [3.9]

(10) $$\nabla \times \mathbf{E} = -\,\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$ [3.10]

The equation [3.9] is known as **Faraday's law.** This is not a special case as postulated before but inherent in the dual space.

The Ampere Maxwell law [1.9] becomes

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$ [1.9]

(11) $$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0$$ [3.11]

(12) $\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ [3.12]

The equation [3.11] in the dual space can be seen as the **DUAL FARADAY LAW**.

Table 3.1

Duality					
Electricity			**Magnetism**		
Yang	Male		Yin	Female	
Positive	$+$		Negative	$-$	
North pole	$\mathbf{N}$		South pole	$\mathbf{S}$	
Direction of movement	$\mathbf{N} \longrightarrow \mathbf{S}$		Direction of movement	$\mathbf{N} \longleftarrow \mathbf{S}$	
Vacuum permittivity	ϵ_0	$\frac{As}{Vm}$	Vacuum permeability	μ_0	$\frac{Vs}{Am}$
Vacuum conductivity	σ_0	$\frac{A}{Vm}$	Vacuum potentivity	η_0	$\frac{V}{Am}$
Vacuum angular frequency for Electric induction	$\omega_p = \frac{\sigma_0}{\epsilon_0}$	$\frac{1}{s}$	Vacuum angular frequency for Magnetic induction	$\omega_e = \frac{\eta_0}{\mu_0}$	$\frac{1}{s}$
Voltage	U	V	Current	I	A
Electric field strength	$\mathbf{E}$	$\frac{V}{m}$	Magnetic field strength	$\mathbf{H}$	$\frac{A}{m}$
Magnetic flux density or Magnetic induction	$\mathbf{B} = \mu_0 \mathbf{H}$	$\frac{Vs}{m^2}$	Dielectric displacement or Electric induction	$\mathbf{D} = \epsilon_0 \mathbf{E}$	$\frac{As}{m^2}$
Elementary electric potential	p	$Wb = Vs$	Elementary magnetic charge	e	$C = As$
Electric potential	ϕ_p	$Wb = Vs$	Magnetic charge	q	$C = As$
Electric potential density	ρ_p	$\frac{Vs}{m^3}$	Magnetic charge density	ρ_e	$\frac{As}{m^3}$

Duality					
Electricity			Magnetism		
Unipolar induction	$\mathbf{E} = \mathbf{v} \times \mathbf{B}$	$\frac{V}{m}$	Equation of convection	$\mathbf{H} = -\mathbf{v} \times \mathbf{D}$	$\frac{A}{m}$
Perpendicular Interaction					
Coulomb law	$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}$	$\frac{V}{m^2}$	Dual Coulomb law	$\nabla \cdot \mathbf{H} = \frac{\rho_p}{\mu_0}$	$\frac{A}{m^2}$
Dual Coulomb law	$\nabla \cdot \mathbf{B} = \rho_p$	$\frac{Vs}{m^3}$	Coulomb law	$\nabla \cdot \mathbf{D} = \rho_e$	$\frac{As}{m^3}$
Electric voltage area density	$\mathbf{V} = \mathbf{v}(\nabla \cdot \mathbf{B})$	$\frac{V}{m^2}$	Magnetic current area density	$\mathbf{J} = -\mathbf{v}(\nabla \cdot \mathbf{D})$	$\frac{A}{m^2}$
Dual ohms law	$\mathbf{V} = \eta_0 \mathbf{H}$		Ohms law	$\mathbf{J} = \sigma_0 \mathbf{E}$	
	$\mathbf{V} = \omega_e \mathbf{B}$			$\mathbf{J} = \omega_p \mathbf{D}$	
Dual Ampère-Maxwell law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{V}$	$\frac{V}{m^2}$	Ampère-Maxwell law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$	$\frac{Vs}{m^3}$		$\nabla \times \mathbf{D} + \frac{1}{c^2}\frac{\partial \mathbf{H}}{\partial t} = -\epsilon_0 \mathbf{V}$	$\frac{As}{m^3}$
	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \omega_e \mathbf{B}$	$\frac{V}{m^2}$		$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \omega_p \mathbf{D}$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{E} = -\mu_0 \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H}\right)$	$\frac{V}{m^2}$		$\nabla \times \mathbf{H} = \epsilon_0 \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E}\right)$	$\frac{A}{m^2}$
	$\nabla \times \mathbf{B} = \frac{1}{c^2}\left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E}\right)$	$\frac{Vs}{m^3}$		$\nabla \times \mathbf{D} = -\frac{1}{c^2}\left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H}\right)$	$\frac{As}{m^3}$
Dual Parallel Interaction					
Dual Gauss law	$\nabla \cdot \mathbf{E} = 0$	$\frac{V}{m^2}$	Gauss law	$\nabla \cdot \mathbf{H} = 0$	$\frac{A}{m^2}$
Gauss law	$\nabla \cdot \mathbf{B} = 0$	$\frac{Vs}{m^3}$	Dual Gauss law	$\nabla \cdot \mathbf{D} = 0$	$\frac{As}{m^3}$

Duality					
Electricity			**Magnetism**		
Electric voltage area density	$\mathbf{V} = 0$	$\frac{V}{m^2}$	Magnetic current area density	$\mathbf{J} = 0$	$\frac{A}{m^2}$
Faraday law	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$	$\frac{V}{m^2}$	Dual Faraday law	$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0$	$\frac{A}{m^2}$

4. Wave Equations

Applying the curl to the Faraday law [1.80] gives

$$\nabla \times \mathbf{E} = -\mu_0 \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H} \right) \tag{1.80}$$

(1) $$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \nabla \times \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H} \right) \tag{4.1}$$

(2) $$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \left(\frac{\partial}{\partial t}(\nabla \times \mathbf{H}) - \omega_e (\nabla \times \mathbf{H}) \right) \tag{4.2}$$

On substituting Ampère-Maxwell law [1.76] gives

$$\nabla \times \mathbf{H} = \epsilon_0 \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E} \right) \tag{1.76}$$

(3) $$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \epsilon_0 \left(\frac{\partial}{\partial t} \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E} \right) - \omega_e \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E} \right) \right) \tag{4.3}$$

(4) $$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2} \left(\frac{\partial}{\partial t} \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E} \right) - \omega_e \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E} \right) \right) \tag{4.4}$$

(5) $$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2} \left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p \frac{\partial \mathbf{E}}{\partial t} + \mathbf{E} \frac{\partial \omega_p}{\partial t} - \omega_e \frac{\partial \mathbf{E}}{\partial t} - \omega_e \omega_p \mathbf{E} \right) \tag{4.5}$$

The angular frequency ω_h of outward spiralling electric induction for a medium is always constant, for example, in vacuum or in any metal.

Therefore

(6) $$\frac{\partial \omega_p}{\partial t} = 0 \tag{4.6}$$

On substituting the equation [4.6] in equation [4.5] gives

(7) $$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p \frac{\partial \mathbf{E}}{\partial t} - \omega_e \frac{\partial \mathbf{E}}{\partial t} - \omega_e \omega_p \mathbf{E}\right)$$ [4.7]

(8) $$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p \frac{\partial \mathbf{E}}{\partial t} - \omega_e \frac{\partial \mathbf{E}}{\partial t} - \omega_e \omega_p \mathbf{E}\right)$$ [4.8]

According to Clifford Algebra

(9) $$\nabla(\nabla \cdot \mathbf{E}) = \nabla \cdot (\nabla \cdot \mathbf{E}) + \nabla \wedge (\nabla \cdot \mathbf{E})$$ [4.9]

(10) $$\nabla(\nabla \cdot \mathbf{E}) = \nabla \cdot (\nabla \cdot \mathbf{E}) + I \nabla \times (\nabla \cdot \mathbf{E})$$ [4.10]

(11) $$\nabla(\nabla \cdot \mathbf{E}) = 0 + I0 = 0$$ [4.11]

The gradient of the divergence always vanishes.

The gradient of the divergence cannot be defined in the Vector algebra because it is the side effect of defining the gradient as a vector. We must correct this error by importing the Clifford algebra identity equation [4.11] into the Vector algebra, therefore $\nabla(\nabla \cdot \mathbf{E})$ always vanishes not just in parallel interaction where $\nabla \cdot \mathbf{E} = 0$ but also in perpendicular interaction where $\nabla \cdot \mathbf{E} \neq 0$.

On substituting equation [4.11] into equation [4.8] gives

(12) $$\nabla^2 \mathbf{E} = \frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p \frac{\partial \mathbf{E}}{\partial t} - \omega_e \frac{\partial \mathbf{E}}{\partial t} - \omega_e \omega_p \mathbf{E}\right)$$ [4.12]

(13) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p \frac{\partial \mathbf{E}}{\partial t} - \omega_e \frac{\partial \mathbf{E}}{\partial t} - \omega_e \omega_p \mathbf{E}\right) - \nabla^2 \mathbf{E} = 0$$ [4.13]

(14) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p \frac{\partial \mathbf{E}}{\partial t} - \omega_e \frac{\partial \mathbf{E}}{\partial t} - \omega_0^2 \mathbf{E}\right) - \nabla^2 \mathbf{E} = 0$$ [4.14]

(15) $$\Box\mathbf{E} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{E}}{\partial t} - \omega_e \frac{\partial \mathbf{E}}{\partial t} - \omega_0^2 \mathbf{E}\right) = 0$$ [4.15]

(16) $$\Box\mathbf{D} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{D}}{\partial t} - \omega_e \frac{\partial \mathbf{D}}{\partial t} - \omega_0^2 \mathbf{D}\right) = 0$$ [4.16]

According to equation [1.55]

$$\mathbf{D} = \frac{\mathbf{J}}{\omega_p}$$ [1.55]

On substituting [1.55] into equation [4.16] gives

(17) $$\Box\mathbf{J} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{J}}{\partial t} - \omega_e \frac{\partial \mathbf{J}}{\partial t} - \omega_0^2 \mathbf{J}\right) = 0$$ [4.17]

Although the equations [4.15], [4.16] and [4.17] look similar, they represent different geometric objects, namely vector, trivector/ bivector and bivector respectively.

For the static case, equation [4.15] becomes

(18) $$\frac{1}{c^2}\left(-\omega_0^2 \mathbf{E}\right) - \nabla^2 \mathbf{E} = 0$$ [4.18]

(19) $$\frac{1}{c^2}\left(-c^2 \kappa_0^2 \mathbf{E}\right) - \nabla^2 \mathbf{E} = 0$$ [4.19]

(20) $$-\kappa_0^2 \mathbf{E} - \nabla^2 \mathbf{E} = 0$$ [4.20]

(21) $$\nabla^2 \mathbf{E} + \kappa_0^2 \mathbf{E} = 0$$ [4.21]

Similarly

(22) $$\nabla^2 \mathbf{D} + \kappa_0^2 \mathbf{D} = 0$$ [4.22]

(23) $$\nabla^2 \mathbf{J} + \kappa_0^2 \mathbf{J} = 0$$ [4.23]

The equations [4.21] and [4.22] are known as **Helmholtz equations**. Although these equations look similar, they represent different geometric objects, namely vector, and trivector/ bivector. The equation [4.22] is a new addition to the Helmholtz equations.

On applying the curl to the Ampère-Maxwell law [1.76] gives

$$\nabla \times \mathbf{H} = \epsilon_0 \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E} \right) \quad [1.76]$$

(24) $$\nabla \times \nabla \times \mathbf{H} = \epsilon_0 \nabla \times \left(\frac{\partial \mathbf{E}}{\partial t} + \omega_p \mathbf{E} \right) \quad [4.24]$$

(25) $$\nabla \times \nabla \times \mathbf{H} = \epsilon_0 \left(\frac{\partial}{\partial t} (\nabla \times \mathbf{E}) + \omega_p (\nabla \times \mathbf{E}) \right) \quad [4.25]$$

On substituting Faraday's law into equation [1.80] gives

$$\nabla \times \mathbf{E} = -\mu_0 \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H} \right) \quad [1.80]$$

(26) $$\nabla \times \nabla \times \mathbf{H} = -\frac{1}{c^2} \left(\frac{\partial}{\partial t} \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H} \right) + \omega_p \left(\frac{\partial \mathbf{H}}{\partial t} - \omega_e \mathbf{H} \right) \right) \quad [4.26]$$

(27) $$\nabla \times \nabla \times \mathbf{H} = -\frac{1}{c^2} \left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_e \frac{\partial \mathbf{H}}{\partial t} - \mathbf{H} \frac{\partial \omega_e}{\partial t} + \omega_p \frac{\partial \mathbf{H}}{\partial t} - \omega_0^2 \mathbf{H} \right) \quad [4.27]$$

The angular frequency ω_h of inward spiralling magnetic induction for a medium is always constant, for example in vacuum or in any metal.

Therefore

(28) $$\frac{\partial \omega_e}{\partial t} = 0 \quad [4.28]$$

On substituting the equation [4.28] in equation [4.27] gives

(29) $$\nabla \times \nabla \times \mathbf{H} = -\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_e \frac{\partial \mathbf{H}}{\partial t} + \omega_p \frac{\partial \mathbf{H}}{\partial t} - \omega_0^2 \mathbf{H}\right)$$ [4.29]

(30) $$\nabla(\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} = -\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_e \frac{\partial \mathbf{H}}{\partial t} + \omega_p \frac{\partial \mathbf{H}}{\partial t} - \omega_0^2 \mathbf{H}\right)$$ [4.30]

According to Clifford algebra

(31) $$\nabla(\nabla \cdot \mathbf{H}) = \nabla \cdot (\nabla \cdot \mathbf{H}) + \nabla \wedge (\nabla \cdot \mathbf{H})$$ [4.31]

(32) $$\nabla(\nabla \cdot \mathbf{H}) = \nabla \cdot (\nabla \cdot \mathbf{H}) + I\nabla \times (\nabla \cdot \mathbf{H})$$ [4.32]

(33) $$\nabla(\nabla \cdot \mathbf{H}) = 0 + I0$$ [4.33]

The gradient of a divergence always vanishes.

The gradient of a divergence cannot be defined in the Vector algebra because it is the side effect of defining the gradient as a vector. We must correct this error by applying the Clifford algebra identity equation [4.33] into Vector algebra, therefore $\nabla(\nabla \cdot \mathbf{H})$ always vanishes not just in parallel interaction where $\nabla \cdot \mathbf{H} = 0$ but also in perpendicular interaction where $\nabla \cdot \mathbf{H} \neq 0$.

On substituting equation [4.33] into equation [4.30] gives

(34) $$\nabla^2 \mathbf{H} = \frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_e \frac{\partial \mathbf{H}}{\partial t} + \omega_p \frac{\partial \mathbf{H}}{\partial t} - \omega_0^2 \mathbf{H}\right)$$ [4.34]

(35) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_e \frac{\partial \mathbf{H}}{\partial t} + \omega_p \frac{\partial \mathbf{H}}{\partial t} - \omega_0^2 \mathbf{H}\right) - \nabla^2 \mathbf{H} = 0$$ [4.35]

(36) $$\Box \mathbf{H} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{H}}{\partial t} - \omega_e \frac{\partial \mathbf{H}}{\partial t} - \omega_0^2 \mathbf{H}\right) = 0$$ [4.36]

(37) $$\Box\mathbf{B}+\frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{B}}{\partial t}-\omega_e\frac{\partial\mathbf{B}}{\partial t}-\omega_0^2\mathbf{B}\right)=0$$ [4.37]

According to equation [1.62]

$$\mathbf{B}=\frac{\mathbf{V}}{\omega_e}$$ [1.62]

On substituting [1.65] into equation [4.37] gives

(38) $$\Box\mathbf{V}+\frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{V}}{\partial t}-\omega_e\frac{\partial\mathbf{V}}{\partial t}-\omega_0^2\mathbf{V}\right)=0$$ [4.38]

Although equations [4.36], [4.37] and [4.38] look similar, they represent different geometric objects namely vector, trivector/ bivector and bivector respectively.

For the static case, the above equation becomes

(39) $$\nabla^2\mathbf{H}+\kappa_0^2\mathbf{H}=0$$ [4.39]

(40) $$\nabla^2\mathbf{B}+\kappa_0^2\mathbf{B}=0$$ [4.40]

(41) $$\nabla^2\mathbf{V}+\kappa_0^2\mathbf{V}=0$$ [4.41]

Although equations [4.39], [4.40] and [4.41] look similar, they represent different geometric objects namely vector, trivector/ bivector and bivector respectively.

Table 4.1

Duality			
Electricity		**Magnetism**	
Perpendicular Interaction			
Wave equation	$\Box\mathbf{E}+\frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{E}}{\partial t}-\omega_e\frac{\partial\mathbf{E}}{\partial t}-\omega_0^2\mathbf{E}\right)=0$		$\Box\mathbf{H}+\frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{H}}{\partial t}-\omega_e\frac{\partial\mathbf{H}}{\partial t}-\omega_0^2\mathbf{H}\right)=0$
	$\Box\mathbf{B}+\frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{B}}{\partial t}-\omega_e\frac{\partial\mathbf{B}}{\partial t}-\omega_0^2\mathbf{B}\right)=0$		$\Box\mathbf{D}+\frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{D}}{\partial t}-\omega_e\frac{\partial\mathbf{D}}{\partial t}-\omega_0^2\mathbf{D}\right)=0$
	$\Box\mathbf{V}+\frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{V}}{\partial t}-\omega_e\frac{\partial\mathbf{V}}{\partial t}-\omega_0^2\mathbf{V}\right)=0$		$\Box\mathbf{J}+\frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{J}}{\partial t}-\omega_e\frac{\partial\mathbf{J}}{\partial t}-\omega_0^2\mathbf{J}\right)=0$

Duality			
Electricity		Magnetism	
Spatial equation (Helmholtz equation)	$\nabla^2\mathbf{E} + \kappa_0^2\mathbf{E} = 0$	Spatial equation (Helmholtz equation)	$\nabla^2\mathbf{H} + \kappa_0^2\mathbf{H} = 0$
	$\nabla^2\mathbf{B} + \kappa_0^2\mathbf{B} = 0$		$\nabla^2\mathbf{D} + \kappa_0^2\mathbf{D} = 0$
	$\nabla^2\mathbf{V} + \kappa_0^2\mathbf{V} = 0$		$\nabla^2\mathbf{J} + \kappa_0^2\mathbf{J} = 0$

4.1. Conductors

For a Conductor, for example a metal,

(42) $$\eta_e \approx 0 \longrightarrow \kappa_e \approx 0 \longrightarrow \omega_e \approx 0$$ [4.42]

Therefore

(43) $$\omega_0 = \omega_p \omega_e \approx 0$$ [4.43]

(44) $$\omega_e \frac{\partial \mathbf{E}}{\partial t} \approx 0$$ [4.44]

On substituting equation [4.43] and [4.44] in wave equation [4.14] gives

(45) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$$ [4.45]

(46) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_p \frac{\partial \mathbf{D}}{\partial t}\right) - \nabla^2 \mathbf{D} = 0$$ [4.46]

(47) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{J}}{\partial t^2} + \omega_p \frac{\partial \mathbf{J}}{\partial t}\right) - \nabla^2 \mathbf{J} = 0$$ [4.47]

Similarly

(48) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_p \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$$ [4.48]

(49) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_p \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0$$ [4.49]

(50) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{V}}{\partial t^2} + \omega_p \frac{\partial \mathbf{V}}{\partial t}\right) - \nabla^2 \mathbf{V} = 0$$ [4.50]

4.2. Non Conductors

For a Non Conductor, for example air,

(51) $$\sigma_p \approx 0 \longrightarrow \kappa_p \approx 0 \longrightarrow \omega_p \approx 0$$ [4.51]

Therefore

(52) $$\omega_0 = \omega_p \omega_e \approx 0$$ [4.52]

(53) $$\omega_p \frac{\partial \mathbf{E}}{\partial t} \approx 0$$ [4.53]

On substituting equation [4.52] and [4.53] in wave equation [4.14] gives

(54) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} - \omega_e \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$$ [4.54]

(55) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{D}}{\partial t^2} - \omega_e \frac{\partial \mathbf{D}}{\partial t}\right) - \nabla^2 \mathbf{D} = 0$$ [4.55]

(56) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{J}}{\partial t^2} - \omega_e \frac{\partial \mathbf{J}}{\partial t}\right) - \nabla^2 \mathbf{J} = 0$$ [4.56]

Similarly

(57) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_e \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$$ [4.57]

(58) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} - \omega_e \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0$$ [4.58]

(59) $$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{V}}{\partial t^2} - \omega_e \frac{\partial \mathbf{V}}{\partial t}\right) - \nabla^2 \mathbf{V} = 0$$ [4.59]

Table 4.2

Duality			
Electricity		**Magnetism**	
Perpendicular Interaction			
Wave equation	$\square \mathbf{E} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{E}}{\partial t} - \omega_e \frac{\partial \mathbf{E}}{\partial t} - \omega_0^2 \mathbf{E}\right) = 0$		$\square \mathbf{H} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{H}}{\partial t} - \omega_e \frac{\partial \mathbf{H}}{\partial t} - \omega_0^2 \mathbf{H}\right) = 0$
	$\square \mathbf{B} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{B}}{\partial t} - \omega_e \frac{\partial \mathbf{B}}{\partial t} - \omega_0^2 \mathbf{B}\right) = 0$		$\square \mathbf{D} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{D}}{\partial t} - \omega_e \frac{\partial \mathbf{D}}{\partial t} - \omega_0^2 \mathbf{D}\right) = 0$
	$\square \mathbf{V} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{V}}{\partial t} - \omega_e \frac{\partial \mathbf{V}}{\partial t} - \omega_0^2 \mathbf{V}\right) = 0$		$\square \mathbf{J} + \frac{1}{c^2}\left(\omega_p \frac{\partial \mathbf{J}}{\partial t} - \omega_e \frac{\partial \mathbf{J}}{\partial t} - \omega_0^2 \mathbf{J}\right) = 0$
Spatial equation (Helmholtz equation)	$\nabla^2 \mathbf{E} + \kappa_0^2 \mathbf{E} = 0$	Spatial equation (Helmholtz equation)	$\nabla^2 \mathbf{H} + \kappa_0^2 \mathbf{H} = 0$
	$\nabla^2 \mathbf{B} + \kappa_0^2 \mathbf{B} = 0$		$\nabla^2 \mathbf{D} + \kappa_0^2 \mathbf{D} = 0$
	$\nabla^2 \mathbf{V} + \kappa_0^2 \mathbf{V} = 0$		$\nabla^2 \mathbf{J} + \kappa_0^2 \mathbf{J} = 0$
Perpendicular Interaction - Conductor			
Wave equation	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$	Wave equation	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_p \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_p \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_p \frac{\partial \mathbf{D}}{\partial t}\right) - \nabla^2 \mathbf{D} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{V}}{\partial t^2} + \omega_p \frac{\partial \mathbf{V}}{\partial t}\right) - \nabla^2 \mathbf{V} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{J}}{\partial t^2} + \omega_p \frac{\partial \mathbf{J}}{\partial t}\right) - \nabla^2 \mathbf{J} = 0$

Duality			
Electricity		**Magnetism**	
Perpendicular Interaction - Non Conductor			
Wave equation	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} - \omega_e \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$	Wave equation	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_e \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} - \omega_e \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} - \omega_e \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{V}}{\partial t^2} - \omega_e \frac{\partial \mathbf{V}}{\partial t}\right) - \nabla^2 \mathbf{V} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{J}}{\partial t^2} - \omega_e \frac{\partial \mathbf{J}}{\partial t}\right) - \nabla^2 \mathbf{J} = 0$

5. Dual Wave Equations

The Faraday law [3.10] is

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \quad [3.10]$$

Applying the curl to Faraday's law [3.10] gives

(1) $$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \nabla \times \frac{\partial \mathbf{H}}{\partial t} \quad [5.1]$$

(2) $$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) \quad [5.2]$$

On substituting the dual Faraday law [3.12] into equation [5.2] gives

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad [3.12]$$

(3) $$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \epsilon_0 \frac{\partial}{\partial t} \frac{\partial \mathbf{E}}{\partial t} \quad [5.3]$$

(4) $$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad [5.4]$$

On substituting equation [3.1] in equation [5.4] gives

$$\nabla \times \nabla \times \mathbf{E} = \kappa_p \nabla \times \mathbf{E} = \kappa_p^2 \mathbf{E} \quad [3.1]$$

(5) $$\kappa_p^2 \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad [5.5]$$

(6) $$\frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2 \kappa_p^2 \mathbf{E} = 0 \quad [5.6]$$

(7) $$\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p^2 \mathbf{E} = 0 \quad [5.7]$$

(8) $$\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_p^2 \mathbf{D} = 0$$ [5.8]

(9) $$\mathbf{J} = 0$$ [5.9]

These are known as **Beltrami Wave Equations.** They can be seen as **DUAL WAVE EQUATIONS.**

For the static case, the equation [5.7] becomes

(10) $$\omega_p^2 \mathbf{E} = 0$$ [5.10]

Since $\mathbf{E}$ is not zero in general, therefore

(11) $$\omega_p = 0$$ [5.11]

On applying the curl to dual Faraday law [3.12] gives

(12) $$\nabla \times \nabla \times \mathbf{H} = \epsilon_0 \nabla \times \frac{\partial \mathbf{E}}{\partial t}$$ [5.12]

(13) $$\nabla \times \nabla \times \mathbf{H} = \epsilon_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{E})$$ [5.13]

On substituting equation [3.10] into equation [5.13] gives

(14) $$\nabla \times \nabla \times \mathbf{H} = -\frac{1}{c^2} \frac{\partial}{\partial t} \frac{\partial \mathbf{H}}{\partial t}$$ [5.14]

(15) $$\nabla \times \nabla \times \mathbf{H} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2}$$ [5.15]

(16) $$\nabla \times \nabla \times \mathbf{B} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2}$$ [5.16]

On substituting equation [3.2] in equation [5.26] gives

$$\nabla \times \nabla \times \mathbf{B} = -\kappa_e \nabla \times \mathbf{B} = \kappa_e^2 \mathbf{B}$$ [3.2]

(17) $$\kappa_e^2\mathbf{B} = -\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}$$ [5.17]

(18) $$\frac{\partial^2\mathbf{B}}{\partial t^2} + c^2\kappa_e^2\mathbf{B} = 0$$ [5.18]

(19) $$\frac{\partial^2\mathbf{B}}{\partial t^2} + \omega_e^2\mathbf{B} = 0$$ [5.19]

(20) $$\frac{\partial^2\mathbf{H}}{\partial t^2} + \omega_e^2\mathbf{H} = 0$$ [5.20]

(21) $$\mathbf{V} = 0$$ [5.21]

For the static case, the equation [5.19] becomes

(22) $$\omega_e^2\mathbf{B} = 0$$ [5.22]

Since $\mathbf{B}$ is not zero in general, therefore

(23) $$\omega_e = 0$$ [5.23]

Table 5.1

Duality			
Electricity		**Magentism**	
Perpendicular Interaction			
Wave equation	$\Box\mathbf{E} + \frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{E}}{\partial t} - \omega_e\frac{\partial\mathbf{E}}{\partial t} - \omega_0^2\mathbf{E}\right) = 0$		$\Box\mathbf{H} + \frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{H}}{\partial t} - \omega_e\frac{\partial\mathbf{H}}{\partial t} - \omega_0^2\mathbf{H}\right) = 0$
	$\Box\mathbf{B} + \frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{B}}{\partial t} - \omega_e\frac{\partial\mathbf{B}}{\partial t} - \omega_0^2\mathbf{B}\right) = 0$		$\Box\mathbf{D} + \frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{D}}{\partial t} - \omega_e\frac{\partial\mathbf{D}}{\partial t} - \omega_0^2\mathbf{D}\right) = 0$
	$\Box\mathbf{V} + \frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{V}}{\partial t} - \omega_e\frac{\partial\mathbf{V}}{\partial t} - \omega_0^2\mathbf{V}\right) = 0$		$\Box\mathbf{J} + \frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{J}}{\partial t} - \omega_e\frac{\partial\mathbf{J}}{\partial t} - \omega_0^2\mathbf{J}\right) = 0$
Spatial equation (Helmholtz equation)	$\nabla^2\mathbf{E} + \kappa_0^2\mathbf{E} = 0$	Spatial equation (Helmholtz equation)	$\nabla^2\mathbf{H} + \kappa_0^2\mathbf{H} = 0$
	$\nabla^2\mathbf{B} + \kappa_0^2\mathbf{B} = 0$		$\nabla^2\mathbf{D} + \kappa_0^2\mathbf{D} = 0$
	$\nabla^2\mathbf{V} + \kappa_0^2\mathbf{V} = 0$		$\nabla^2\mathbf{J} + \kappa_0^2\mathbf{J} = 0$

Duality			
Electricity		Magentism	
Dual Parallel Interaction			
Dual wave equation (Beltrami equation)	$\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p^2 \mathbf{E} = 0$	Dual wave equation (Beltrami equation)	$\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_e^2 \mathbf{H} = 0$
	$\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_e^2 \mathbf{B} = 0$		$\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_p^2 \mathbf{D} = 0$
	$\mathbf{V} = 0$		$\mathbf{J} = 0$
Dual spatial equation (No wave)	$\omega_p = \omega_e = 0$	Dual spatial equation (No wave)	$\omega_p = \omega_e = 0$

5.1. Conductors

For a conductor, for example a metal [4.42]

$$\eta_e \approx 0 \longrightarrow \kappa_e \approx 0 \longrightarrow \omega_e \approx 0 \quad [4.42]$$

Therefore

(24) $$\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p^2 \mathbf{E} = 0 \quad [5.24]$$

(25) $$\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_p^2 \mathbf{D} = 0 \quad [5.25]$$

(26) $$\mathbf{J} = 0 \quad [5.26]$$

The dual wave equations for $\mathbf{E}$ and $\mathbf{D}$ fields remain unchanged.

(27) $$\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0 \quad [5.27]$$

(28) $$\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \quad [5.28]$$

(29) $\mathbf{V} = 0$ [5.29]

The dual equations [5.27] to [5.29] mean that $\mathbf{H}$ and $\mathbf{B}$ fields are at an inflection point i.e. either at maxima or minima states in the conductor.

5.2. Non Conductors

For a Non Conductor, for example air [4.51]

$$\sigma_p \approx 0 \longrightarrow \kappa_p \approx 0 \longrightarrow \omega_p \approx 0 \quad [4.51]$$

Therefore

(30) $$\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad [5.30]$$

(31) $$\frac{\partial^2 \mathbf{D}}{\partial t^2} = 0 \quad [5.31]$$

(32) $\mathbf{J} = 0$ [5.32]

The dual equations [5.30] to [5.32] means that $\mathbf{E}$ and $\mathbf{D}$ fields are at an inflection point i.e. either at maxima or minima states in the non conductor.

(33) $$\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_e^2 \mathbf{H} = 0 \quad [5.33]$$

(34) $$\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_e^2 \mathbf{B} = 0 \quad [5.34]$$

(35) $\mathbf{V} = 0$ [5.35]

The dual wave equations for $\mathbf{H}$ and $\mathbf{B}$ fields remain unchanged.

Table 5.2

Duality			
Electricity		**Magnetism**	
Perpendicular Interaction - Conductor			
Wave equation	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_e \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$	Wave equation	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_e \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_e \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_e \frac{\partial \mathbf{D}}{\partial t}\right) - \nabla^2 \mathbf{D} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{V}}{\partial t^2} + \omega_e \frac{\partial \mathbf{V}}{\partial t}\right) - \nabla^2 \mathbf{V} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{J}}{\partial t^2} + \omega_e \frac{\partial \mathbf{J}}{\partial t}\right) - \nabla^2 \mathbf{J} = 0$
Perpendicular Interaction - Non Conductor			
Wave equation	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} - \omega_p \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$	Wave equation	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_p \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} - \omega_p \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{D}}{\partial t^2} - \omega_p \frac{\partial \mathbf{D}}{\partial t}\right) - \nabla^2 \mathbf{D} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{V}}{\partial t^2} - \omega_p \frac{\partial \mathbf{V}}{\partial t}\right) - \nabla^2 \mathbf{V} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{J}}{\partial t^2} - \omega_p \frac{\partial \mathbf{J}}{\partial t}\right) - \nabla^2 \mathbf{J} = 0$
Dual Parallel Interaction - Conductor			
Dual wave equation - Remains unchanged - Harmonic	$\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p^2 \mathbf{E} = 0$	Dual wave equation - Linear	$\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$
State of Infection	$\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$	Remains unchanged - Harmonic	$\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_p^2 \mathbf{D} = 0$
	$\mathbf{V} = 0$		$\mathbf{J} = 0$
Dual Parallel Interaction - Non Conductor			
Dual wave equation - Linear	$\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$	Dual wave equation - Remains unchanged - Harmonic	$\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_e^2 \mathbf{H} = 0$
Remains unchanged - Harmonic	$\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_e^2 \mathbf{B} = 0$	State of Inflection	$\frac{\partial^2 \mathbf{D}}{\partial t^2} = 0$
	$\mathbf{V} = 0$		$\mathbf{J} = 0$

6. Wave Velocity

The time derivatives of a field can be rewritten as

(1) $$\frac{\partial \mathbf{E}}{\partial t} = \frac{\partial \mathbf{E}}{\partial r}\frac{\partial r}{\partial t} = \mathbf{v}\frac{\partial \mathbf{E}}{\partial r}$$ [6.1]

(2) $$\begin{aligned}\frac{\partial^2 \mathbf{E}}{\partial t^2} &= \mathbf{v}\left(\frac{\partial^2 \mathbf{E}}{\partial r^2}\frac{\partial r}{\partial t}\right) + \frac{\partial \mathbf{E}}{\partial r}\frac{\partial \mathbf{v}}{\partial t} \\ &= \mathbf{v}\left(\frac{\partial^2 \mathbf{E}}{\partial r^2}\mathbf{v}\right) + \frac{\partial \mathbf{E}}{\partial r}\frac{\partial \mathbf{v}}{\partial t} \\ &= \mathbf{v}^2\left(\frac{\partial^2 \mathbf{E}}{\partial r^2}\right) + \frac{\partial \mathbf{E}}{\partial r}\frac{\partial \mathbf{v}}{\partial t}\end{aligned}$$ [6.2]

In accordance with special relativity, we have to assume that $\mathbf{v}$ is a constant. Therefore

(3) $$\frac{\partial \mathbf{v}}{\partial t} = 0$$ [6.3]

On substituting equation [6.3] in equation [6.2] gives

(4) $$\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mathbf{v}^2\frac{\partial^2 \mathbf{E}}{\partial r^2}$$ [6.4]

(5) $$\nabla^2 \mathbf{E} = \frac{\partial^2 \mathbf{E}}{\partial r^2}$$ [6.5]

Similarly

(6) $$\frac{\partial \mathbf{H}}{\partial t} = \mathbf{v}\frac{\partial \mathbf{H}}{\partial r}$$ [6.6]

(7) $$\frac{\partial^2 \mathbf{H}}{\partial t^2} = \mathbf{v}^2\frac{\partial^2 \mathbf{H}}{\partial r^2}$$ [6.7]

(8) $$\nabla^2 \mathbf{H} = \frac{\partial^2 \mathbf{H}}{\partial r^2}$$ [6.8]

(9) $$\frac{\partial \mathbf{D}}{\partial t} = \mathbf{v}\frac{\partial \mathbf{D}}{\partial r}$$ [6.9]

(10) $$\frac{\partial^2 \mathbf{D}}{\partial t^2} = \mathbf{v}^2\frac{\partial^2 \mathbf{D}}{\partial r^2}$$ [6.10]

(11) $$\nabla^2 \mathbf{D} = \frac{\partial^2 \mathbf{D}}{\partial r^2}$$ [6.11]

(12) $$\frac{\partial \mathbf{B}}{\partial t} = \mathbf{v}\frac{\partial \mathbf{B}}{\partial r}$$ [6.12]

(13) $$\frac{\partial^2 \mathbf{B}}{\partial t^2} = \mathbf{v}^2\frac{\partial^2 \mathbf{B}}{\partial r^2}$$ [6.13]

(14) $$\nabla^2 \mathbf{B} = \frac{\partial^2 \mathbf{B}}{\partial r^2}$$ [6.14]

Consider both an decay and growth exponential approach for vectors

(15) $$\mathbf{E} = \mathbf{E}_0\ e^{-\kappa_p \cdot r}$$ [6.15]

(16) $$\frac{\partial \mathbf{E}}{\partial r} = -\,\kappa_p \mathbf{E}_0\ e^{-\kappa_p \cdot r} = -\,\kappa_p \mathbf{E}$$ [6.16]

(17) $$\frac{\partial^2 \mathbf{E}}{\partial r^2} = \kappa_p^2 \mathbf{E}_0\ e^{-\kappa_p \cdot r} = \kappa_p^2 \mathbf{E}$$ [6.17]

(18) $$\mathbf{H} = \mathbf{H}_0\ e^{-\kappa_e \cdot r}$$ [6.18]

(19) $$\frac{\partial \mathbf{H}}{\partial r} = -\,\kappa_e \mathbf{H}_0\ e^{-\boldsymbol{\kappa}_e \cdot r} = -\,\kappa_e \mathbf{H}$$ [6.19]

(20) $$\frac{\partial^2 \mathbf{H}}{\partial r^2} = \kappa_e^2 \mathbf{H}_0\ e^{-\boldsymbol{\kappa}_e \cdot r} = \kappa_e^2 \mathbf{H}$$ [6.20]

(21) $$\mathbf{E} = \mathbf{E}_0\ e^{\boldsymbol{\kappa}_p \cdot r}$$ [6.21]

(22) $$\frac{\partial \mathbf{E}}{\partial r} = \kappa_p \mathbf{E}_0\ e^{\boldsymbol{\kappa}_p \cdot r} = \kappa_p \mathbf{E}$$ [6.22]

(23) $$\frac{\partial^2 \mathbf{E}}{\partial r^2} = \kappa_p^2 \mathbf{E}_0\ e^{\boldsymbol{\kappa}_p \cdot r} = \kappa_p^2 \mathbf{E}$$ [6.23]

(24) $$\mathbf{H} = \mathbf{H}_0\ e^{\boldsymbol{\kappa}_e \cdot r}$$ [6.24]

(25) $$\frac{\partial \mathbf{H}}{\partial r} = \kappa_e \mathbf{H}_0\ e^{\boldsymbol{\kappa}_e \cdot r} = \kappa_e \mathbf{H}$$ [6.25]

(26) $$\frac{\partial^2 \mathbf{H}}{\partial r^2} = \kappa_e^2 \mathbf{H}_0\ e^{\boldsymbol{\kappa}_e \cdot r} = \kappa_e^2 \mathbf{H}$$ [6.26]

Consider a harmonic approach for bivectors

(27) $$\mathbf{D} = \mathbf{D}_0\ e^{i\boldsymbol{\kappa}_p \cdot r}$$ [6.27]

(28) $$\frac{\partial \mathbf{D}}{\partial r} = i\kappa_p \mathbf{D}_0\ e^{i\boldsymbol{\kappa}_p \cdot r} = i\kappa_p \mathbf{D}$$ [6.28]

(29)
$$\begin{aligned}\frac{\partial^2 \mathbf{D}}{\partial r^2} &= i^2\kappa_p^2\mathbf{D}_0\ e^{i\kappa_p\cdot r}\\ &= i^2\kappa_p^2\mathbf{D}\\ &= -\kappa_p^2\mathbf{D}\end{aligned}$$ [6.29]

(30) $$\mathbf{B} = \mathbf{B}_0\ e^{-i\kappa_e\cdot r}$$ [6.30]

The minus sign in the exponent signifies that $\mathbf{D}$ and $\mathbf{B}$ fields rotates in opposite direction.

(31)
$$\begin{aligned}\frac{\partial \mathbf{B}}{\partial r} &= -i\kappa_e\mathbf{B}_0\ e^{-i\kappa_e\cdot r}\\ &= -i\kappa_e\mathbf{B}\end{aligned}$$ [6.31]

(32)
$$\begin{aligned}\frac{\partial^2 \mathbf{B}}{\partial r^2} &= i^2\kappa_e^2\mathbf{D}_0\ e^{-i\kappa_e\cdot r}\\ &= i^2\kappa_e^2\mathbf{B}\\ &= -\kappa_e^2\mathbf{B}\end{aligned}$$ [6.32]

6.1. Conductors

The wave equation for the electric field in the conductor [4.45] is

$$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p\frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2\mathbf{E} = 0$$ [4.45]

On substituting equation [6.1], [6.4] and [6.5] in equation [4.45] gives

(33) $$\frac{1}{c^2}\left(\mathbf{v}^2\frac{\partial^2 \mathbf{E}}{\partial r^2} + \omega_p\mathbf{v}\frac{\partial \mathbf{E}}{\partial r}\right) - \frac{\partial^2 \mathbf{E}}{\partial r^2} = 0$$ [6.33]

The velocity $\mathbf{v}$ in equation [6.33] represents the velocity of the electric field in the conductor $\mathbf{v}_{Ec}$, therefore

(34) $$\frac{1}{c^2}\left(\mathbf{v}_{Ec}^2\frac{\partial^2\mathbf{E}}{\partial r^2}+\omega_p\mathbf{v}_{Ec}\frac{\partial\mathbf{E}}{\partial r}\right)-\frac{\partial^2\mathbf{E}}{\partial r^2}=0$$ [6.34]

On substituting equations [6.16], [6.17] (decaying approach) in equation [6.34] gives

(35) $$\mathbf{v}_{Ec}^2(\kappa_p^2\mathbf{E})+\omega_p\mathbf{v}_{Ec}(-\kappa_p\mathbf{E})-c^2(\kappa_p^2\mathbf{E})=0$$ [6.35]

(36) $$\kappa_p^2\mathbf{v}_{Ec}^2-c\kappa_p^2\mathbf{v}_{Ec}-c^2\kappa_p^2=0$$ [6.36]

(37) $$\mathbf{v}_{Ec}^2-c\,\mathbf{v}_{Ec}-c^2=0$$ [6.37]

(38) $$\begin{aligned}\mathbf{v}_{Ec}&=\frac{c\pm\sqrt{(-c)^2+4c^2}}{2}\\&=\frac{c\pm\sqrt{5c^2}}{2}\\&=\frac{c\pm\sqrt{5}c}{2}\\&=\frac{1\pm\sqrt{5}}{2}c\end{aligned}$$ [6.38][6.32]

(39) $$\mathbf{v}_{Ec(decay)}=1.618c,\ -0.618c$$ [6.39]

Therefore

(40) $$\mathbf{v}_{Ec(decay)}=1.618c$$ [6.40]

The multiplier of the speed of light in the two roots the of equation [6.33] are in the golden ratio i.e. $\frac{1}{1.618}=0.618.$

Thus, an electric field strength $\mathbf{E}$ wave decays rapidly with 1.618 times speed of light in the conductor.

On substituting equations [6.22], [6.23] (growth approach) in equation [6.34] gives

(41) $$\mathbf{v}_{Ec}^2(\kappa_p^2\mathbf{E}) + \omega_p\mathbf{v}_{Ec}(\kappa_p\mathbf{E}) - c^2(\kappa_p^2\mathbf{E}) = 0$$ [6.41]

(42) $$\kappa_p^2\mathbf{v}_{Ec}^2 + c\kappa_p^2\mathbf{v}_{Ec} - c^2\kappa_p^2 = 0$$ [6.42]

(43) $$\mathbf{v}_{Ec}^2 + c\,\mathbf{v}_{Ec} - c^2 = 0$$ [6.43]

(44) $$\begin{aligned}\mathbf{v}_{Ec} &= \frac{-c \pm \sqrt{c^2+4c^2}}{2}\\ &= \frac{-c \pm \sqrt{5c^2}}{2}\\ &= \frac{-c \pm \sqrt{5}c}{2}\\ &= \frac{-1 \pm \sqrt{5}}{2}c\end{aligned}$$ [6.44]

(45) $$\mathbf{v}_{Ec(growth)} = -\,1.618c,\ 0.618c$$ [6.45]

Therefore

(46) $$\mathbf{v}_{Ec(growth)} = 0.618c$$ [6.46]

Thus, an electric field strength $\mathbf{E}$ wave grows slowly with 0.618 times speed of light in the conductor.

The wave equation for the electric displacement in the conductor [4.46] is

$$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{D}}{\partial t^2} + \omega_p\frac{\partial\mathbf{D}}{\partial t}\right) - \nabla^2\mathbf{D} = 0$$ [4.46]

On substituting equation [6.9], [6.10] and [6.11] in equation [4.46] gives

(47) $$\frac{1}{c^2}\left(\mathbf{v}^2\frac{\partial^2\mathbf{D}}{\partial r^2} + \omega_p\mathbf{v}\frac{\partial\mathbf{D}}{\partial r}\right) - \frac{\partial^2\mathbf{D}}{\partial r^2} = 0$$ [6.47]

The velocity $\mathbf{v}$ in equation [6.47] represents the velocity of the electric displacement field in the conductor $\mathbf{v}_{Dc}$, therefore

(48) $$\frac{1}{c^2}\left(\mathbf{v}_{Dc}^2\frac{\partial^2\mathbf{D}}{\partial r^2}+\omega_p\mathbf{v}_{Dc}\frac{\partial\mathbf{D}}{\partial r}\right)-\frac{\partial^2\mathbf{D}}{\partial r^2}=0$$ [6.48]

On substituting equation [6.22], [6.23] (harmonic approach) in equation [6.48] gives

(49) $$\mathbf{v}_{Dc}^2(-\kappa_h^2\mathbf{D})+\omega_p\mathbf{v}_{Dc}(i\kappa_p\mathbf{D})-c^2(-\kappa_p^2\mathbf{D})=0$$ [6.49]

(50) $$-\kappa_p^2\mathbf{v}_{Dc}^2+ic\kappa_p^2\mathbf{v}_{Dc}+c^2\kappa_p^2=0$$ [6.50]

(51) $$\mathbf{v}_{Dc}^2-ic\,\mathbf{v}_{Dc}-c^2=0$$ [6.51]

(52) $$\begin{aligned}\mathbf{v}_{Dc}&=\frac{ic\pm\sqrt{(-ic)^2+4c^2}}{2}\\&=\frac{ic\pm\sqrt{-c^2+4c^2}}{2}\\&=\frac{ic\pm\sqrt{3c^2}}{2}\\&=\frac{i\pm\sqrt{3}}{2}c\end{aligned}$$ [6.52]

(53) $$\mathbf{v}_{Dc}=(0.866+0.5i)c,\ (-0.866+0.5i)c$$ [6.53]

Therefore

(54) $$\mathbf{v}_{Dc}=(0.866+0.5i)c$$ [6.54]

(55) $$\mathbf{v}_{Dc}=\left(cos\frac{\pi}{6}+i\ sin\frac{\pi}{6}\right)c$$ [6.55]

(56) $$\mathbf{v}_{Dc}=c\ e^{\frac{\pi}{6}i}$$ [6.56]

(57) $\mathbf{v}_{Dc} = c\ R\left(\frac{\pi}{6}\right)$ [6.57]

Similarly

(58) $\mathbf{v}_{Jc} = c\ R\left(\frac{\pi}{6}\right)$ [6.58]

The electric induction field $\mathbf{D}$ and Current area density field $\mathbf{J}$ rotate harmonically with rotor angle of $\frac{\pi}{6}$ with the speed of light. This implies that the $\mathbf{B}$ and $\mathbf{V}$ fields wave number and hence their wave length shortens by a factor of 3 in the conductors. This is the precise reason for adopting the 3 phase AC electric power system where each phase lags $\frac{\pi}{6}$.

The wave equation for the magnetic field strength in the conductor [4.48] is

$$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_p \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0 \qquad [4.48]$$

On substituting equation [6.6], [6.7] and [6.8] in equation [4.48] gives

(59) $\frac{1}{c^2}\left(\mathbf{v}^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} + \omega_p \mathbf{v} \frac{\partial \mathbf{H}}{\partial r}\right) - \frac{\partial^2 \mathbf{H}}{\partial r^2} = 0$ [6.59]

The velocity $\mathbf{v}$ in equation [6.59] represents the velocity of the magnetic field strength in the conductor $\mathbf{v}_{Hc}$, therefore

(60) $\frac{1}{c^2}\left(\mathbf{v}_{Hc}^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} + \omega_p \mathbf{v}_{Hc} \frac{\partial \mathbf{H}}{\partial r}\right) - \frac{\partial^2 \mathbf{H}}{\partial r^2} = 0$ [6.60]

On substituting equation [6.19], [6.20] (decaying approach) in equation [6.60] gives

(61) $\mathbf{v}_{Hc}^2(\kappa_e^2 \mathbf{H}) + \omega_p \mathbf{v}_{Hc}(-\kappa_e \mathbf{H}) - c^2(\kappa_e^2 \mathbf{H}) = 0$ [6.61]

(62) $\kappa_e^2 \mathbf{v}_{Hc}^2 - c\kappa_p \kappa_e \mathbf{v}_{Hc} - c^2 \kappa_e^2 = 0$ [6.62]

(63) $\kappa_e \mathbf{v}_{Hc}^2 - c\kappa_p \mathbf{v}_{Hc} - c^2 \kappa_e = 0$ [6.63]

For a conductor, for example, a metal [4.42]

$$\eta_e \approx 0 \longrightarrow \kappa_e \approx 0 \longrightarrow \omega_e \approx 0$$ [4.42]

On substituting equation [4.42] in equation [6.51] gives

(64) $-c\kappa_p \mathbf{v}_{Hc} = 0$ [6.64]

(65) $\mathbf{v}_{Hc(decay)} = 0$ [6.65]

On substituting equation [6.25], [6.26] (growth approach) in equation [6.60] gives

(66) $\mathbf{v}_{Hc}^2(\kappa_e^2 \mathbf{H}) + \omega_p \mathbf{v}_{Hc}(\kappa_e \mathbf{H}) - c^2(\kappa_e^2 \mathbf{H}) = 0$ [6.66]

(67) $\kappa_e^2 \mathbf{v}_{Hc}^2 + c\kappa_p \kappa_e \mathbf{v}_{Hc} - c^2 \kappa_e^2 = 0$ [6.67]

(68) $\kappa_e \mathbf{v}_{Hc}^2 + c\kappa_p \mathbf{v}_{Hc} - c^2 \kappa_e = 0$ [6.68]

For a conductor, for example a metal, [4.42]

$$\eta_e \approx 0 \longrightarrow \kappa_e \approx 0 \longrightarrow \omega_e \approx 0$$ [4.42]

On substituting equation [4.42] in equation [6.68] gives

(69) $c\kappa_p \mathbf{v}_{Hc} = 0$ [6.69]

(70) $\mathbf{v}_{Hc(growth)} = 0$ [6.70]

Thus, magnetic field strength $\mathbf{H}$ is damped in the conductor.

The wave equation for the magnetic induction field in the conductor [4.49] is

$$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_p \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0 \quad [4.49]$$

On substituting equation [6.12], [6.13] and [6.14] in equation [4.49] gives

(71) $$\frac{1}{c^2}\left(\mathbf{v}^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} + \omega_p \mathbf{v} \frac{\partial \mathbf{B}}{\partial r}\right) - \frac{\partial^2 \mathbf{B}}{\partial r^2} = 0 \quad [6.71]$$

The velocity $\mathbf{v}$ in equation [6.71] represents the velocity of the magnetic induction field in the conductor $\mathbf{v}_{Bc}$, therefore

(72) $$\frac{1}{c^2}\left(\mathbf{v}_{Bc}^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} + \omega_p \mathbf{v}_{Bc} \frac{\partial \mathbf{B}}{\partial r}\right) - \frac{\partial^2 \mathbf{B}}{\partial r^2} = 0 \quad [6.72]$$

On substituting equation [6.31], [6.32] (harmonic approach) in equation [6.72] gives

(73) $$\mathbf{v}_{Bc}^2(-\kappa_e^2 \mathbf{B}) + \omega_p \mathbf{v}_{Bc}(-i\kappa_e \mathbf{B}) - c^2(-\kappa_e^2 \mathbf{B}) = 0 \quad [6.73]$$

(74) $$-\kappa_e^2 \mathbf{v}_{Bc}^2 - ic\kappa_p \kappa_e \mathbf{v}_{Bc} + c^2 \kappa_e^2 = 0 \quad [6.74]$$

(75) $$\kappa_e \mathbf{v}_{Bc}^2 + ic\kappa_p \mathbf{v}_{Bc} - c^2 \kappa_e = 0 \quad [6.75]$$

For a conductor [4.42]

$$\eta_e \approx 0 \longrightarrow \kappa_e \approx 0 \longrightarrow \omega_e \approx 0 \quad [4.42]$$

On substituting equation [6.42] in equation [6.75] gives

(76) $$ic\kappa_p \mathbf{v}_{Bc} = 0 \quad [6.76]$$

(77) $$\mathbf{v}_{Bc} = 0 \quad [6.77]$$

Therefore, the magnetic induction field $\mathbf{B}$ wave is damped in the conductor.

Similarly

(78) $\mathbf{v}_{Vc} = 0$ [6.78]

Thus, $\mathbf{H}$, $\mathbf{B}$ and $\mathbf{V}$ fields are damped in the conductor.

6.2. Non Conductors

The wave equation for the electric field in the non conductor [4.54] is

$$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} - \omega_e \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0 \quad [4.54]$$

On substituting equation [6.1], [6.4] and [6.5] in equation [4.54] gives

(79) $$\frac{1}{c^2}\left(\mathbf{v}^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} - \omega_e \mathbf{v} \frac{\partial \mathbf{E}}{\partial r}\right) - \frac{\partial^2 \mathbf{E}}{\partial r^2} = 0 \quad [6.79]$$

The velocity $\mathbf{v}$ in equation [6.79] represents the velocity of the electric field in the non conductor $\mathbf{v}_{Enc}$, therefore

(80) $$\frac{1}{c^2}\left(\mathbf{v}_{Enc}^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} - \omega_e \mathbf{v}_{Enc} \frac{\partial \mathbf{E}}{\partial r}\right) - \frac{\partial^2 \mathbf{E}}{\partial r^2} = 0 \quad [6.80]$$

On substituting equation [6.16] and [6.17] (decaying approach) in equation [6.80] gives

(81) $$\mathbf{v}_{Enc}^2(\kappa_p^2 \mathbf{E}) - \omega_e \mathbf{v}_{Enc}(-\kappa_p \mathbf{E}) - c^2(\kappa_p^2 \mathbf{E}) = 0 \quad [6.81]$$

(82) $$\kappa_p^2 \mathbf{v}_{Enc}^2 - c\kappa_e \kappa_p \mathbf{v}_{Enc} - c^2 \kappa_p^2 = 0 \quad [6.82]$$

(83) $$\kappa_p \mathbf{v}_{Enc}^2 - c\kappa_e \mathbf{v}_{Enc} - c^2 \kappa_p = 0 \quad [6.83]$$

For a Non Conductor [4.51]

$$\sigma_p \approx 0 \longrightarrow \kappa_p \approx 0 \longrightarrow \omega_p \approx 0 \quad [4.51]$$

On substituting equation [4.51] in equation [6.83] gives

(84) $$-c\kappa_e \mathbf{v}_{Enc} = 0$$ [6.84]

(85) $$\mathbf{v}_{Enc(decay)} = 0$$ [6.85]

On substituting equation [6.22] and [6.23] (decaying approach) in equation [6.80] gives

(86) $$\mathbf{v}_{Enc}^2(\kappa_p^2\mathbf{E}) - \omega_e \mathbf{v}_{Enc}(\kappa_p \mathbf{E}) - c^2(\kappa_p^2\mathbf{E}) = 0$$ [6.86]

(87) $$\kappa_p^2 \mathbf{v}_{Enc}^2 + c\kappa_e\kappa_p \mathbf{v}_{Enc} - c^2\kappa_p^2 = 0$$ [6.87]

(88) $$\kappa_p \mathbf{v}_{Enc}^2 + c\kappa_e \mathbf{v}_{Enc} - c^2\kappa_p = 0$$ [6.88]

For a non conductor [4.51]

$$\sigma_p \approx 0 \longrightarrow \kappa_p \approx 0 \longrightarrow \omega_p \approx 0$$ [4.51]

On substituting equation [4.51] in equation [6.88] gives

(89) $$c\kappa_e \mathbf{v}_{Enc} = 0$$ [6.89]

(90) $$\mathbf{v}_{Enc(growth)} = 0$$ [6.90]

Therefore an electric field strength $\mathbf{E}$ wave is damped in the non conductor.

The wave equation for the electric displacement in the non conductor [4.55] is

$$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{D}}{\partial t^2} - \omega_e \frac{\partial \mathbf{D}}{\partial t}\right) - \nabla^2 \mathbf{D} = 0$$ [4.55]

On substituting equation [6.9], [6.10] and [6.11] in equation [4.55] gives

(91) $$\frac{1}{c^2}\left(\mathbf{v}^2 \frac{\partial^2 \mathbf{D}}{\partial r^2} - \omega_e \mathbf{v} \frac{\partial \mathbf{D}}{\partial r}\right) - \frac{\partial^2 \mathbf{D}}{\partial r^2} = 0$$ [6.91]

The velocity $\mathbf{v}$ in equation [6.91] represents the velocity of the electric displacement field in the non conductor $\mathbf{v}_{Dnc}$, therefore

(92) $$\frac{1}{c^2}\left(\mathbf{v}_{Dnc}^2\frac{\partial^2\mathbf{D}}{\partial r^2} - \omega_e\mathbf{v}_{Dnc}\frac{\partial\mathbf{D}}{\partial r}\right) - \frac{\partial^2\mathbf{D}}{\partial r^2} = 0$$ [6.92]

On substituting equation [6.28], [6.29] (harmonic approach) in equation [6.92] gives

(93) $$\mathbf{v}_{Dnc}^2(-\kappa_p^2\mathbf{D}) - \omega_e\mathbf{v}_{Dnc}(i\kappa_p\mathbf{D}) - c^2(-\kappa_p^2\mathbf{D}) = 0$$ [6.93]

(94) $$-\kappa_p^2\mathbf{v}_{Dnc}^2 - ic\kappa_e\kappa_p\mathbf{v}_{Dnc} + c^2\kappa_p^2 = 0$$ [6.94]

(95) $$\kappa_p\mathbf{v}_{Dnc}^2 + ic\kappa_e\ \mathbf{v}_{Dnc} - c^2\kappa_p = 0$$ [6.95]

For a non conductor [4.51]

$$\sigma_p \approx 0 \longrightarrow \kappa_p \approx 0 \longrightarrow \omega_p \approx 0$$ [4.51]

On substituting equation [4.51] in equation [6.95] gives

(96) $$ic\kappa_e\ \mathbf{v}_{Dnc} = 0$$ [6.96]

(97) $$\mathbf{v}_{Dnc} = 0$$ [6.97]

Therefore the electric induction field $\mathbf{D}$ wave is damped in the non conductor.

Similarly

(98) $$\mathbf{v}_{Jnc} = 0$$ [6.98]

Thus, $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{J}$ fields are damped in the non conductor.

The wave equation for the magnetic field strength in the non conductor [4.57] is

$$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{H}}{\partial t^2} - \omega_e\frac{\partial\mathbf{H}}{\partial t}\right) - \nabla^2\mathbf{H} = 0$$ [4.57]

On substituting equation [6.6], [6.7] and [6.8] in equation [4.57] gives

(99) $$\frac{1}{c^2}\left(\mathbf{v}^2\frac{\partial^2\mathbf{H}}{\partial r^2} - \omega_e\mathbf{v}\frac{\partial\mathbf{H}}{\partial r}\right) - \frac{\partial^2\mathbf{H}}{\partial r^2} = 0$$ [6.99]

The velocity $\mathbf{v}$ in equation [6.99] represents the velocity of the magnetic field strength in the non conductor $\mathbf{v}_{Hnc}$, therefore

(100) $$\frac{1}{c^2}\left(\mathbf{v}_{Hnc}^2\frac{\partial^2\mathbf{H}}{\partial r^2} - \omega_e\mathbf{v}_{Hnc}\frac{\partial\mathbf{H}}{\partial r}\right) - \frac{\partial^2\mathbf{H}}{\partial r^2} = 0$$ [6.100]

On substituting equation [6.19], [6.20] (decaying approach) in equation [6.100] gives

(101) $$\mathbf{v}_{Hnc}^2(-\kappa_e^2\mathbf{H}) - \omega_e\mathbf{v}_{Hnc}(-\kappa_e\mathbf{H}) - c^2(-\kappa_e^2\mathbf{H}) = 0$$ [6.101]

(102) $$\kappa_e^2\mathbf{v}_{Hnc}^2 - c\kappa_e^2\mathbf{v}_{Hnc} - c^2\kappa_e^2 = 0$$ [6.102]

(103) $$\mathbf{v}_{Hnc}^2 - c\,\mathbf{v}_{Hnc} - c^2 = 0$$ [6.103]

(104) $$\begin{aligned}\mathbf{v}_{Hnc} &= \frac{c \pm \sqrt{(-c)^2 + 4c^2}}{2}\\ &= \frac{c \pm \sqrt{5c^2}}{2}\\ &= \frac{c \pm \sqrt{5}c}{2}\\ &= \frac{1 \pm \sqrt{5}}{2}c\end{aligned}$$ [6.104]

(105) $$\mathbf{v}_{Hnc(decay)} = 1.618c,\ -0.618c$$ [6.105]

Therefore

(106) $$\mathbf{v}_{Hnc(decay)} = 1.618c$$ [6.106]

Thus, a magnetic field strength $\mathbf{H}$ wave decays rapidly with 1.618 times speed of light in the non conductor.

On substituting equation [6.25], [6.26] (growth approach) in equation [6.100] gives

(107) $\mathbf{v}_{Hnc}^2(-\kappa_e^2\mathbf{H}) - \omega_e\mathbf{v}_{Hnc}(\kappa_e\mathbf{H}) - c^2(-\kappa_e^2\mathbf{H}) = 0$ [6.107]

(108) $\kappa_e^2\mathbf{v}_{Hnc}^2 + c\kappa_e^2\mathbf{v}_{Hnc} - c^2\kappa_e^2 = 0$ [6.108]

(109) $\mathbf{v}_{Hnc}^2 + c\,\mathbf{v}_{Hnc} - c^2 = 0$ [6.109]

(110)
$$\begin{aligned}\mathbf{v}_{Hnc} &= \frac{-c \pm \sqrt{c^2 + 4c^2}}{2} \\ &= \frac{-c \pm \sqrt{5c^2}}{2} \\ &= \frac{-c \pm \sqrt{5}c}{2} \\ &= \frac{-1 \pm \sqrt{5}}{2}c\end{aligned}$$
[6.110]

(111) $\mathbf{v}_{Hnc(growth)} = -\ 1.618c,\ 0.618c$ [6.111]

Therefore

(112) $\mathbf{v}_{Hnc(growth)} = 0.618c$ [6.112]

Thus, a magnetic field strength $\mathbf{H}$ wave grows slowly with 0.618 times speed of light in the non conductor.

The wave equation for the magnetic induction field in the non conductor [4.58] is

$$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{B}}{\partial t^2} - \omega_e\frac{\partial\mathbf{B}}{\partial t}\right) - \nabla^2\mathbf{B} = 0$$ [4.58]

On substituting equation [6.12], [6.13] and [6.14] in equation [4.58] gives

(113) $$\frac{1}{c^2}\left(\mathbf{v}^2\frac{\partial^2\mathbf{B}}{\partial r^2}-\omega_e\mathbf{v}\frac{\partial\mathbf{B}}{\partial r}\right)-\frac{\partial^2\mathbf{B}}{\partial r^2}=0$$ [6.113]

The velocity $\mathbf{v}$ in equation [6.113] represents the velocity of the magnetic induction field in the conductor $\mathbf{v}_{Bnc}$, therefore

(114) $$\frac{1}{c^2}\left(\mathbf{v}_{Bnc}^2\frac{\partial^2\mathbf{B}}{\partial r^2}-\omega_e\mathbf{v}_{Bnc}\frac{\partial\mathbf{B}}{\partial r}\right)-\frac{\partial^2\mathbf{B}}{\partial r^2}=0$$ [6.114]

On substituting equation [6.31], [6.32] (harmonic approach) in equation [6.114] gives

(115) $$\mathbf{v}_{Bnc}^2(-\kappa_e^2\mathbf{B})-\omega_e\mathbf{v}_{Bnc}(-i\kappa_e\mathbf{B})-c^2(-\kappa_e^2\mathbf{B})=0$$ [6.115]

(116) $$\kappa_e^2\mathbf{v}_{Bnc}^2-ic\kappa_e^2\mathbf{v}_{Bnc}-c^2\kappa_e^2=0$$ [6.116]

(117) $$\mathbf{v}_{Bnc}^2-ic\mathbf{v}_{Bnc}-c^2=0$$ [6.117]

(118) $$\begin{aligned}\mathbf{v}_{Bnc}&=\frac{ic\pm\sqrt{(-ic)^2+4c^2}}{2}\\&=\frac{ic\pm\sqrt{-c^2+4c^2}}{2}\\&=\frac{ic\pm\sqrt{3c^2}}{2}\\&=\frac{i\pm\sqrt{3}}{2}c\end{aligned}$$ [6.118]

(119) $$\mathbf{v}_{Bnc}=(0.866+0.5i)c,\ (-0.866+0.5i)c$$ [6.119]

Therefore

(120) $$\mathbf{v}_{Bnc}=(0.866+0.5i)c$$ [6.120]

(121) $\mathbf{v}_{Bnc} = \left(cos\frac{\pi}{6} + i\ sin\frac{\pi}{6} \right) c$ [6.121]

(122) $\mathbf{v}_{Bnc} = c\ e^{\frac{\pi}{6}i}$ [6.122]

(123) $\mathbf{v}_{Bnc} = c\ R\left(\frac{\pi}{6}\right)$ [6.123]

Similarly

(124) $\mathbf{v}_{Vnc} = c\ R\left(\frac{\pi}{6}\right)$ [6.124]

The magnetic induction field $\mathbf{B}$ and voltage area density field $\mathbf{V}$ rotate harmonically with rotor angle of $\frac{\pi}{6}$ with the speed of light. This implies that the $\mathbf{B}$ and $\mathbf{V}$ fields wave number and hence their wave length shortens by factor of 3 in the non conductors.

Table 6.1

Duality					
Electricity			**Magnetism**		
Perpendicular Interaction - Conductor					
Rapid decay	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{E}}{\partial t^2} - \omega_p\frac{\partial\mathbf{E}}{\partial t}\right) - \nabla^2\mathbf{E} = 0$	$\mathbf{v}_{Ec(decay)} = 1.618c$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{H}}{\partial t^2} - \omega_p\frac{\partial\mathbf{H}}{\partial t}\right) - \nabla^2\mathbf{H} = 0$	$\mathbf{v}_{Hc(decay)} = 0$
Slow growth	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{E}}{\partial t^2} - \omega_p\frac{\partial\mathbf{E}}{\partial t}\right) - \nabla^2\mathbf{E} = 0$	$\mathbf{v}_{Ec(growth)} = 0.618c$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{H}}{\partial t^2} - \omega_p\frac{\partial\mathbf{H}}{\partial t}\right) - \nabla^2\mathbf{H} = 0$	$\mathbf{v}_{Hc(growth)} = 0$
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{B}}{\partial t^2} - \omega_p\frac{\partial\mathbf{B}}{\partial t}\right) - \nabla^2\mathbf{B} = 0$	$\mathbf{v}_{Bc} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{D}}{\partial t^2} - \omega_p\frac{\partial\mathbf{D}}{\partial t}\right) - \nabla^2\mathbf{D} = 0$	$\mathbf{v}_{Dc} = c\ e^{\frac{\pi}{6}i}$
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{V}}{\partial t^2} - \omega_p\frac{\partial\mathbf{V}}{\partial t}\right) - \nabla^2\mathbf{V} = 0$	$\mathbf{v}_{Vc} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{J}}{\partial t^2} - \omega_p\frac{\partial\mathbf{J}}{\partial t}\right) - \nabla^2\mathbf{J} = 0$	$\mathbf{v}_{Jc} = c\ e^{\frac{\pi}{6}i}$
Perpendicular Interaction - Non Conductor					
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{E}}{\partial t^2} + \omega_e\frac{\partial\mathbf{E}}{\partial t}\right) - \nabla^2\mathbf{E} = 0$	$\mathbf{v}_{Enc(decay)} = 0$	Rapid decay	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{H}}{\partial t^2} + \omega_e\frac{\partial\mathbf{H}}{\partial t}\right) - \nabla^2\mathbf{H} = 0$	$\mathbf{v}_{Hnc(decay)} = 1.618c$
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{E}}{\partial t^2} + \omega_e\frac{\partial\mathbf{E}}{\partial t}\right) - \nabla^2\mathbf{E} = 0$	$\mathbf{v}_{Enc(growth)} = 0$	Slow growth	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{H}}{\partial t^2} + \omega_e\frac{\partial\mathbf{H}}{\partial t}\right) - \nabla^2\mathbf{H} = 0$	$\mathbf{v}_{Hnc(growth)} = 0.618c$

Duality					
Electricity			Magnetism		
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2}+\omega_e\frac{\partial \mathbf{B}}{\partial t}\right)-\nabla^2\mathbf{B}=0$	$\mathbf{v}_{Bnc}=c\ e^{\frac{\pi}{6}i}$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{D}}{\partial t^2}+\omega_e\frac{\partial \mathbf{D}}{\partial t}\right)-\nabla^2\mathbf{D}=0$	$\mathbf{v}_{Dnc}=0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{V}}{\partial t^2}+\omega_e\frac{\partial \mathbf{V}}{\partial t}\right)-\nabla^2\mathbf{V}=0$	$\mathbf{v}_{Vnc}=c\ e^{\frac{\pi}{6}i}$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{J}}{\partial t^2}+\omega_e\frac{\partial \mathbf{J}}{\partial t}\right)-\nabla^2\mathbf{J}=0$	$\mathbf{v}_{Jnc}=0$

On the closer inspection of Table 6.1, it can be clearly observed that

i) In the perpendicular interaction $\mathbf{E} \perp \mathbf{H}$, the electric field vector rapidly decays whereas the magnetic field vector is instantly damped in the conductor and vice versa in the non conductor. Although the $\mathbf{E}$ and $\mathbf{H}$ waves are perpendicular, it does not mean that they are tied to each other and follow the same pattern.

ii) In the perpendicular interaction $\mathbf{E} \perp \mathbf{H}$, the magnetic field vector slowly grows whereas the electric field vector is instantly damped in the conductor and vice versa in the non conductor. In the perpendicular interaction $\langle \mathbf{B} \rangle_2 \perp \langle \mathbf{D} \rangle_2$, the magnetic induction field bivector is instantly damped whereas the electric induction field bivector rotates harmonically in the conductor and vice versa in the non conductor. Although the $\langle \mathbf{B} \rangle_2$ and $\langle \mathbf{D} \rangle_2$ waves waves are perpendicular, it does not mean that they are tied to each other and follow the same pattern.

7. Dual Wave Velocity

7.1. Conductors

The dual wave equation for the Electric field in the conductor [5.24] is

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p^2 \mathbf{E} = 0 \qquad [5.24]$$

On substituting equation [6.4] in equation [5.24] gives

(1) $$\mathbf{v}^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} + \omega_p^2 \mathbf{E} = 0 \qquad [7.1]$$

The velocity $\mathbf{v}$ in equation [7.1] represents the velocity of the dual wave of the electric field strength in the conductor $\mathbf{v}_{Ec(dual)}$, therefore

(2) $$\mathbf{v}_{Ec(dual)}^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} + \omega_p^2 \mathbf{E} = 0 \qquad [7.2]$$

On substituting equation [6.17] or [6.23] (decaying or growth approach) in equation [7.2] gives

(3) $$\mathbf{v}_{Ec(dual)}^2 (\kappa_p^2 \mathbf{E}) + \omega_p^2 \mathbf{E} = 0 \qquad [7.3]$$

(4) $$\mathbf{v}_{Ec(dual)}^2 \kappa_p^2 + c^2 \kappa_p^2 = 0 \qquad [7.4]$$

(5) $$\mathbf{v}_{Ec(dual)}^2 + c^2 = 0 \qquad [7.5]$$

(6) $$\mathbf{v}_{Ec(dual)} = \pm\, ic \qquad [7.6]$$

Therefore

(7) $$\mathbf{v}_{Ec(dual)} = ic \qquad [7.7]$$

(8) $\mathbf{v}_{Ec(dual)} = \left(cos\frac{\pi}{2} + i\ sin\frac{\pi}{2} \right) c$ [7.8]

(9) $\mathbf{v}_{Enc(dual)} = c\ e^{\frac{\pi}{2}i}$ [7.9]

(10) $\mathbf{v}_{Enc(dual)} = c\ R\left(\frac{\pi}{2}\right)$ [7.10]

The electric field $\mathbf{E}$ vector decays or grows with speed of light with rotor angle of $\frac{\pi}{2}$ in the conductor in parallel interaction.

The dual wave equation for the electric displacement field in the non conductor [5.25] is

$$\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_p^2 \mathbf{D} = 0$$ [5.25]

On substituting equation [6.10] in equation [5.25] gives

(11) $\mathbf{v}^2 \frac{\partial^2 \mathbf{D}}{\partial r^2} + \omega_p^2 \mathbf{D} = 0$ [7.11]

The velocity $\mathbf{v}$ in equation [7.11] represents the velocity of a dual wave for the electric induction field in the conductor $\mathbf{v}_{Dc(dual)}$, therefore

(12) $\mathbf{v}_{Dc(dual)}^2 \frac{\partial^2 \mathbf{D}}{\partial r^2} + \omega_p^2 \mathbf{D} = 0$ [7.12]

On substituting equation [6.23] (harmonic approach) in equation [7.31] gives

(13) $\mathbf{v}_{Dc(dual)}^2 (-\kappa_p^2 \mathbf{D}) + \omega_p^2 \mathbf{D} = 0$ [7.13]

(14) $-\mathbf{v}_{Dc(dual)}^2 \kappa_p^2 + c^2 \kappa_p^2 = 0$ [7.14]

(15) $\mathbf{v}_{Dc(dual)}^2 - c^2 = 0$ [7.15]

(16) $\mathbf{v}_{Dc(dual)} = \pm\, c$ [7.16]

Therefore

(17) $\mathbf{v}_{Dc(dual)} = c$ [7.17]

(18) $\mathbf{v}_{Dc(dual)} = (cos2\pi + i\ sin2\pi)\, c$ [7.18]

(19) $\mathbf{v}_{Dc(dual)} = c\ e^{2\pi i}$ [7.19]

(20) $\mathbf{v}_{Dc(dual)} = c\ R\,(2\pi)$ [7.20]

The electric induction field $\mathbf{D}$ bivector spins harmonically with rotor angle of 2π with speed of light in the conductor in parallel interaction. It is a spinor and takes a rotation of 4π to go back to its original state.

Since $\mathbf{J} = 0$

(21) $\mathbf{v}_{Jc(dual)} = 0$ [7.21]

According to equations [5.27] to [5.29]

$$\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0 \qquad [5.27]$$

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \qquad [5.28]$$

$$\mathbf{V} = 0 \qquad [5.29]$$

Therefore,

(22) $\mathbf{v}_{Hc(dual)} = 0$ [7.22]

(23) $\mathbf{v}_{Bc(dual)} = 0$ [7.23]

(24) $\mathbf{v}_{Vc(dual)} = 0$ [7.24]

There is no dual wave for the $\mathbf{H}$, $\mathbf{B}$ and $\mathbf{V}$ field in the conductor in parallel interaction.

7.2. Non Conductors

According to equations [5.30] to [5.32]

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \qquad [5.30]$$

$$\frac{\partial^2 \mathbf{D}}{\partial t^2} = 0 \qquad [5.31]$$

$$\mathbf{J} = 0 \qquad [5.32]$$

Therefore

(25) $$\mathbf{v}_{Enc(dual)} = 0 \qquad [7.25]$$

(26) $$\mathbf{v}_{Dnc(dual)} = 0 \qquad [7.26]$$

(27) $$\mathbf{v}_{Jnc(dual)} = 0 \qquad [7.27]$$

There is no dual wave for the $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{J}$ field in the non conductor in parallel interaction.

The dual wave equation for the magnetic field in the non conductor [5.33] is

$$\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_e^2 \mathbf{H} = 0 \qquad [5.33]$$

On substituting equation [6.7] in equation [5.33] gives

(28) $$\mathbf{v}^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} + \omega_e^2 \mathbf{H} = 0 \qquad [7.28]$$

The velocity $\mathbf{v}$ in equation [7.28] represents the velocity of dual wave of the magnetic field strength in the non conductor $\mathbf{v}_{Hc(dual)}$, therefore

(29) $\mathbf{v}_{Hnc(dual)}^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} + \omega_e^2 \mathbf{H} = 0$ [7.29]

On substituting equation [6.20] or [6.26] (decaying or growth approach) in equation [7.28] gives

(30) $\mathbf{v}_{Hnc(dual)}^2 (\kappa_e^2 \mathbf{H}) + \omega_e^2 \mathbf{H} = 0$ [7.30]

(31) $\mathbf{v}_{Hnc(dual)}^2 \kappa_e^2 + c^2 \kappa_e^2 = 0$ [7.31]

(32) $\mathbf{v}_{Hnc(dual)}^2 + c^2 = 0$ [7.32]

(33) $\mathbf{v}_{Hnc(dual)} = \pm\, ic$ [7.33]

Therefore

(34) $\mathbf{v}_{Hnc(dual)} = ic$ [7.34]

(35) $\mathbf{v}_{Hnc(dual)} = \left(cos \frac{\pi}{2} + i\ sin \frac{\pi}{2} \right) c$ [7.35]

(36) $\mathbf{v}_{Hnc(dual)} = c\ e^{\frac{\pi}{2} i}$ [7.36]

(37) $\mathbf{v}_{Hnc(dual)} = c\ R \left(\frac{\pi}{2} \right)$ [7.37]

The magnetic field $\mathbf{H}$ vector decays or grows with speed of light with rotor angle of $\frac{\pi}{2}$ in the conductor in parallel interaction.

The dual wave equation for the magnetic induction field in the conductor [5.34] is

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_e^2 \mathbf{B} = 0$$ [5.34]

On substituting equation [6.13] in equation [5.34] gives

(38) $$\mathbf{v}^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} + \omega_e^2 \mathbf{B} = 0$$ [7.38]

The velocity $\mathbf{v}$ in equation [7.38] represents the velocity of a dual wave for the magnetic induction field in the non conductor $\mathbf{v}_{Bnc(dual)}$, therefore

(39) $$\mathbf{v}_{Bnc(dual)}^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} + \omega_e^2 \mathbf{B} = 0$$ [7.39]

On substituting equation [6.26] (harmonic approach) in equation [7.39] gives

(40) $$\mathbf{v}_{Bnc(dual)}^2 (-\kappa_e^2 \mathbf{B}) + \omega_e^2 \mathbf{B} = 0$$ [7.40]

(41) $$-\mathbf{v}_{Bnc(dual)}^2 \kappa_e^2 + c^2 \kappa_e^2 = 0$$ [7.41]

(42) $$\mathbf{v}_{Bnc(dual)}^2 - c^2 = 0$$ [7.42]

(43) $$\mathbf{v}_{Bnc(dual)} = \pm c$$ [7.43]

Therefore

(44) $$\mathbf{v}_{Bnc(dual)} = c$$ [7.44]

(45) $$\mathbf{v}_{Bnc(dual)} = (cos 2\pi + i \ sin 2\pi)\, c$$ [7.45]

(46) $$\mathbf{v}_{Bnc(dual)} = c \ e^{2\pi i}$$ [7.45]

(47) $$\mathbf{v}_{Bnc(dual)} = c \ R\,(2\pi)$$ [7.46]

The electric induction field $\mathbf{B}$ bivector spins harmonically with rotor angle of 2π with speed of light in the non conductor in parallel interaction. It is a spinor and takes a rotation of 4π to go back to its original state.

Since $\mathbf{V} = 0$

(48) $\mathbf{v}_{Vnc(dual)} = 0$ [7.46]

Table 7.1

Duality					
Electricity			**Magnetism**		
Perpendicular Interaction - Conductor					
Rapid decay	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{E}}{\partial t^2} - \omega_p \frac{\partial\mathbf{E}}{\partial t}\right) - \nabla^2\mathbf{E} = 0$	$\mathbf{v}_{Ec(decay)} = 1.618c$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{H}}{\partial t^2} - \omega_p \frac{\partial\mathbf{H}}{\partial t}\right) - \nabla^2\mathbf{H} = 0$	$\mathbf{v}_{Hc(decay)} = 0$
Slow growth	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{E}}{\partial t^2} - \omega_p \frac{\partial\mathbf{E}}{\partial t}\right) - \nabla^2\mathbf{E} = 0$	$\mathbf{v}_{Ec(growth)} = 0.618c$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{H}}{\partial t^2} - \omega_p \frac{\partial\mathbf{H}}{\partial t}\right) - \nabla^2\mathbf{H} = 0$	$\mathbf{v}_{Hc(growth)} = 0$
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{B}}{\partial t^2} - \omega_p \frac{\partial\mathbf{B}}{\partial t}\right) - \nabla^2\mathbf{B} = 0$	$\mathbf{v}_{Bc} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{D}}{\partial t^2} - \omega_p \frac{\partial\mathbf{D}}{\partial t}\right) - \nabla^2\mathbf{D} = 0$	$\mathbf{v}_{Dc} = c\ e^{\frac{\pi}{6}i}$
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{V}}{\partial t^2} - \omega_p \frac{\partial\mathbf{V}}{\partial t}\right) - \nabla^2\mathbf{V} = 0$	$\mathbf{v}_{Vc} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{J}}{\partial t^2} - \omega_p \frac{\partial\mathbf{J}}{\partial t}\right) - \nabla^2\mathbf{J} = 0$	$\mathbf{v}_{Jc} = c\ e^{\frac{\pi}{6}i}$
Perpendicular Interaction - Non Conductor					
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{E}}{\partial t^2} + \omega_e \frac{\partial\mathbf{E}}{\partial t}\right) - \nabla^2\mathbf{E} = 0$	$\mathbf{v}_{Enc(decay)} = 0$	Rapid decay	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{H}}{\partial t^2} + \omega_e \frac{\partial\mathbf{H}}{\partial t}\right) - \nabla^2\mathbf{H} = 0$	$\mathbf{v}_{Hnc(decay)} = 1.618c$
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{E}}{\partial t^2} + \omega_e \frac{\partial\mathbf{E}}{\partial t}\right) - \nabla^2\mathbf{E} = 0$	$\mathbf{v}_{Enc(growth)} = 0$	Slow growth	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{H}}{\partial t^2} + \omega_e \frac{\partial\mathbf{H}}{\partial t}\right) - \nabla^2\mathbf{H} = 0$	$\mathbf{v}_{Hnc(growth)} = 0.618c$
	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{B}}{\partial t^2} + \omega_e \frac{\partial\mathbf{B}}{\partial t}\right) - \nabla^2\mathbf{B} = 0$	$\mathbf{v}_{Bnc} = c\ e^{\frac{\pi}{6}i}$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{D}}{\partial t^2} + \omega_e \frac{\partial\mathbf{D}}{\partial t}\right) - \nabla^2\mathbf{D} = 0$	$\mathbf{v}_{Dnc} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{V}}{\partial t^2} + \omega_e \frac{\partial\mathbf{V}}{\partial t}\right) - \nabla^2\mathbf{V} = 0$	$\mathbf{v}_{Vnc} = c\ e^{\frac{\pi}{6}i}$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2\mathbf{J}}{\partial t^2} + \omega_e \frac{\partial\mathbf{J}}{\partial t}\right) - \nabla^2\mathbf{J} = 0$	$\mathbf{v}_{Jnc} = 0$
Parallel Interaction - Conductor					
Normal decay and growth	$\frac{\partial^2\mathbf{E}}{\partial t^2} + \omega_p^2\mathbf{E} = 0$	$\mathbf{v}_{Ec(dual)} = c\ e^{\frac{\pi}{2}i}$	Maxima or Minima	$\frac{\partial^2\mathbf{H}}{\partial t^2} = 0$	$\mathbf{v}_{Hc(dual)} = 0$
Maxima or Minima	$\frac{\partial^2\mathbf{B}}{\partial t^2} = 0$	$\mathbf{v}_{Bc(dual)} = 0$	Spinor	$\frac{\partial^2\mathbf{D}}{\partial t^2} + \omega_p^2\mathbf{D} = 0$	$\mathbf{v}_{Dc(dual)} = c\ e^{2\pi i}$
	$\mathbf{V} = 0$			$\mathbf{J} = 0$	
Parallel Interaction - Non Conductor					
Maxima or Minima	$\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$	$\mathbf{v}_{Enc(dual)} = 0$	Normal decay and growth	$\frac{\partial^2\mathbf{H}}{\partial t^2} + \omega_e^2\mathbf{H} = 0$	$\mathbf{v}_{Hnc(dual)} = c\ e^{\frac{\pi}{2}i}$
Spinor	$\frac{\partial^2\mathbf{B}}{\partial t^2} + \omega_e^2\mathbf{B} = 0$	$\mathbf{v}_{Bnc(dual)} = c\ e^{2\pi i}$	Maxima or Minima	$\frac{\partial^2\mathbf{D}}{\partial t^2} = 0$	$\mathbf{v}_{Dnc(dual)} = 0$

Duality					
Electricity			Magnetism		
	$\mathbf{V} = 0$			$\mathbf{J} = 0$	

On the closer inspection of Table 7.1, it can clearly be observed that

i) In the perpendicular interaction $\mathbf{E} \perp \mathbf{H}$, the electric field vector rapidly decays whereas the magnetic field vector is instantly damped in the conductor and vice versa in the non conductor. Although the $\mathbf{E}$ and $\mathbf{H}$ waves are perpendicular, it does not mean that they are tied to each other and follow the same pattern.

ii) In the perpendicular interaction $\mathbf{E} \perp \mathbf{H}$, the magnetic field vector slowly grows whereas the electric field vector is instantly damped in the conductor and vice versa in the non conductor. In the perpendicular interaction $\langle \mathbf{B} \rangle_2 \perp \langle \mathbf{D} \rangle_2$, the magnetic induction field bivector is instantly damped whereas the electric induction field bivector rotates harmonically in the conductor and vice versa in the non conductor. Although the $\langle \mathbf{B} \rangle_2$ and $\langle \mathbf{D} \rangle_2$ waves are perpendicular, it does not mean that they are tied to each other and follow the same pattern.

iii) In the parallel interaction $\mathbf{E} \parallel \langle \mathbf{D} \rangle_2$, the electric field vector decays or grows with rotor angle of $\frac{\pi}{2}$ and the electric induction bivector spins harmonically (rotor angle of 2π) in the conductor but both of them are damped in the non conductor. Although the $\mathbf{E}$ and $\langle \mathbf{D} \rangle_2$waves are parallel, it does not mean that they are tied to each other and follow the same pattern.

iv) In the parallel interaction $\langle \mathbf{B} \rangle_2 \parallel \mathbf{H}$, both the magnetic field vector and the magnetic induction bivector are damped in the conductor but the magnetic field vector decays or grows with rotor angle of $\frac{\pi}{2}$ and the magnetic induction bivector spins harmonically (rotor angle of 2π) in the non conductor. Although the $\mathbf{E}$ and $\langle \mathbf{D} \rangle_2$waves are parallel, it does not mean that they are tied to each other and follow the same pattern.

8. Equilibrium Wave Equations

The wave equation for the electric field [4.15] is

$$\Box\mathbf{E} + \frac{1}{c^2}\left(\omega_p\frac{\partial\mathbf{E}}{\partial t} - \omega_e\frac{\partial\mathbf{E}}{\partial t} - \omega_0^2\mathbf{E}\right) = 0 \quad [4.15]$$

For the special case of equilibrium in the quantum realm

(1) $$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e \quad [8.1]$$

On substituting the equation [8.1] in the wave equation [4.15], this becomes

(2) $$\Box\mathbf{E} - \kappa_0^2\mathbf{E} = 0 \quad [8.2]$$

(3) $$\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \nabla^2\mathbf{E} - \kappa_0^2\mathbf{E} = 0 \quad [8.3]$$

(4) $$\frac{\partial^2\mathbf{E}}{\partial t^2} - c^2\nabla^2\mathbf{E} - c^2\kappa_0^2\mathbf{E} = 0 \quad [8.4]$$

Similarly

(5) $$\frac{\partial^2\mathbf{D}}{\partial t^2} - c^2\nabla^2\mathbf{D} - c^2\kappa_0^2\mathbf{D} = 0 \quad [8.5]$$

(6) $$\frac{\partial^2\mathbf{J}}{\partial t^2} - c^2\nabla^2\mathbf{J} - c^2\kappa_0^2\mathbf{J} = 0 \quad [8.6]$$

(7) $$\frac{\partial^2\mathbf{H}}{\partial t^2} - c^2\nabla^2\mathbf{H} - c^2\kappa_0^2\mathbf{H} = 0 \quad [8.7]$$

(8) $$\frac{\partial^2\mathbf{B}}{\partial t^2} - c^2\nabla^2\mathbf{B} - c^2\kappa_0^2\mathbf{B} = 0 \quad [8.8]$$

(9) $$\frac{\partial^2\mathbf{V}}{\partial t^2} - c^2\nabla^2\mathbf{V} - c^2\kappa_0^2\mathbf{V} = 0 \quad [8.9]$$

According to the Einstein/ de Broglie equation for a rest particle:

(10) $\hbar\omega_0 = mc^2$ [8.10]

(11) $\omega_0 = \dfrac{mc^2}{\hbar}$ [8.11]

(12) $\kappa_0 = \dfrac{mc}{\hbar}$ [8.12]

(13) On substituting the equation [8.12] in equation [8.2] gives

(14) $\Box \mathbf{E} - \left(\dfrac{mc}{\hbar}\right)^2 \mathbf{E} = 0$ [8.13]

In this way, the equilibrium wave equation is connected with the quantum-mechanical realm. The equilibrium wave equation [8.13] is known as the **Klein–Gordon equation.**

Table 8.1

Duality			
Electricity		**Magnetism**	
Perpendicular Interaction			
Equilibrium wave equation	$\dfrac{\partial^2 \mathbf{E}}{\partial t^2} - c^2\nabla^2\mathbf{E} - c^2\kappa_0^2\mathbf{E} = 0$	Equilibrium wave equation	$\dfrac{\partial^2 \mathbf{H}}{\partial t^2} - c^2\nabla^2\mathbf{H} - c^2\kappa_0^2\mathbf{H} = 0$
	$\dfrac{\partial^2 \mathbf{B}}{\partial t^2} - c^2\nabla^2\mathbf{B} - c^2\kappa_0^2\mathbf{B} = 0$		$\dfrac{\partial^2 \mathbf{D}}{\partial t^2} - c^2\nabla^2\mathbf{D} - c^2\kappa_0^2\mathbf{D} = 0$
	$\dfrac{\partial^2 \mathbf{V}}{\partial t^2} - c^2\nabla^2\mathbf{V} - c^2\kappa_0^2\mathbf{V} = 0$		$\dfrac{\partial^2 \mathbf{J}}{\partial t^2} - c^2\nabla^2\mathbf{J} - c^2\kappa_0^2\mathbf{J} = 0$

9. Dual Equilibrium Wave Equations

The dual/ Beltrami wave equation for the electric field [5.7] is

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p^2 \mathbf{E} = 0 \qquad [5.7]$$

For the special case of equilibrium in the quantum realm [8.1]

$$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e$$

On substituting the equation [8.1] in the wave equation [5.7] becomes

(1) $$\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_0^2 \mathbf{E} = 0 \qquad [9.1]$$

(2) $$\frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{E} = 0 \qquad [9.2]$$

Similarly

(3) $$\frac{\partial^2 \mathbf{D}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{D} = 0 \qquad [9.3]$$

(4) $$\mathbf{J} = 0 \qquad [9.4]$$

(5) $$\frac{\partial^2 \mathbf{H}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{H} = 0 \qquad [9.5]$$

(6) $$\frac{\partial^2 \mathbf{B}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{B} = 0 \qquad [9.6]$$

(7) $$\mathbf{V} = 0 \qquad [9.7]$$

On substituting the equation [8.10] in equation [9.2] gives

(8) $$\frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2 \left(\frac{mc}{\hbar}\right)^2 \mathbf{E} = 0$$ [9.8]

In this way, the dual equilibrium wave equation is connected with the quantum-mechanical realm. The equation [9.8] can be seen as the Klein–Gordon equation in the dual space**.**

Table 9.1

Duality			
Electricity		**Magnetism**	
Perpendicular Interaction			
Equilibrium wave equation	$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \nabla^2 \mathbf{E} - c^2 \kappa_0^2 \mathbf{E} = 0$	Equilibrium wave equation	$\frac{\partial^2 \mathbf{H}}{\partial t^2} - c^2 \nabla^2 \mathbf{H} - c^2 \kappa_0^2 \mathbf{H} = 0$
	$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c^2 \nabla^2 \mathbf{B} - c^2 \kappa_0^2 \mathbf{B} = 0$		$\frac{\partial^2 \mathbf{D}}{\partial t^2} - c^2 \nabla^2 \mathbf{D} - c^2 \kappa_0^2 \mathbf{D} = 0$
	$\frac{\partial^2 \mathbf{V}}{\partial t^2} - c^2 \nabla^2 \mathbf{V} - c^2 \kappa_0^2 \mathbf{V} = 0$		$\frac{\partial^2 \mathbf{J}}{\partial t^2} - c^2 \nabla^2 \mathbf{J} - c^2 \kappa_0^2 \mathbf{J} = 0$
Parallel Interaction			
Dual equilibrium wave equation	$\frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{E} = 0$	Dual equilibrium wave equation	$\frac{\partial^2 \mathbf{H}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{H} = 0$
	$\frac{\partial^2 \mathbf{B}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{B} = 0$		$\frac{\partial^2 \mathbf{D}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{D} = 0$
	$\mathbf{V} = 0$		$\mathbf{J} = 0$

10. Equilibrium Wave Velocity

The equilibrium wave equation for the electric field [8.4] is

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \nabla^2 \mathbf{E} - c^2 \kappa_0^2 \mathbf{E} = 0 \qquad [8.4]$$

On substituting equation [6.4] and [6.5] in equation [8.4] gives

(1) $$\mathbf{v}^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} - c^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} - c^2 \kappa_0^2 \mathbf{E} = 0 \qquad [10.1]$$

The velocity $\mathbf{v}$ in equation [10.1] represents the velocity of the electric field in the equilibrium $\mathbf{v}_{Ee}$, therefore

(2) $$\mathbf{v}_{Ee}^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} - c^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} - c^2 \kappa_0^2 \mathbf{E} = 0 \qquad [10.2]$$

On substituting equation [6.17] or [6.23] (decaying or growth approach) in equation [10.2] gives

(3) $$\mathbf{v}_{Ee}^2 (\kappa_p^2 \mathbf{E}) - c^2 (\kappa_p^2 \mathbf{E}) - c^2 \kappa_0^2 \mathbf{E} = 0 \qquad [10.3]$$

For the special case of equilibrium in the quantum realm [8.1]

$$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e \qquad [8.1]$$

Therefore

(4) $$\mathbf{v}_{Ee}^2 (\kappa_0^2 \mathbf{E}) - c^2 (\kappa_0^2 \mathbf{E}) - c^2 \kappa_0^2 \mathbf{E} = 0 \qquad [10.4]$$

(5) $$\mathbf{v}_{Ee}^2 - 2c^2 = 0 \qquad [10.5]$$

(6) $$\mathbf{v}_{Ee} = \pm \sqrt{2} c \qquad [10.6]$$

Therefore

(7) $\mathbf{v}_{Ee} = \sqrt{2}c$ [10.7]

The electric field strength $\mathbf{E}$ decays or grows faster than speed of light in the equilibrium state.

The equilibrium wave equation for the electric displacement [8.5] is

$$\frac{\partial^2 \mathbf{D}}{\partial t^2} - c^2 \nabla^2 \mathbf{D} - c^2 \kappa_0^2 \mathbf{D} = 0 \qquad [8.5]$$

On substituting equation [6.10] and [6.11] in equation [8.5] gives

(8) $\mathbf{v}^2 \frac{\partial^2 \mathbf{D}}{\partial r^2} - c^2 \frac{\partial^2 \mathbf{D}}{\partial r^2} - c^2 \kappa_0^2 \mathbf{D} = 0$ [10.8]

The velocity $\mathbf{v}$ in equation [10.8] represents the velocity of the electric displacement field in equilibrium $\mathbf{v}_{De}$, therefore

(9) $\mathbf{v}_{De}^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} - c^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} - c^2 \kappa_0^2 \mathbf{E} = 0$ [10.9]

On substituting equation [6.23] (harmonic approach) in equation [10.12] gives

(10) $\mathbf{v}_{De}^2(-\kappa_p^2 \mathbf{E}) - c^2(-\kappa_p^2 \mathbf{E}) - c^2 \kappa_0^2 \mathbf{E} = 0$ [10.10]

For the special case of equilibrium in the quantum realm [8.1]

$$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e \qquad [8.1]$$

Therefore

(11) $\mathbf{v}_{De}^2(-\kappa_0^2 \mathbf{E}) + c^2(\kappa_0^2 \mathbf{E}) - c^2 \kappa_0^2 \mathbf{E} = 0$ [10.11]

(12) $-\kappa_p^2 \mathbf{v}_{De}^2 = 0$ [10.12]

(13) $\mathbf{v}_{De} = 0$ [10.13]

Similarly

(14) $\mathbf{v}_{Je} = 0$ [10.14][374]

Therefore, the $\mathbf{D}$ and $\mathbf{J}$ fields are damped in the equilibrium state.

The equilibrium wave equation for the magnetic field [8.7] is

$$\frac{\partial^2 \mathbf{H}}{\partial t^2} - c^2 \nabla^2 \mathbf{H} - c^2 \kappa_0^2 \mathbf{H} = 0$$ [8.7]

On substituting equation [6.7] and [6.8] in equation [8.7] gives

(15) $\mathbf{v}^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} - c^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} - c^2 \kappa_0^2 \mathbf{H} = 0$ [10.15]

The velocity $\mathbf{v}$ in equation [10.15] represents the velocity of the magnetic field in the equilibrium $\mathbf{v}_{He}$, therefore

(16) $\mathbf{v}_{He}^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} - c^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} - c^2 \kappa_0^2 \mathbf{H} = 0$ [10.16]

On substituting equation [6.20] or [6.26] (decaying or growth approach) in equation [10.16] gives

(17) $\mathbf{v}_{He}^2 (\kappa_e^2 \mathbf{E}) - c^2 (\kappa_e^2 \mathbf{E}) - c^2 \kappa_0^2 \mathbf{E} = 0$ [10.17]

For the special case of equilibrium in the quantum realm [8.1]

$$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e$$ [8.1]

Therefore

(18) $\mathbf{v}_{He}^2 (\kappa_0^2 \mathbf{E}) - c^2 (\kappa_0^2 \mathbf{E}) - c^2 \kappa_0^2 \mathbf{E} = 0$ [10.18]

(19) $\mathbf{v}_{He}^2 - 2c^2 = 0$ [10.19]

(20) $\mathbf{v}_{He} = \pm \sqrt{2} c$ [10.20]

Therefore

(21) $\mathbf{v}_{He} = \sqrt{2}c$ [10.21]

The magnetic field strength $\mathbf{H}$ decays or grows faster than speed of light in equilibrium state.

The equilibrium wave equation for the magnetic induction [8.8] is

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c^2 \nabla^2 \mathbf{B} - c^2 \kappa_0^2 \mathbf{B} = 0 \quad [8.8]$$

On substituting equation [6.13] and [6.14] in equation [8.8] gives

(22) $\mathbf{v}^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} - c^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} - c^2 \kappa_0^2 \mathbf{B} = 0$ [10.22]

The velocity $\mathbf{v}$ in equation [10.21] represents the velocity of the magnetic induction field in equilibrium $\mathbf{v}_{Be}$, therefore

(23) $\mathbf{v}_{Be}^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} - c^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} - c^2 \kappa_0^2 \mathbf{B} = 0$ [10.23]

On substituting equation [6.26] (harmonic approach) in equation [10.23] gives

(24) $\mathbf{v}_{Be}^2(-\kappa_e^2 \mathbf{E}) - c^2(-\kappa_e^2 \mathbf{E}) - c^2 \kappa_0^2 \mathbf{E} = 0$ [10.24]

For the special case of equilibrium in the quantum realm [8.1]

$$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e \quad [8.1]$$

Therefore

(25) $\mathbf{v}_{Be}^2(-\kappa_0^2 \mathbf{E}) + c^2(\kappa_0^2 \mathbf{E}) - c^2 \kappa_0^2 \mathbf{E} = 0$ [10.25]

(26) $-\kappa_e^2 \mathbf{v}_{Be}^2 = 0$ [10.26]

(27) $\mathbf{v}_{Be} = 0$ [10.27]

Similarly

(28) $\mathbf{v}_{Ve} = 0$ [10.28][392]

Therefore, the $\mathbf{B}$ and $\mathbf{V}$ fields are damped in the equilibrium state.

Table 10.1

Duality					
Electricity			**Magnetism**		
Perpendicular Interaction					
Equilibrium state - Faster decay or growth	$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2\nabla^2\mathbf{E} - c^2\kappa_0^2\mathbf{E} = 0$	$\mathbf{v}_{Ee} = \sqrt{2}c$	Equilibrium state - Faster decay or growth	$\frac{\partial^2 \mathbf{H}}{\partial t^2} - c^2\nabla^2\mathbf{H} - c^2\kappa_0^2\mathbf{H} = 0$	$\mathbf{v}_{He} = \sqrt{2}c$
Damped	$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c^2\nabla^2\mathbf{B} - c^2\kappa_0^2\mathbf{B} = 0$	$\mathbf{v}_{Be} = 0$	Damped	$\frac{\partial^2 \mathbf{D}}{\partial t^2} - c^2\nabla^2\mathbf{D} - c^2\kappa_0^2\mathbf{D} = 0$	$\mathbf{v}_{De} = 0$
Damped	$\frac{\partial^2 \mathbf{V}}{\partial t^2} - c^2\nabla^2\mathbf{V} - c^2\kappa_0^2\mathbf{V} = 0$	$\mathbf{v}_{Ve} = 0$	Damped	$\frac{\partial^2 \mathbf{J}}{\partial t^2} - c^2\nabla^2\mathbf{J} - c^2\kappa_0^2\mathbf{J} = 0$	$\mathbf{v}_{Je} = 0$

On the closer inspection of Table 10.1, it can be clearly observed that

i) In the perpendicular interaction $\mathbf{E} \perp \mathbf{H}$ in the equilibrium state, the electric field vector and the magnetic field decay or grow rapidly and follow the same pattern.

ii) In the perpendicular interaction $\langle \mathbf{B} \rangle_2 \perp \langle \mathbf{D} \rangle_2$ in the equilibrium state, both the magnetic induction field bivector and the electric induction field bivector are instantly damped and follow the same pattern.

11. Dual Equilibrium Wave Velocity

The dual equilibrium wave equation for the electric field [9.2] is

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{E} = 0 \quad [9.2]$$

On substituting equation [6.4] in equation [9.2] gives

(1) $$\mathbf{v}^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} + c^2 \kappa_0^2 \mathbf{E} = 0 \quad [11.1]$$

The velocity $\mathbf{v}$ in equation [11.1] represents the dual wave velocity of the electric field in the equilibrium $\mathbf{v}_{Ee(dual)}$, therefore

(2) $$\mathbf{v}_{Ee(dual)}^2 \frac{\partial^2 \mathbf{E}}{\partial r^2} + c^2 \kappa_0^2 \mathbf{E} = 0 \quad [11.2]$$

On substituting equation [6.17] or [6.23] (decaying or growth approach) in equation [10.2] gives

(3) $$\mathbf{v}_{Ee}^2(\kappa_p^2 \mathbf{E}) + c^2 \kappa_0^2 \mathbf{E} = 0 \quad [11.3]$$

For the special case of equilibrium in the quantum realm [8.1]

$$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e \quad [8.1]$$

Therefore

(4) $$\mathbf{v}_{Ee(dual)}^2(\kappa_0^2 \mathbf{E}) + c^2 \kappa_0^2 \mathbf{E} = 0 \quad [11.4]$$

(5) $$\mathbf{v}_{Ee(dual)}^2 + c^2 = 0 \quad [11.5]$$

(6) $$\mathbf{v}_{Ee(dual)} = \pm\, ic \quad [11.6]$$

Therefore

(7) $\mathbf{v}_{Ee(dual)} = ic$ [11.7]

(8) $\mathbf{v}_{Ee(dual)} = \left(cos\frac{\pi}{2} + i\ sin\frac{\pi}{2} \right) c$ [11.8]

(9) $\mathbf{v}_{Ee(dual)} = c\ e^{\frac{\pi}{2}i}$ [11.9]

(10) $\mathbf{v}_{Ee(dual)} = c\ R\left(\frac{\pi}{2}\right)$ [11.10]

The electric field $\mathbf{E}$ vector decays or grows with speed of light with rotor angle of $\frac{\pi}{2}$ in equilibrium state in parallel interaction.

The dual equilibrium wave equation for the electric displacement [9.3] is

$$\frac{\partial^2 \mathbf{D}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{D} = 0$$ [9.3]

On substituting equation [6.10] in equation [9.3] gives

(11) $\mathbf{v}^2 \frac{\partial^2 \mathbf{D}}{\partial r^2} + c^2 \kappa_0^2 \mathbf{D} = 0$ [11.11]

The velocity $\mathbf{v}$ in equation [11.11] represents the dual wave velocity of the electric displacement field in equilibrium $\mathbf{v}_{De(dual)}$, therefore

(12) $\mathbf{v}^2_{De(dual)} \frac{\partial^2 \mathbf{D}}{\partial r^2} + c^2 \kappa_0^2 \mathbf{D} = 0$ [11.12]

On substituting equation [6.23] (harmonic approach) in equation [11.12] gives

(13) $\mathbf{v}^2_{De(dual)}(-\kappa_p^2 \mathbf{D}) + c^2 \kappa_0^2 \mathbf{D} = 0$ [11.13]

For the special case of equilibrium in the quantum realm [8.1]

$$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e \quad [8.1]$$

Therefore

(14) $\mathbf{v}_{De}^2(-\kappa_0^2\mathbf{D}) + c^2\kappa_0^2\mathbf{D} = 0$ [11.14]

(15) $\mathbf{v}_{De(dual)}^2 - c^2 = 0$ [11.15]

(16) $\mathbf{v}_{De(dual)} = \pm\, c$ [11.16]

Therefore

(17) $\mathbf{v}_{De(dual)} = c$ [11.17]

(18) $\mathbf{v}_{De(dual)} = (cos2\pi + i\ sin2\pi)\,c$ [11.18]

(19) $\mathbf{v}_{De(dual)} = c\ e^{2\pi i}$ [11.19]

(20) $\mathbf{v}_{De(dual)} = c\ R\,(2\pi)$ [11.20]

The electric induction field $\mathbf{D}$ bivector spins harmonically with rotor angle of 2π with speed of light in the equilibrium state in parallel interaction. It is a spinor and takes a rotation of 4π to go back to its original state.

Since $\mathbf{J} = 0$

(21) $\mathbf{v}_{Je(dual)} = 0$ [11.21]

The dual equilibrium wave equation for the magnetic field [9.5] is

$$\frac{\partial^2\mathbf{H}}{\partial t^2} + c^2\kappa_0^2\mathbf{H} = 0 \quad [9.5]$$

On substituting equation [6.7] in equation [9.5] gives

(22) $$\mathbf{v}^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} + c^2 \kappa_0^2 \mathbf{H} = 0$$ [11.22]

The velocity $\mathbf{v}$ in equation [11.22] represents the dual wave velocity of the magnetic field in the equilibrium $\mathbf{v}_{He(dual)}$, therefore

(23) $$\mathbf{v}_{He(dual)}^2 \frac{\partial^2 \mathbf{H}}{\partial r^2} + c^2 \kappa_0^2 \mathbf{H} = 0$$ [11.23]

On substituting equation [6.20] or [6.26] (decaying or growth approach) in equation [11.23] gives

(24) $$\mathbf{v}_{He(dual)}^2 (\kappa_e^2 \mathbf{E}) + c^2 \kappa_0^2 \mathbf{E} = 0$$ [11.24]

For the special case of equilibrium in the quantum realm [8.1]

$$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e$$ [8.1]

Therefore

(25) $$\mathbf{v}_{He(dual)}^2 (\kappa_0^2 \mathbf{E}) + c^2 \kappa_0^2 \mathbf{E} = 0$$ [11.25]

(26) $$\mathbf{v}_{He(dual)}^2 + c^2 = 0$$ [11.26]

(27) $$\mathbf{v}_{He(dual)} = \pm\, ic$$ [11.27]

Therefore

(28) $$\mathbf{v}_{He(dual)} = ic$$ [11.28]

(29) $$\mathbf{v}_{He(dual)} = \left(cos \frac{\pi}{2} + i\ sin \frac{\pi}{2} \right) c$$ [11.29]

(30) $$\mathbf{v}_{He(dual)} = c\ e^{\frac{\pi}{2} i}$$ [11.30]

(31) $\mathbf{v}_{He(dual)} = c\ R\left(\frac{\pi}{2}\right)$ [11.31]

The magnetic field $\mathbf{H}$ vector decays or grows with speed of light with rotor angle of $\frac{\pi}{2}$ in equilibrium state in parallel interaction.

The dual equilibrium wave equation for the magnetic induction [9.6] is

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{B} = 0$$ [9.6]

On substituting equation [6.13] in equation [9.6] gives

(32) $\mathbf{v}^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} + c^2 \kappa_0^2 \mathbf{B} = 0$ [11.32]

The velocity $\mathbf{v}$ in equation [11.32] represents the dual wave velocity of the magnetic induction field in equilibrium $\mathbf{v}_{Be(dual)}$, therefore

(33) $\mathbf{v}_{Be(dual)}^2 \frac{\partial^2 \mathbf{B}}{\partial r^2} + c^2 \kappa_0^2 \mathbf{B} = 0$ [11.33]

On substituting equation [6.26] (harmonic approach) in equation [11.33] gives

(34) $\mathbf{v}_{Be(dual)}^2(-\kappa_e^2 \mathbf{E}) - c^2 \kappa_0^2 \mathbf{E} = 0$ [11.34]

For the special case of equilibrium in the quantum realm [8.1]

$$\omega_0 = \omega_p = \omega_e \longrightarrow \kappa_0 = \kappa_p = \kappa_e$$ [8.1]

Therefore

(35) $\mathbf{v}_{Be(dual)}^2(-\kappa_0^2 \mathbf{E}) + c^2 \kappa_0^2 \mathbf{E} = 0$ [11.35]

(36) $\mathbf{v}_{Be(dual)}^2 - c^2 = 0$ [11.36]

(37) $\mathbf{v}_{Be(dual)} = \pm\, c$ [11.37]

Therefore

(38) $\mathbf{v}_{Be(dual)} = c$ [11.38]

(39) $\mathbf{v}_{Be(dual)} = (cos 2\pi + i\ sin 2\pi)\, c$ [11.39]

(40) $\mathbf{v}_{Be(dual)} = c\ e^{2\pi i}$ [11.40]

(41) $\mathbf{v}_{Be(dual)} = c\ R\,(2\pi)$ [11.41]

The electric induction field $\mathbf{B}$ bivector spins harmonically with rotor angle of 2π with speed of light in the equilibrium state in parallel interaction. It is a spinor and takes a rotation of 4π to go back to its original state.

Since $\mathbf{V} = 0$

(42) $\mathbf{v}_{Ve(dual)} = 0$ [11.42]

Table 11.1

Duality					
Electricity			**Magnetism**		
Perpendicular Interaction					
Equilibrium state - Faster decay or growth	$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2\nabla^2\mathbf{E} - c^2\kappa_0^2\mathbf{E} = 0$	$\mathbf{v}_{Ee} = \sqrt{2}c$	Equilibrium state - Faster decay or growth	$\frac{\partial^2 \mathbf{H}}{\partial t^2} - c^2\nabla^2\mathbf{H} - c^2\kappa_0^2\mathbf{H} = 0$	$\mathbf{v}_{He} = \sqrt{2}c$
Damped	$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c^2\nabla^2\mathbf{B} - c^2\kappa_0^2\mathbf{B} = 0$	$\mathbf{v}_{Be} = 0$	Damped	$\frac{\partial^2 \mathbf{D}}{\partial t^2} - c^2\nabla^2\mathbf{D} - c^2\kappa_0^2\mathbf{D} = 0$	$\mathbf{v}_{De} = 0$
Damped	$\frac{\partial^2 \mathbf{V}}{\partial t^2} - c^2\nabla^2\mathbf{V} - c^2\kappa_0^2\mathbf{V} = 0$	$\mathbf{v}_{Ve} = 0$	Damped	$\frac{\partial^2 \mathbf{J}}{\partial t^2} - c^2\nabla^2\mathbf{J} - c^2\kappa_0^2\mathbf{J} = 0$	$\mathbf{v}_{Je} = 0$
Parallel Interaction					
Equilibrium state - Normal decay or growth	$\frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2\kappa_0^2\mathbf{E} = 0$	$\mathbf{v}_{Ee(dual)} = c\ e^{\frac{\pi}{2}i}$	Equilibrium state - Normal decay or growth	$\frac{\partial^2 \mathbf{H}}{\partial t^2} + c^2\kappa_0^2\mathbf{H} = 0$	$\mathbf{v}_{He(dual)} = c\ e^{\frac{\pi}{2}i}$

Duality					
Electricity			**Magnetism**		
Spinor	$\frac{\partial^2 \mathbf{B}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{B} = 0$	$\mathbf{v}_{Be(dual)} = c\ e^{2\pi i}$	Spinor	$\frac{\partial^2 \mathbf{D}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{D} = 0$	$\mathbf{v}_{De(dual)} = c\ e^{2\pi i}$
	$\mathbf{V} = 0$			$\mathbf{J} = 0$	

On the closer inspection of Table 11.1, it can be clearly observed that

i) In the perpendicular interaction $\mathbf{E} \perp \mathbf{H}$ in the equilibrium state, the electric field vector and the magnetic field decay or grow rapidly and follow the same pattern.

ii) In the perpendicular interaction $\langle \mathbf{B} \rangle_2 \perp \langle \mathbf{D} \rangle_2$ in the equilibrium state, both the magnetic induction field bivector and the electric induction field bivector are instantly damped and follow the same pattern.

iii) In the parallel interaction $\mathbf{E} \parallel \langle \mathbf{D} \rangle_2$ in the equilibrium state, the electric field vector decays or grow with rotor angle of $\frac{\pi}{2}$ and the electric induction bivector spins (rotor angle of 2π) harmonically.

iv) In the parallel interaction $\langle \mathbf{B} \rangle_2 \parallel \mathbf{H}$ in the equilibrium state, the magnetic field vector decays or grow with rotor angle of $\frac{\pi}{2}$ and the magnetic induction bivector spins (rotor angle of 2π) harmonically.

v) The only waves in equilibrium state are due to parallel interaction. The dual wave equations are independent of the spatial component, without assumptions to that effect.

12. Wave Function

The total energy W with kinetic energy T and potential energy U is

(1) $$W = T + U$$ [12.1]

The energy and momentum of a quantum-mechanical particle due to the wave-particle duality is

(2) $$W = \hbar\omega$$ [12.2]

(3) $$p = \hbar\kappa$$ [12.3]

(4) $$\begin{aligned} T &= \frac{1}{2}mv^2 \\ &= \frac{1}{2m}(mv)^2 \\ &= \frac{p^2}{2m} \end{aligned}$$ [12.4]

On substituting equation [12.3] in equation [12.4] gives

(5) $$T = \frac{\hbar^2\kappa^2}{2m}$$ [12.5]

(6) $$\kappa^2 = \frac{2m}{\hbar^2}T$$ [12.6]

Consider a wave function

(7) $$\Psi(r,t) = Ae^{i(\kappa\cdot\mathbf{r}-\omega t)}$$ [12.7]

(8) $$\begin{aligned} \frac{\partial^2\Psi}{\partial r^2} &= i^2\kappa^2\mathbf{A}\ e^{i(\kappa\cdot\mathbf{r}-\omega t)} \\ &= -\kappa^2\Psi \end{aligned}$$ [12.8]

On multiplying the equation [12.1] with equation [12.7] gives

(9) $$W\Psi = T\Psi + U\Psi$$ [12.9]

On substituting equation [12.6] in equation [12.8] gives

(10) $$\frac{\partial^2 \Psi}{\partial r^2} = -\frac{2m}{\hbar^2} T\Psi$$ [12.10]

(11) $$\nabla^2 \Psi = -\frac{2m}{\hbar^2} T\psi$$ [12.11]

(12) $$T\psi = -\frac{\hbar^2}{2m}\nabla^2 \Psi$$ [12.12]

On substituting equation [12.12] with wave function [12.9] gives

(13) $$W\Psi = -\frac{\hbar^2}{2m}\nabla^2 \Psi + U\Psi$$ [12.13]

(14) $$W\Psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + U\right)\Psi$$ [12.14]

(15) $$W\Psi = \hat{H}\Psi$$ [12.15]

In the equilibrium state, the particle has no potential energy. Therefore the equation [12.9] becomes

(16) $$W\Psi = T\Psi$$ [12.16]

(17) $$E\Psi = -\frac{\hbar^2}{2m}\nabla^2 \Psi$$ [12.17]

(18) $$\frac{\partial \Psi}{\partial t} = -i\omega \mathbf{A}\ e^{i(\omega \cdot \mathbf{r} - \omega t)} = -i\omega\Psi$$ [12.18]

(19) $$\frac{\partial \Psi}{\partial t} = -i\frac{\hbar\omega}{\hbar}\Psi = -i\frac{W}{\hbar}\Psi$$ [12.19]

(20) $$W\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$ [12.20]

(21) $$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi$$ [12.21]

Under external influence, the equation [12.21] becomes

(22) $$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + U\Psi$$ [12.22]

As we have observed earlier, the only wave equations in equilibrium state/quantum realm are due to parallel interaction. The dual wave equations are independent of the spatial component, i.e., the time and space variables are naturally separated.

Consider the separation of variables approach

(23) $$\Psi(r,t) = \psi(t)\phi(r)$$ [12.23]

(24) $$\frac{\partial \Psi}{\partial t} = \frac{d\psi}{dt}\phi$$ [12.24]

(25) $$\frac{\partial^2 \Psi}{\partial r^2} = \psi\frac{d^2\phi}{dt^2}$$ [12.25]

On substituting equations [12.23], [12.24] and [12.25] in equation [12.22] gives

(26) $$i\hbar\frac{d\psi}{dt}\phi = -\frac{\hbar^2}{2m}\boldsymbol{\psi}\frac{d^2\phi}{dt^2} + U\psi\phi$$ [12.26]

(27) $$i\hbar\frac{1}{\psi}\frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\frac{\mathbf{1}}{\boldsymbol{\phi}}\frac{d^2\phi}{dt^2} + U$$ [12.27]

Therefore

(28) $$i\hbar \frac{1}{\psi} \frac{d\psi}{dt} = W$$ [12.28]

(29) $$\frac{1}{\psi} \frac{d\psi}{dt} = \frac{1}{i\hbar} W$$ [12.29]

(30) $$\frac{1}{\psi} \frac{d\psi}{dt} = -i \frac{W}{\hbar}$$ [12.30]

(31) $$\psi = e^{-i \frac{W}{\hbar} t}$$ [12.31]

The only possible wave functions in equilibrium state are decay and growth wave function for vectors and spinor wave function for bivectors.

Therefore, the equation [12.31] becomes

(32) $$\psi = ce^{\pm i \frac{W}{\hbar} \frac{\pi}{2} t}$$ [12.32]

(33) $$\psi = ce^{\pm i \frac{W}{\hbar} 2\pi t}$$ [12.33]

13. Electric Potential and Magnetic Charge

According to fluid dynamics, the current density of mass transport is

(1) $$\mathbf{J}_{fluid\ dynamics} = \mathbf{v}\rho_m$$ [13.1]

where ρ_m.is a mass density

According to equation [1.35]

$$\mathbf{J} = -\mathbf{v}(\nabla \cdot \mathbf{D})$$ [1.35]

(2) $$\mathbf{J}_{electro\ dynamics} = -\mathbf{v}(\nabla \cdot \mathbf{D})$$ [13.2]

On comparing equations [13.1] and [13.2], it can be concluded that

(3) $$\mathbf{J}_{fluid\ dynamics} \equiv \mathbf{J}_{electro\ dynamics}$$ [13.3]

Therefore

(4) $$\rho_e \equiv \nabla \cdot \mathbf{D}$$ [13.4]

The equation [13.4] is a proper definition — though not of the electric induction $\mathbf{D}$ field, but of the magnetic charge, given that electric induction bivector $\mathbf{D}$ exists and is primary. It states that the magnetic charge is the divergence of the bivector $\mathbf{D}$, i.e., the notion of the magnetic charge captures the divergence aspect of the spinning bivector $\mathbf{D}$ field. Thus, the magnetic charge particle is a bivector/ trivector element and represents nothing but the spinning spacetime structure. It flows analogously to that of the the electric induction $\mathbf{D}$ field. The negative sign in equation [1.32] means that the flow of the magnetic charge is in the opposite direction to the flow of the current area density $\mathbf{J}$.

The equation [13.4] can be rewritten mathematically as

(5) $$\rho_e = \nabla \cdot \mathbf{D}$$ [13.5]

The equation [13.5] is known as the **Coulomb law.**

Therefore, the current area density equation [1.35] becomes

(6) $$\mathbf{J} = -\,\mathbf{v}\rho_e$$ [13.6]

The dual of a magnetic charge density ρ_e is an electric potential density ρ_p.

(7) $$\rho_e\left(\frac{As}{m^3}\right) \longleftarrow dual \longrightarrow \rho_p\left(\frac{Vs}{m^3}\right)$$ [13.7]

The dual of an electric current area density $\mathbf{J}$ is a magnetic voltage area density $\mathbf{V}$.

(8) $$\mathbf{J} = -\,\mathbf{v}\rho_e\left(\frac{A}{m^2}\right) \longleftarrow dual \longrightarrow \mathbf{V} = \mathbf{v}\rho_p\left(\frac{V}{m^2}\right)$$ [1.48]

Therefore

(9) $$\rho_p \equiv \nabla \cdot \mathbf{B}$$ [13.9]

The equation [13.9] is a proper definition — though not of the magnetic induction $\mathbf{B}$ field, but of the electric potential, given that the magnetic induction bivector $\mathbf{B}$ exists and is primary. It states that the electric potential is the divergence of the bivector $\mathbf{B}$, i.e., the notion of the electric potential captures the divergence aspect of the spinning bivector $\mathbf{B}$ field. Thus, the electric potential particle is a bivector/ trivector element and represents nothing but the spinning spacetime structure. It flows analogously to that of the electric induction $\mathbf{B}$ field. The flow of the electric potential is in the same direction to the flow of voltage area density $\mathbf{V}$.

The equation [13.9] can be rewritten mathematically as

(10) $$\rho_p = \nabla \cdot \mathbf{B}$$ [13.10]

The equation [13.2] can be seen as the **Dual Coulomb law.**

In the parallel interaction

$$\nabla \cdot \mathbf{D} = 0 \tag{2.15}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.17}$$

This does not mean that there are no magnetic charge e or of electric potential p in parallel dual interaction. It means that there is a constant flow of magnetic charges and electric potentials in parallel dual interaction.

Table 13.1

Duality					
Electricity			**Magnetism**		
Perpendicular Interaction - Conductor					
Rapid decay	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} - \omega_p \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$	$\mathbf{v}_{Ec(decay)} = 1.618c$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_p \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$	$\mathbf{v}_{Hc(decay)} = 0$
Slow growth	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} - \omega_p \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$	$\mathbf{v}_{Ec(growth)} = 0.618c$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} - \omega_p \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$	$\mathbf{v}_{Hc(growth)} = 0$
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} - \omega_p \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0$	$\mathbf{v}_{Bc} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{D}}{\partial t^2} - \omega_p \frac{\partial \mathbf{D}}{\partial t}\right) - \nabla^2 \mathbf{D} = 0$	$\mathbf{v}_{Dc} = c\ e^{\frac{\pi}{6}i}$
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{V}}{\partial t^2} - \omega_p \frac{\partial \mathbf{V}}{\partial t}\right) - \nabla^2 \mathbf{V} = 0$	$\mathbf{v}_{Vc} = 0$		$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{J}}{\partial t^2} - \omega_p \frac{\partial \mathbf{J}}{\partial t}\right) - \nabla^2 \mathbf{J} = 0$	$\mathbf{v}_{Jc} = c\ e^{\frac{\pi}{6}i}$
	ρ_p	$\mathbf{v}_{\rho_p c} = 0$		ρ_e	$\mathbf{v}_{\rho_e c} = c\ e^{\frac{\pi}{6}i}$
Perpendicular Interaction - Non Conductor					
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_e \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$	$\mathbf{v}_{Enc(decay)} = 0$	Rapid decay	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_e \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$	$\mathbf{v}_{Hnc(decay)} = 1.618c$
Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_e \frac{\partial \mathbf{E}}{\partial t}\right) - \nabla^2 \mathbf{E} = 0$	$\mathbf{v}_{Enc(growth)} = 0$	Slow growth	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_e \frac{\partial \mathbf{H}}{\partial t}\right) - \nabla^2 \mathbf{H} = 0$	$\mathbf{v}_{Hnc(growth)} = 1.618c$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_e \frac{\partial \mathbf{B}}{\partial t}\right) - \nabla^2 \mathbf{B} = 0$	$\mathbf{v}_{Bnc} = c\ e^{\frac{\pi}{6}i}$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_e \frac{\partial \mathbf{D}}{\partial t}\right) - \nabla^2 \mathbf{D} = 0$	$\mathbf{v}_{Dnc} = 0$
	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{V}}{\partial t^2} + \omega_e \frac{\partial \mathbf{V}}{\partial t}\right) - \nabla^2 \mathbf{V} = 0$	$\mathbf{v}_{Vnc} = c\ e^{\frac{\pi}{6}i}$	Damped wave	$\frac{1}{c^2}\left(\frac{\partial^2 \mathbf{J}}{\partial t^2} + \omega_e \frac{\partial \mathbf{J}}{\partial t}\right) - \nabla^2 \mathbf{J} = 0$	$\mathbf{v}_{Jnc} = 0$
	ρ_p	$\mathbf{v}_{\rho_p nc} = c\ e^{\frac{\pi}{6}i}$		ρ_e	$\mathbf{v}_{\rho_e nc} = 0$

Duality					
Electricity			Magnetism		
Parallel Interaction - Conductor					
Normal decay and growth	$\frac{\partial^2 \mathbf{E}}{\partial t^2} + \omega_p^2 \mathbf{E} = 0$	$\mathbf{v}_{Ec(dual)} = c\ e^{\frac{\pi}{2}i}$	Maxima or Minima	$\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$	$\mathbf{v}_{Hc(dual)} = 0$
Maxima or Minima	$\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$	$\mathbf{v}_{Bc(dual)} = 0$	Spinor	$\frac{\partial^2 \mathbf{D}}{\partial t^2} + \omega_p^2 \mathbf{D} = 0$	$\mathbf{v}_{Dc(dual)} = c\ e^{2\pi i}$
	$\mathbf{V} = 0$			$\mathbf{J} = 0$	
	ρ_p	$\mathbf{v}_{\rho_p c(dual)} = 0$		ρ_e	$\mathbf{v}_{\rho_e c(dual)} = c\ e^{2\pi i}$
Parallel Interaction - Non Conductor					
Maxima or Minima	$\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$	$\mathbf{v}_{Enc(dual)} = 0$	Normal decay and growth	$\frac{\partial^2 \mathbf{H}}{\partial t^2} + \omega_e^2 \mathbf{H} = 0$	$\mathbf{v}_{Hnc(dual)} = c\ e^{\frac{\pi}{2}i}$
Spinor	$\frac{\partial^2 \mathbf{B}}{\partial t^2} + \omega_e^2 \mathbf{B} = 0$	$\mathbf{v}_{Bnc(dual)} = c\ e^{2\pi i}$	Maxima or Minima	$\frac{\partial^2 \mathbf{D}}{\partial t^2} = 0$	$\mathbf{v}_{Dnc(dual)} = 0$
	$\mathbf{V} = 0$			$\mathbf{J} = 0$	
	ρ_p	$\mathbf{v}_{\rho_p nc(dual)} = c\ e^{2\pi i}$		ρ_e	$\mathbf{v}_{\rho_e nc(dual)} = 0$

Table 13.2

Duality					
Electricity			Magnetism		
Perpendicular Interaction					
Equilibrium state - Faster decay or growth	$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \nabla^2 \mathbf{E} - c^2 \kappa_0^2 \mathbf{E} = 0$	$\mathbf{v}_{Ee} = \sqrt{2}c$	Equilibrium state - Faster decay or growth	$\frac{\partial^2 \mathbf{H}}{\partial t^2} - c^2 \nabla^2 \mathbf{H} - c^2 \kappa_0^2 \mathbf{H} = 0$	$\mathbf{v}_{He} = \sqrt{2}c$
Damped	$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c^2 \nabla^2 \mathbf{B} - c^2 \kappa_0^2 \mathbf{B} = 0$	$\mathbf{v}_{Be} = 0$	Damped	$\frac{\partial^2 \mathbf{D}}{\partial t^2} - c^2 \nabla^2 \mathbf{D} - c^2 \kappa_0^2 \mathbf{D} = 0$	$\mathbf{v}_{De} = 0$
Damped	$\frac{\partial^2 \mathbf{V}}{\partial t^2} - c^2 \nabla^2 \mathbf{V} - c^2 \kappa_0^2 \mathbf{V} = 0$	$\mathbf{v}_{Ve} = 0$	Damped	$\frac{\partial^2 \mathbf{J}}{\partial t^2} - c^2 \nabla^2 \mathbf{J} - c^2 \kappa_0^2 \mathbf{J} = 0$	$\mathbf{v}_{Je} = 0$
	ρ_p	$\mathbf{v}_{\rho_p e} = 0$		ρ_e	$\mathbf{v}_{\rho_e e} = 0$

Duality					
Electricity			**Magnetism**		
Parallel Interaction					
Equilibrium state - Normal decay or growth	$\frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{E} = 0$	$\mathbf{v}_{Ee(dual)} = c \ e^{\frac{\pi}{2} i}$	Equilibrium state - Normal decay or growth	$\frac{\partial^2 \mathbf{H}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{H} = 0$	$\mathbf{v}_{He(dual)} = c \ e^{\frac{\pi}{2} i}$
Spinor	$\frac{\partial^2 \mathbf{B}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{B} = 0$	$\mathbf{v}_{Be(dual)} = c \ e^{2\pi i}$	Spinor	$\frac{\partial^2 \mathbf{D}}{\partial t^2} + c^2 \kappa_0^2 \mathbf{D} = 0$	$\mathbf{v}_{De(dual)} = c \ e^{2\pi i}$
	$\mathbf{V} = 0$			$\mathbf{J} = 0$	
	ρ_p	$\mathbf{v}_{\rho_p e(dual)} = c \ e^{2\pi i}$		ρ_e	$\mathbf{v}_{\rho_e e(dual)} = c \ e^{2\pi i}$

14. Equations of Transformation

Unipolar induction by Faraday [1.13] is expressed by

$$\mathbf{E} = \mathbf{v} \times \mathbf{B} \quad [1.13]$$

The equation of convection [1.14], formulated by rules of duality from the equation of unipolar induction, is

$$\mathbf{H} = -\mathbf{v} \times \mathbf{D} \quad [1.14]$$

The equations [1.13] and [1.14] together are called equations of transformation.

1) $$\mathbf{H} = -\epsilon_0(\mathbf{v} \times \mathbf{E}) \quad [14.1]$$

On substituting equation [1.13] into equation [14.1] gives

2) $$\mathbf{H}' = -\epsilon_0(\mathbf{v} \times \mathbf{E}) \quad [14.2]$$

3) $$\mathbf{H}' = -\epsilon_0(\mathbf{v} \times \mathbf{v} \times \mathbf{B}) \quad [14.3]$$

4) $$\mathbf{H}' = -\epsilon_0\mu_0(\mathbf{v} \times \mathbf{v} \times \mathbf{H}) \quad [14.4]$$

According to the Vector algebra

5) $$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C} \quad [14.5]$$

Therefore

6) $$\mathbf{H}' = -\epsilon_0\mu_0\left((\mathbf{v} \cdot \mathbf{H})\mathbf{v} - (\mathbf{v} \cdot \mathbf{v})\mathbf{H}\right) \quad [15.6]$$

Since $(\mathbf{v} \cdot \mathbf{H})\mathbf{v}$ always vanishes, therefore

7) $$\mathbf{H}' = -\epsilon_0\mu_0\left(-v^2\mathbf{H}\right) \quad [15.7]$$

8) $$\mathbf{H}' = \frac{v^2}{c^2}\mathbf{H} \quad [15.8]$$

Alternatively

9) $\mathbf{E} = \mu_0 \mathbf{v} \times \mathbf{H}$ [14.9]

On substituting equation [1.14] into equation {14.9] gives

10) $\mathbf{E}' = -\,\mu_0(\mathbf{v} \times \mathbf{v} \times \mathbf{D})$ [14.10]

11) $\mathbf{E}' = -\,\epsilon_0\mu_0(\mathbf{v} \times \mathbf{v} \times \mathbf{E})$ [14.11]

According to the Vector algebra

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$ [14.5]

Therefore

12) $\mathbf{E}' = -\,\epsilon_0\mu_0\left((\mathbf{v} \cdot \mathbf{E})\mathbf{v} - (\mathbf{v} \cdot \mathbf{v})\mathbf{E}\right)$ [14.12]

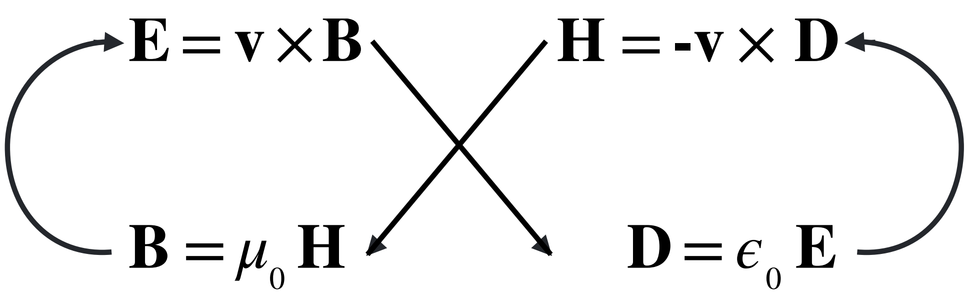
Since $(\mathbf{v} \cdot \mathbf{E})\mathbf{v}$ always vanishes, therefore

13) $\mathbf{E}' = -\,\epsilon_0\mu_0\left(-v^2\mathbf{E}\right)$ [14.13]

14) $\mathbf{E}' = \dfrac{v^2}{c^2}\mathbf{E}$ [14.14]

The equation [14.8] and [14.14} imply that the equations of transformation convert into one another at $v = c$ and there is a constant flow of the electric and magnetic fields ad infinitum as shown in Figure 14.1.

Figure 14.1



Infinite Loop

For $v \neq c$ there will be infinite cause and effect feed back and feed forward non-linear cycles.

Let's consider starting electric and magnetic fields as $\mathbf{E}_0$ and $\mathbf{H}_0$ respectively.

15) $\mathbf{E} = \mathbf{E}_0$ [14.15]

16) $\mathbf{H} = \mathbf{H}_0$ [14.16]

For 1st Cycle

17) $\mathbf{D}_1 = \epsilon_0 \mathbf{E}_0$ [14.17]

On substituting equation [14.17] into equation [1.14] gives

$$\mathbf{H} = -\,\mathbf{v} \times \mathbf{D} \quad [1.14]$$

18) $\mathbf{H}_1 = -\,\epsilon_0(\mathbf{v} \times \mathbf{E}_0)$ [14.18]

19) $\mathbf{E}_0 = \mathbf{v} \times \mathbf{B}_0$ [14.19]

On substituting equation [14.19] into equation {14.18] gives

20) $\mathbf{H}_1 = -\,\epsilon_0(\mathbf{v} \times \mathbf{v} \times \mathbf{B}_0)$ [14.20]

21) $\mathbf{B}_0 = \mu_0 \mathbf{H}_0$ [14.21]

On substituting equation [14.21] into equation {14.20] gives

22) $\mathbf{H}_1 = -\,\epsilon_0 \mu_0(\mathbf{v} \times \mathbf{v} \times \mathbf{H}_0)$ [14.22]

Therefore

23) $\mathbf{H}_1 = \dfrac{v^2}{c^2}\mathbf{H}_0$ [14.23]

24) $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_1$ [14.24]

25) $$\mathbf{H} = \mathbf{H}_0 + \frac{v^2}{c^2}\mathbf{H}_0$$ [14.25]

For 2nd Cycle

26) $$\mathbf{H}_2 = \frac{v^2}{c^2}\mathbf{H}_1$$ [14.26]

On substituting equation [14.23] into equation [14.26] gives

27) $$\mathbf{H}_2 = \frac{v^4}{c^4}\mathbf{H}_0$$ [14.27]

28) $$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_1 + \mathbf{H}_2$$ [14.28]

29) $$\mathbf{H} = \mathbf{H}_0 + \frac{v^2}{c^2}\mathbf{H}_0 + \frac{v^4}{c^4}\mathbf{H}_0$$ [14.29]

For Infinite cycles:

30) $$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_1 + \mathbf{H}_2 + \mathbf{H}_3 \ldots$$ [14.30]

31) $$\mathbf{H} = \mathbf{H}_0 + \frac{v^2}{c^2}\mathbf{H}_0 + \frac{v^4}{c^4}\mathbf{H}_0 + \frac{v^6}{c^6}\mathbf{H}_0 \ldots$$ [14.31]

32) $$\mathbf{H} = \left(1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} + \frac{v^6}{c^6} \ldots\right)\mathbf{H}_0$$ [14.32]

33) $$\mathbf{H} = \left(1 + \frac{v^2}{c^2} + \left(\frac{v^2}{c^2}\right)^2 + \left(\frac{v^2}{c^2}\right)^3 \ldots\right)\mathbf{H}_0$$ [14.33]

Similarly

34) $$\mathbf{E} = \left(1 + \frac{v^2}{c^2} + \left(\frac{v^2}{c^2}\right)^2 + \left(\frac{v^2}{c^2}\right)^3 \ldots\right)\mathbf{E}_0$$ [14.34]

For $v > c \longrightarrow \frac{v^2}{c^2} > 1$ in case of general relativity in the higher dimensions, there will be an ever increasing electric and magnetic field, which generates energy for example in the case of a decaying electric field in the conductor [6.40] and a magnetic field in the non conductor [6.106]. This is precisely what happens in the RA device during the trailing edge of the pulse in the input coil.

$$\mathbf{v}_{Ec(decay)} = 1.618c \qquad [6.40]$$

$$\mathbf{v}_{Hnc(decay)} = 1.618c \qquad [6.106]$$

For $v < c \longrightarrow \frac{v^2}{c^2} < 1$ in case of special relativity the equations [14.33] and [14.34] become a converging series.

Therefore

35) $$\mathbf{H} = \left(\frac{1}{1 - \frac{v^2}{c^2}} \right) \mathbf{H}_0 \qquad [14.35]$$

Similarly

36) $$\mathbf{E} = \left(\frac{1}{1 - \frac{v^2}{c^2}} \right) \mathbf{E}_0 \qquad [14.36]$$

Therefore

37) $$\frac{\mathbf{H}_0}{\mathbf{H}} = 1 - \frac{v^2}{c^2} \qquad [14.37]$$

38) $$\frac{\mathbf{E}_0}{\mathbf{E}} = 1 - \frac{v^2}{c^2} \qquad [14.38]$$

The length contraction in special relativity is given by

39) $$L = \sqrt{1 - \frac{v^2}{c^2}}\ L_0$$ [14.39]

40) $$\left(\frac{L}{L_0}\right)^2 = 1 - \frac{v^2}{c^2}$$ [14.40]

On comparing equations [14.37], [14.38] and [14.40] gives

41) $$\frac{\mathbf{H}_0}{\mathbf{H}} = \frac{\mathbf{E}_0}{\mathbf{E}} = \left(\frac{L}{L_0}\right)^2 = 1 - \frac{v^2}{c^2}$$ [14.41]

Therefore

42) $$L \propto \frac{1}{\sqrt{\mathbf{E}}}, \frac{1}{\sqrt{\mathbf{H}}}$$ [14.42]

43) $$L_0 \propto \frac{1}{\sqrt{\mathbf{E}_0}}, \frac{1}{\sqrt{\mathbf{H}_0}}$$ [14.43]

The equations [14.42] and [14.43] mean that **the electric and the magnetic field determine the length measurement**.

This has profound implications. If the observer is exposed to the same fields, in which the object being observed is situated, then then observer will not be able to perceive the influence of the fields on the object. For example, if we would sit inside a rocket, the rocket will become smaller with faster velocity, and we would notice nothing since we also would contract to the same extent.

For a wave with wavelength λ and frequency f we have

44) $$c = \lambda f$$ [14.44]

From equation [14.42] it follows that

45) $$c \propto \lambda \propto L \propto \frac{1}{\sqrt{\mathbf{E}}}, \frac{1}{\sqrt{\mathbf{H}}}$$ [14.45]

46) $$c^2 \propto \lambda^2 \propto L^2 \propto \frac{1}{\mathbf{E}}, \frac{1}{\mathbf{H}}$$ [14.46]

It is evident from equation [14.45] that every measurement of a velocity and the speed of light which are measured in meters per second, the field determines the length measurement and hence speed of light. This means it is not possible to measure the speed of light correctly as the metre has been internationally defined as the length of the path travelled by light in vacuum for during a time interval of 1/299792458 of a second. If speed of light is changed, then this applies to the measurement path in the same way. The variable is measured by itself, and as a result the speed of light is always measured as a constant value.

15. Speed of Light

In the previous chapters, we have seen that the electric and magnetic field can have velocities other than the speed of light with rotations.

This can also be easily visualised geometrically with the Clifford algebra. In the Vector algebra, the velocity is deemed as a vector.

The geometric product of two vectors is given by equation [2.7]

$$\mathbf{AB} = \langle \mathbf{A} \cdot \mathbf{B} \rangle_0 + \langle \mathbf{A} \wedge \mathbf{B} \rangle_2 \qquad [2.7]$$

In both perpendicular and parallel interaction, the velocity of the electric field $\mathbf{v}_E$ and the velocity of the magnetic field $\mathbf{v}_H$ are parallel i.e. in the same direction. The wedge product of parallel vectors is zero, therefore

1) $$\mathbf{v} = \langle \mathbf{v}_E \cdot \mathbf{v}_H \rangle_0 \qquad [15.1]$$

The term $\langle \mathbf{v}_E \cdot \mathbf{v}_H \rangle_0$ is a scalar value which is Lorentz invariant. This represents the scalar speed of light c which is observed everywhere.

In higher dimensions, the velocity is a bivector. The geometric product of two bivectors is given by equation [2.20]

$$\langle \mathbf{A} \rangle_2 \langle \mathbf{B} \rangle_2 = \langle \mathbf{A} \cdot\cdot\, \mathbf{B} \rangle_0 + \langle \mathbf{A} \cdot \mathbf{B} \rangle_2 + \langle \mathbf{A} \wedge \mathbf{B} \rangle_4 \qquad [2.20]$$

Therefore, the velocity clif is

2) $$\mathbf{v} = \langle \mathbf{v}_E \cdot\cdot\, \mathbf{v}_B \rangle_0 + \langle \mathbf{v}_E \cdot \mathbf{v}_B \rangle_2 + \langle \mathbf{v}_E \wedge \mathbf{v}_B \rangle_4 \qquad [15.2]$$

The wedge product of parallel bivectors is zero, therefore

3) $$\mathbf{v} = \langle \mathbf{v}_E \cdot\cdot\, \mathbf{v}_B \rangle_0 + \langle \mathbf{v}_E \cdot \mathbf{v}_B \rangle_2 \qquad [15.3]$$

In reality, the first term $\langle \mathbf{v}_E \cdot\cdot\, \mathbf{v}_B \rangle_0$ is a scalar value which is Lorentz invariant and represents the scalar speed of light c which is observed everywhere although this is attributed to the $\langle \mathbf{v}_E \cdot \mathbf{v}_H \rangle_0$ in equation [15.1] in the Vector algebra which only distorts our understanding. It does not describe the workings of the reality which remain unchanged irrespective of our view.

The second term $\langle \mathbf{v}_E \cdot \mathbf{v}_H \rangle_2$ is a bivector and is the reason for the velocities other than the invariant scalar c.

A velocity bivector always has a rotation. The rotation doesn't change, only the velocity or speed with which rotation happens change. This means the angular frequency of the current area density or voltage area density remains constant for the medium but the rotational velocity changes. The velocity of the outward spiralling current area density increases whereas the velocity of the inward spiralling voltage area density decreases. When the velocity of the outward spiralling current area density increases, more energy is consumed which produces heat, whereas when the velocity of the inward spiralling voltage area density decreases, more energy is released which cools. This is what some have called the cold electricity.

16. Continuity Equations

In the perpendicular interaction, on applying the divergence to the Ampère-Maxwell law [1.9] gives

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \quad [1.9]$$

1) $$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \quad [16.1]$$

2) $$\nabla \cdot \nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial}{\partial t}(\nabla \cdot \mathbf{E}) = \mu_0(\nabla \cdot \mathbf{J}) \quad [16.2]$$

3) $$\frac{\partial}{\partial t}(\nabla \cdot \mathbf{E}) = -c^2 \mu_0 (\nabla \cdot \mathbf{J}) \quad [16.3\}$$

Applying the time derivative of the Coulomb law [1.1]gives

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0} \quad [1.1]$$

(4) $$\frac{\partial}{\partial t}(\nabla \cdot \mathbf{E}) = \frac{1}{\epsilon_0}\frac{\partial \rho_e}{\partial t} \quad [16.4]$$

On substituting equation [16.4] in equation [16.3] gives

(5) $$\frac{1}{\epsilon_0}\frac{\partial \rho_e}{\partial t} = -c^2 \mu_0 (\nabla \cdot \mathbf{J}) \quad [16.5]$$

(6) $$\frac{\partial \rho_e}{\partial t} = -c^2 \epsilon_0 \mu_0 (\nabla \cdot \mathbf{J}) \quad [16.6]$$

(7) $$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{J} = 0 \quad [16.7]$$

The equation [15.7] is known as the **continuity equation**. The correct terminology would be **MAGNETIC CONTINUITY EQUATION.**

Applying the divergence to the dual Ampère-Maxwell law [1.50] gives

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{V} \qquad [1.50]$$

8) $$\nabla \times \mathbf{D} + \frac{1}{c^2}\frac{\partial \mathbf{H}}{\partial t} = \epsilon_0 \mathbf{V} \qquad [16.8]$$

9) $$\nabla \cdot \nabla \times \mathbf{D} + \frac{1}{c^2}\frac{\partial}{\partial t}(\nabla \cdot \mathbf{H}) = \epsilon_0(\nabla \cdot \mathbf{V}) \qquad [16.9]$$

10) $$\frac{\partial}{\partial t}(\nabla \cdot \mathbf{H}) = c^2 \epsilon_0(\nabla \cdot \mathbf{V}) \qquad [16.10]$$

Applying the time derivative of the dual Coulomb law [1.48] gives

$$\nabla \cdot \mathbf{H} = \frac{\rho_p}{\mu_0} \qquad [1.48]$$

11) $$\frac{\partial}{\partial t}(\nabla \cdot \mathbf{H}) = \frac{1}{\mu_0}\frac{\partial \rho_p}{\partial t} \qquad [16.11]$$

On substituting equation [16.11] in equation [16.10] gives

12) $$\frac{1}{\mu_0}\frac{\partial \rho_p}{\partial t} = c^2 \epsilon_0(\nabla \cdot \mathbf{V}) \qquad [16.12]$$

13) $$\frac{\partial \rho_p}{\partial t} = c^2 \epsilon_0 \mu_0(\nabla \cdot \mathbf{V}) \qquad [16.13]$$

14) $$\frac{\partial \rho_p}{\partial t} - \nabla \cdot \mathbf{V} = 0 \qquad [16.14]$$

The equation [16.14] can be seen as the **DUAL POTENTIAL CONTINUITY EQUATION.**

In the parallel dual interaction, similarly

15) $$\frac{\partial \rho_e}{\partial t} = 0$$ [15.15]

16) $$\frac{\partial \rho_p}{\partial t} = 0$$ [15.16]

The equations [15.15] and [15.16] mean that there is a constant flow of magnetic charges and electric potentials in parallel dual interaction.

Table 15.1

Duality					
Electricity			**Magnetism**		
Perpendicular Interaction					
Potential Continuity	$\frac{\partial \rho_p}{\partial t} - \nabla \cdot \mathbf{V} = 0$		Magnetic Continuity	$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{J} = 0$	
Parallel Interaction					
Potential Continuity	$\frac{\partial \rho_e}{\partial t} = 0$		Magnetic Continuity	$\frac{\partial \rho_p}{\partial t} = 0$	

17. Electric and Magnetic Force

In the perpendicular interaction

1) $\mathbf{F}_{\perp} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ [17.1]

The equation [16.1] is known as the **Lorentz force**. This is an **ELECTRIC FORCE.**

It is evident from equation [16.1] (the second term on the RHS) that the Faraday unipolar induction is inherent in the Lorentz force.

The dual of an electric force would be a **MAGNETIC FORCE**.

2) $\mathbf{G}_{\perp} = -\phi_p(\mathbf{H} + \mathbf{v} \times \mathbf{D})$ [17.2]

The equation [17.12] can be seen as the **DUAL LORENTZ FORCE**.

It is evident from equation [16.2] (the second term on the RHS) that the equation of convection is inherent in the magnetic force.

In the parallel dual interaction, the second terms in equations [17.1] and first term in equation [17.3] are exchanged.

Therefore

3) $\mathbf{F}_{\parallel} = q\mathbf{E} - \phi_p\mathbf{H}$ [17.3]

4) $\mathbf{G}_{\parallel} = q\mathbf{v} \times \mathbf{B} - \phi_p\mathbf{v} \times \mathbf{D}$ [17.4]

Table 16.1

Duality					
Electricity			Magnetism		
Perpendicular Interaction					
Potential Continuity	$\frac{\partial \rho_p}{\partial t} - \nabla \cdot \mathbf{V} = 0$		Magnetic Continuity	$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{J} = 0$	

Duality					
Electricity			**Magnetism**		
Electric Force (Lorentz Force)	$\mathbf{F}_{\perp} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	$\frac{AVs}{m}$	Magnetic Force (Dual Lorentz Force)	$\mathbf{G}_{\perp} = -\phi_p(\mathbf{H} + \mathbf{v} \times \mathbf{D})$	$\frac{AVs}{m}$
Parallel Interaction					
Potential Continuity	$\frac{\partial \rho_e}{\partial t} = 0$		Magnetic Continuity	$\frac{\partial \rho_p}{\partial t} = 0$	
Electric Force	$\mathbf{F}_{\parallel} = q\mathbf{E} - \phi_p \mathbf{H}$	$\frac{AVs}{m}$	Magnetic Force	$\mathbf{G}_{\parallel} = q\mathbf{v} \times \mathbf{B} - \phi_p \mathbf{v} \times \mathbf{D}$	$\frac{AVs}{m}$

18. Power Area Density & Energy Volume Density

In the perpendicular interaction, the Poynting vector is

(1) $$\mathbf{S}_\perp = \mathbf{E} \times \mathbf{H}$$ [18.1]

In the Clifford algebra, the Poynting vector would be the geometric product between $\mathbf{E}$ and $\mathbf{H}$.

2) $$\mathbf{S}_\perp = \mathbf{E}\mathbf{H}$$ [18.2]

The geometric product of two vectors is given by equation [2.7]

$$\mathbf{A}\mathbf{B} = \langle \mathbf{A} \cdot \mathbf{B} \rangle_0 + \langle \mathbf{A} \wedge \mathbf{B} \rangle_2$$ [2.7]

Therefore

3) $$\mathbf{S}_\perp = \langle \mathbf{E} \cdot \mathbf{H} \rangle_0 + \langle \mathbf{E} \wedge \mathbf{H} \rangle_2$$ [18.3]

The dot product of perpendicular vectors is zero, therefore

4) $$\langle \mathbf{E} \cdot \mathbf{H} \rangle_0 = 0$$ [18.4]

On substituting equation [18.4] in equation[18.3] gives

5) $$\mathbf{S}_\perp = \langle \mathbf{E} \wedge \mathbf{H} \rangle_2$$ [18.5]

Like the Beltrami condition, the Poynting vector uses the original interpretation of the direct representation of a curl of a field from the Clifford algebra and imports it into the Vector algebra, i.e., applies it to the indirect representation of a curl of a field.

6) $$\mathbf{S}_\perp = \langle \mathbf{E} \wedge \mathbf{H} \rangle_2 \longrightarrow \mathbf{S}_\perp = \langle \mathbf{E} \times \mathbf{H} \rangle_1$$ [18.6]

The cross product is defined

7) $$\mathbf{A} \times \mathbf{B} = AB\ sin\theta$$ [18.7]

For perpendicular vector, the cross product becomes

8) $$\mathbf{A} \times \mathbf{B} = AB\ sin\frac{\pi}{2} \\ = AB$$ [18.8]

Therefore

9) $$\mathbf{S}_{\perp} = \mathbf{E} \times \mathbf{H} \\ = \text{EH}$$ [18.9]

The dual of the the Poynting vector would be a **DUAL POYNTING VECTOR**.

10) $$\mathbf{S}_{\perp} = \mathbf{EH} \longleftarrow dual \longrightarrow \mathbf{S}_{\perp(dual)} = \mathbf{DB}$$ [18.10]

The geometric product of two bivectors is given by equation [2.20]

$$\langle \mathbf{A} \rangle_2 \langle \mathbf{B} \rangle_2 = \langle \mathbf{A} \cdot\cdot\ \mathbf{B} \rangle_0 + \langle \mathbf{A} \cdot \mathbf{B} \rangle_2 + \langle \mathbf{A} \wedge \mathbf{B} \rangle_4$$ [2.20]

Therefore

11) $$\mathbf{S}_{\perp(dual)} = \langle \mathbf{D} \cdot\cdot\ \mathbf{B} \rangle_0 + \langle \mathbf{D} \cdot \mathbf{B} \rangle_2 + \langle \mathbf{D} \wedge \mathbf{B} \rangle_4$$ [18.11]

The double dot product of perpendicular bivectors is zero and the quadvector vanishes in $D = 3$, therefore

12) $$\langle \mathbf{D} \cdot\cdot\ \mathbf{B} \rangle_0 = 0$$ [18.12]

13) $$\langle \mathbf{D} \wedge \mathbf{B} \rangle_4 = 0$$ [18.13]

On substituting equations [18.12] and [18.13] in equation[18.11] gives

14) $$\mathbf{S}_{\perp(dual)} = \langle \mathbf{D} \cdot \mathbf{B} \rangle_2$$ [18.14]

Like the Poynting vector, we must import the dual Poynting vector into the Vector algebra. We apply the direct representation of the dot product to the indirect representation of a curl of a field.

15) $$\mathbf{S}_{\perp(dual)} = \langle \mathbf{D} \cdot \mathbf{B} \rangle_2 \longrightarrow \mathbf{S}_{\perp(dual)} = \langle \mathbf{D} \times \mathbf{B} \rangle_1$$ [18.15]

We know that $\mathbf{S}_{\perp(dual)}$ is a bivector, thus its dimension would be the same as that of $\mathbf{S}_{\perp}\left(\frac{AV}{m^2}\right)$. This is achieved by multiplying $\mathbf{S}_{\perp(dual)}$ by c^2.

Therefore

16) $\mathbf{S}_{\perp(dual)} = c^2\mathbf{D} \times \mathbf{B}$ [18.16]

In the parallel dual interaction $\mathbf{E} \parallel \langle \mathbf{D} \rangle_2$, the Poynting vector is

17) $\mathbf{S}_{\parallel} = \mathbf{ED}$ [18.17]

The geometric product of a vector and a bivector is given by equation [2.12]

$$\mathbf{A}\langle \mathbf{B} \rangle_2 = \langle \mathbf{A} \cdot \mathbf{B} \rangle_1 + \langle \mathbf{A} \wedge \mathbf{B} \rangle_3 \quad [2.12]$$

Therefore

18) $\mathbf{S}_{\parallel} = \langle \mathbf{E} \cdot \mathbf{D} \rangle_1 + \langle \mathbf{E} \wedge \mathbf{D} \rangle_3$ [18.18]

The wedge product of parallel vector and bivector is zero, therefore

19) $\langle \mathbf{E} \wedge \mathbf{D} \rangle_3 = 0$ [18.19]

On substituting equation [18.19] in equation[18.18] gives

20) $\mathbf{S}_{\parallel} = \langle \mathbf{E} \cdot \mathbf{D} \rangle_1$ [18.20]

Again, we must import the original interpretation of the direct representation of the Poynting vector of a parallel interaction from the Clifford algebra and apply it into the Vector algebra.

21) $\mathbf{S}_{\parallel} = \langle \mathbf{E} \cdot \mathbf{D} \rangle_1 \longrightarrow \mathbf{S}_{\parallel} = \langle \mathbf{E} \cdot \mathbf{D} \rangle_0$ [18.21]

The dot product of two vectors is given by equation [2.10]

$$\langle \mathbf{A} \cdot \mathbf{B} \rangle_0 = \frac{\mathbf{AB} + \mathbf{BA}}{2} \quad [2.10]$$

Therefore

22) $\langle \mathbf{E} \cdot \mathbf{D} \rangle_0 = \dfrac{\mathbf{ED} + \mathbf{DE}}{2}$ [18.22]

23) $\langle \mathbf{E} \cdot \mathbf{D} \rangle_0 = \dfrac{\mathbf{E}(\epsilon_0 \mathbf{E}) + (\epsilon_0 \mathbf{E})\mathbf{E}}{2}$ [18.23]

24) $\langle \mathbf{E} \cdot \mathbf{D} \rangle_0 = \epsilon_0 \mathrm{E}^2$ [18.24]

Therefore

25) $\mathbf{S}_{\|} = \epsilon_0 \mathrm{E}^2$ [18.25]

The dot product of a vector and a bivector is given by equation [2.15]

$$\langle \mathbf{A} \cdot \langle \mathbf{B} \rangle_{\mathbf{2}} \rangle_1 = \frac{\mathbf{AB} - \mathbf{BA}}{2} \quad [2.15]$$

Therefore

26) $\langle \mathbf{E} \cdot \mathbf{D} \rangle_1 = \dfrac{\mathbf{E}(\epsilon_0 \mathbf{E}) - (\epsilon_0 \mathbf{E})\mathbf{E}}{2}$ [18.26]

27) $\langle \mathbf{E} \cdot \mathbf{D} \rangle_1 = 0$ [18.27]

The discrepancy in equations [18.24] and [18.27] is due to the fact that in reality $\langle \mathbf{D} \rangle_3$ is a trivector but we assume it as a bivector $\langle \mathbf{D} \rangle_2$. This means that we must also count the time dimension in the unit of the object while ascertaining the grade of the object even in three dimensions.

The dot product of a vector and a trivector is given by equation [2.18]

$$\langle \mathbf{A} \cdot \langle \mathbf{B} \rangle_{\mathbf{r}} \rangle_{(r-1)} = \frac{\mathbf{AB} + (-1)^{r-1}\mathbf{BA}}{2} \quad [2.18]$$

Therefore

28) $\langle \mathbf{E} \cdot \mathbf{D} \rangle_2 = \dfrac{\mathbf{ED} + \mathbf{DE}}{2}$ [18.28]

29) $\langle \mathbf{E} \cdot \mathbf{D} \rangle_2 = \epsilon_0 \mathrm{E}^2$ [18.29]

Therefore equation [18.21] becomes

30) $\mathbf{S}_{\|} = \langle \mathbf{E} \cdot \mathbf{D} \rangle_2 \longrightarrow \mathbf{S}_{\|} = \langle \mathbf{E} \cdot \mathbf{D} \rangle_0$ [18.30]

Similarly

31) $\mathbf{S}_{\|} = \langle \mathbf{H} \cdot \mathbf{B} \rangle_2 \longrightarrow \mathbf{S}_{\|} = \langle \mathbf{H} \cdot \mathbf{B} \rangle_0$ [18.31]

32) $\langle \mathbf{H} \cdot \mathbf{B} \rangle_0 = \mu_0 \mathrm{H}^2$ [18.32]

33) $\mathbf{S}_{\|(dual)} = \mu_0 \mathrm{H}^2$ [18.33]

The Vector algebra is grade blind and defined the energy density bivector from perpendicular interaction as the power area density vector based on the dimension $\dfrac{AV}{m^2}$ and the energy volume density bivector from parallel interaction as the energy volume density scalar based on the dimension $\dfrac{AVs}{m^3}$.

34) $\langle \mathbf{S} \rangle_2 = \langle \mathbf{S}_{\perp} \rangle_2 + \langle \mathbf{S}_{\perp(dual)} \rangle_2 \longrightarrow \langle \mathbf{P} \rangle_1 = \langle \mathbf{S}_{\|} \rangle_1 + \langle \mathbf{S}_{\|(dual)} \rangle_1$ [18.34]

35) $\langle \mathbf{P} \rangle_1 = \mathbf{EH} + c^2 \mathbf{DB}$ [18.35]

36) $\langle \mathbf{S} \rangle_1 = \langle \mathbf{S}_{\|} \rangle_2 + \langle \mathbf{S}_{\|(dual)} \rangle_2 \longrightarrow \langle \mathbf{W} \rangle_0 = \langle \mathbf{S}_{\|} \rangle_0 + \langle \mathbf{S}_{\|(dual)} \rangle_0$ [18.36]

37) $\langle \mathbf{W} \rangle_0 = \mathbf{ED} + \mathbf{HB}$ [18.37]

As already pointed out earlier, the parallel interaction is a transverse wave and not a longitudinal wave. The parallel interaction is also not a scalar wave as the energy density is not a scalar quantity. The correct terminology is the Beltrami wave as detailed perviously.

Table 18.1

Duality					
Electricity			**Magnetism**		
Perpendicular Interaction					
Potential Continuity	$\frac{\partial \rho_p}{\partial t} - \nabla \cdot \mathbf{V} = 0$		Magnetic Continuity	$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{J} = 0$	
Electric Force (Lorentz Force)	$\mathbf{F}_\perp = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	$\frac{AVs}{m}$	Magnetic Force (Dual Lorentz Force)	$\mathbf{G}_\perp = -\phi_p(\mathbf{H} + \mathbf{v} \times \mathbf{D})$	$\frac{AVs}{m}$
Power Area Density	$\mathbf{S}_\perp = \mathbf{EH}$	$\frac{AV}{m^2}$	Dual Power Area Density	$\mathbf{S}_{\perp(dual)} = c^2\mathbf{DB}$	$\frac{AV}{m^2}$
	$\mathbf{S}_\perp = \mathbf{E} \times \mathbf{H}$	$\frac{AV}{m^2}$		$\mathbf{S}_{\perp(dual)} = c^2\mathbf{D} \times \mathbf{B}$	$\frac{AV}{m^2}$
Parallel Interaction					
Potential Continuity	$\frac{\partial \rho_e}{\partial t} = 0$		Magnetic Continuity	$\frac{\partial \rho_p}{\partial t} = 0$	
Electric Force	$\mathbf{F}_\parallel = q\mathbf{E} - \phi_p\mathbf{H}$	$\frac{AVs}{m}$	Magnetic Force	$\mathbf{G}_\parallel = q\mathbf{v} \times \mathbf{B} - \phi_p\mathbf{v} \times \mathbf{D}$	$\frac{AVs}{m}$
Energy Volume Density	$\mathbf{S}_\parallel = \mathbf{ED}$	$\frac{AVs}{m^3}$	Dual Energy Volume Density	$\mathbf{S}_{\parallel(dual)} = \mathbf{HB}$	$\frac{AVs}{m^3}$
	$\mathbf{S}_\parallel = \epsilon_0 \mathrm{E}^2$	$\frac{AVs}{m^3}$		$\mathbf{S}_{\parallel(dual)} = \mu_0 \mathrm{H}^2$	$\frac{AVs}{m^3}$

In higher dimension, the $\mathbf{D}$ and $\mathbf{B}$ are trivectors.

The geometric product of two trivectors is given by equation [2.24]

$$\langle \mathbf{A} \rangle_3 \langle \mathbf{B} \rangle_3 = \langle \mathbf{A} \cdot\cdot\cdot \mathbf{B} \rangle_0 + \langle \mathbf{A} \cdot\cdot \mathbf{B} \rangle_2 + \langle \mathbf{A} \cdot \mathbf{B} \rangle_4 + \langle \mathbf{A} \wedge \mathbf{B} \rangle_6 \quad [2.24]$$

Therefore

38) $\mathbf{S}_{\perp(dual)} = \langle \mathbf{D} \cdot\cdot\cdot \mathbf{B} \rangle_0 + \langle \mathbf{D} \cdot\cdot\ \mathbf{B} \rangle_2 + \langle \mathbf{D} \cdot \mathbf{B} \rangle_4 + \langle \mathbf{D} \wedge \mathbf{B} \rangle_6$ [18.38]

The triple dot product of perpendicular trivectors is zero, the quadvector is a pseudo scalar and the sextvector vanishes in $D = 4$, therefore

39) $\langle \mathbf{D} \cdot\cdot\cdot \mathbf{B} \rangle_0 = 0$ [18.39]

40) $\langle \mathbf{D} \wedge \mathbf{B} \rangle_6 = 0$ [18.40]

On substituting equations [18.39] and [18.40] in equation[18.38] gives

41) $\mathbf{S}_{\perp(dual)} = \langle \mathbf{D} \cdot\cdot\ \mathbf{B} \rangle_2 + \langle \mathbf{D} \cdot \mathbf{B} \rangle_4$ [18.41]

It is evident from equation [18.38] (the second term on the RHS) that there are additional higher grade power terms inherent in the dual power.

The geometric product of a vector and a trivector is given by equation [2.17]

$$\mathbf{A}\langle \mathbf{B} \rangle_r = \langle \mathbf{A} \cdot \langle \mathbf{B} \rangle_r \rangle_{(r-1)} + \langle \mathbf{A} \wedge \langle \mathbf{B} \rangle_r \rangle_{(r+1)} \quad [2.17]$$

Therefore

42) $\mathbf{S}_{||} = \langle \mathbf{E} \cdot \mathbf{D} \rangle_2 + \langle \mathbf{E} \wedge \mathbf{D} \rangle_4$ [1842]

The wedge product of a parallel vector and a trivector is zero, therefore

43) $\langle \mathbf{E} \wedge \mathbf{D} \rangle_4 = 0$ [18.43]

On substituting equation [18.43] in equation [18.42] gives

44) $\mathbf{S}_{||} = \langle \mathbf{E} \cdot \mathbf{D} \rangle_2$ [18.44]

The dot product of a vector and a trivector is given by equation [2.18]

$$\langle \mathbf{A} \cdot \langle \mathbf{B} \rangle_{\mathbf{r}} \rangle_{(r-1)} = \frac{\mathbf{AB} + (-1)^{r-1}\mathbf{BA}}{2} \quad [2.18]$$

Therefore

45) $$\langle \mathbf{E} \cdot \mathbf{D} \rangle_2 = \frac{\mathbf{ED} + \mathbf{DE}}{2}$$ [18.45]

46) $$\langle \mathbf{E} \cdot \mathbf{D} \rangle_2 = \epsilon_0 \mathrm{E}^2$$ [18.46]

47) $$\mathbf{S}_{\|} = \epsilon_0 \mathrm{E}^2$$ [18.47]

Similarly

48) $$\mathbf{S}_{\|(dual)} = \langle \mathbf{H} \cdot \mathbf{B} \rangle_2$$ [18.48]

49) $$\langle \mathbf{H} \cdot \mathbf{B} \rangle_2 = \mu_0 \mathrm{H}^2$$ [18.49]

50) $$\mathbf{S}_{\|(dual)} = \mu_0 \mathrm{H}^2$$ [18.50]

Therefore

51) $$\mathbf{P} = \langle \mathbf{EH} \rangle_2 + c^2 \langle \mathbf{DB} \rangle_2 + c^2 \langle \mathbf{DB} \rangle_4$$ [18.51]

52) $$\mathbf{W} = \langle \mathbf{ED} \rangle_2 + \langle \mathbf{HB} \rangle_2$$ [18.52]

In $D = 4$, both power area density and energy volume density are bivectors and in addition there is an additional pseudo scalar quadvector power term.

We know that the equations of transformation, i.e., the unipolar induction by Faraday and the equation of convection, are valid at the same time.

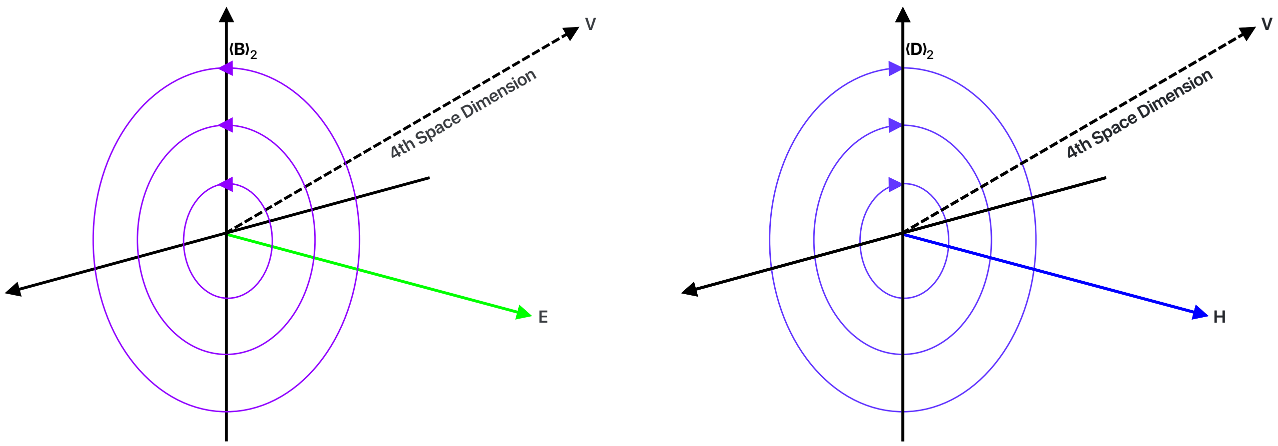
$$\mathbf{E} = \mathbf{v} \times \langle \mathbf{B} \rangle_2 \longrightarrow \mathbf{v} \perp \mathbf{E} \perp \langle \mathbf{B} \rangle_2$$ [1.13]

$$\mathbf{H} = -\mathbf{v} \times \langle \mathbf{D} \rangle_2 \longrightarrow \mathbf{v} \perp \mathbf{H} \perp \langle \mathbf{D} \rangle_2$$ [1.14]

This means that the velocity $\mathbf{v}$ is perpendicular to $\mathbf{E}$ and $\langle \mathbf{B} \rangle_2$ and $\mathbf{H}$ and $\langle \mathbf{D} \rangle_2$ at the same time. This implies that there must exist a 4th space dimension even though we cannot visualise it. This is exactly the same situation with the time dimension in

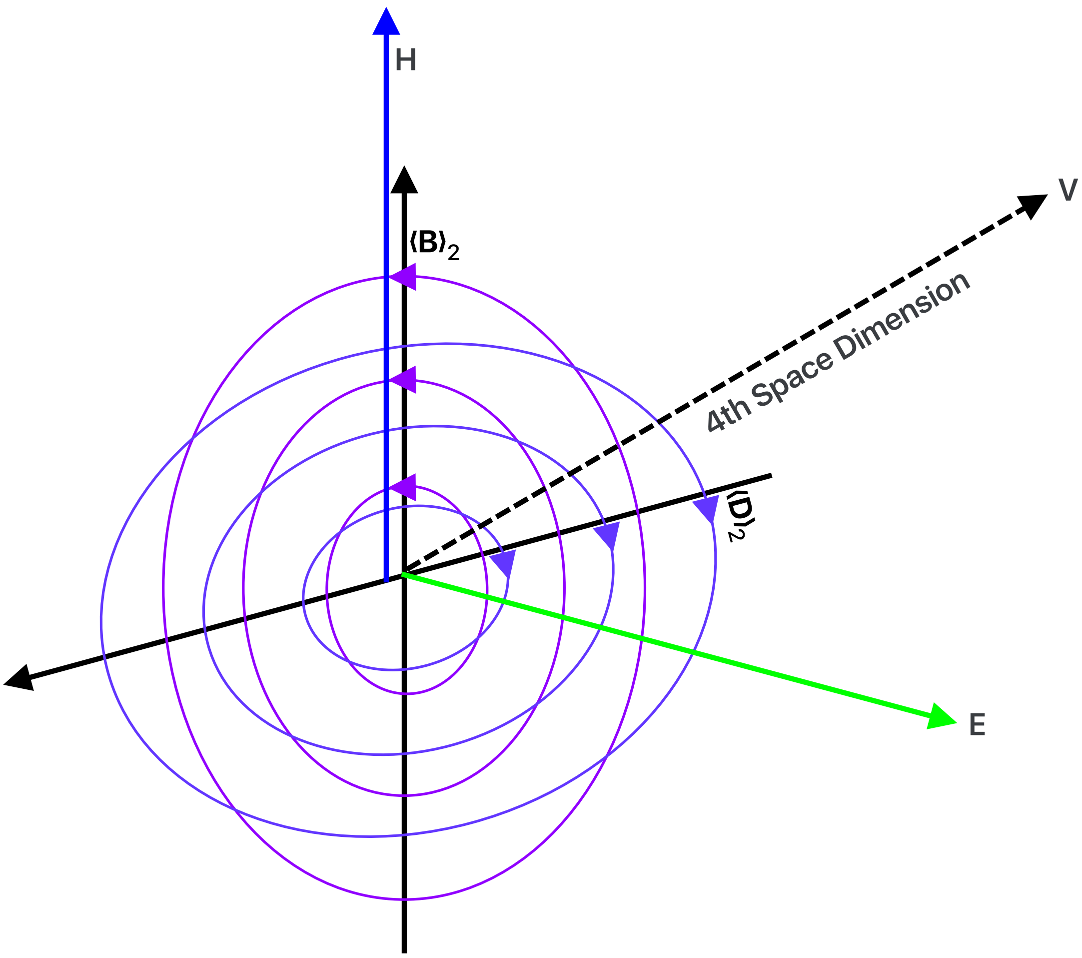
Minkowski spacetime which is perpendicular to three space dimensions which has been widely accepted even though we cannot visualise it.

Figure 18.1



Fragmented Equations of Transformation

Figure 18.2



Unified Equations of Transformation

The real power clif is

53) $$\mathbf{P} = \langle \mathbf{EH} \rangle_2 + c^2 \langle \mathbf{DB} \rangle_2 + c^2 \langle \mathbf{DB} \rangle_4 + c^2 \langle \mathbf{DB} \rangle_6$$ [18.53]

The real energy clif is

54) $$\mathbf{W} = \langle \mathbf{ED} \rangle_2 + \langle \mathbf{HB} \rangle_2$$ [18.54]

The quadvector and sextvector power terms are always present in reality irrespective of our view and our belief. This is the reason that we are unable to explain certain phenomena.

It is necessary to acknowledge the existence of the 4th space dimension and to move to $5D$ spacetime in order to better understand reality. The $5D$ spacetime is called the “conformal space” or “conformal geometry”. The basic geometric elements of this space are spheres and circles: a point is regarded as a sphere of radius 0, a plane as a sphere of radius ∞ passing through ∞, and a line as a circle of radius ∞ passing through ∞. This means that the conformal geometry is a geometry of curved spacetime and hence general relativity.

Since the last term of equation [18.53] is a sextvector, even the $5D$ spacetime is not sufficient and it is necessary to work with $7D$ spacetime to get closer to the true nature of reality.

19. Unaccounted Power Volume Density

The Poynting theorem is

(1) $$\nabla \cdot \mathbf{P} = -\frac{\partial \mathbf{W}}{\partial t} - \mathbf{J} \cdot \mathbf{E}$$ [19.1]

The power area density [18.35] is

$$\langle \mathbf{P} \rangle_1 = \mathbf{EH} + c^2 \mathbf{DB}$$ [18.35]

2) $$\mathbf{P} = \mathbf{E} \times \mathbf{H} + c^2 (\mathbf{D} \times \mathbf{B})$$ [19.2]

3) $$\nabla \cdot \mathbf{P} = \nabla \cdot (\mathbf{E} \times \mathbf{H}) + c^2 \nabla \cdot (\mathbf{D} \times \mathbf{B})$$ [19.3]

According to Vector algebra

4) $$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = (\nabla \times \mathbf{A}) \cdot \mathbf{B} - (\nabla \times \mathbf{B}) \cdot \mathbf{A}$$ [19.4]

Therefore

5) $$\nabla \cdot \mathbf{P} = (\nabla \times \mathbf{E}) \cdot \mathbf{H} - (\nabla \times \mathbf{H}) \cdot \mathbf{E} + c^2 \left((\nabla \times \mathbf{D}) \cdot \mathbf{B} - (\nabla \times \mathbf{B}) \cdot \mathbf{D} \right)$$ [19.5]

On substituting the Ampère-Maxwell law [1.9] and [1.9a], and the dual Ampère-Maxwell law [1.50] and [1.50a] into equation [19.5] gives

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$ [1.9]

$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$ [1.9a]

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{V}$$ [1.50]

$$\nabla \times \mathbf{D} + \frac{1}{c^2} \frac{\partial \mathbf{H}}{\partial t} = \epsilon_0 \mathbf{V}$$ {1.50a]

(6) $$\nabla \cdot \mathbf{P} = \left(-\frac{\partial \mathbf{B}}{\partial t} + \mathbf{V}\right) \cdot \mathbf{H} - \left(\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}\right) \cdot \mathbf{E} + c^2\left(\left(-\frac{1}{c^2}\frac{\partial \mathbf{H}}{\partial t} + \epsilon_0 \mathbf{V}\right) \cdot \mathbf{B} - \left(\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}\right) \cdot \mathbf{D}\right)$$ [19.6]

(7) $$\nabla \cdot \mathbf{P} = -\frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{H} + \mathbf{V} \cdot \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{E} - \mathbf{J} \cdot \mathbf{E} - \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{B} + \epsilon_0 c^2 \mathbf{V} \cdot \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{D} - \mu_0 c^2 \mathbf{J} \cdot \mathbf{D}$$ [19.7]

(8) $$\nabla \cdot \mathbf{P} = -\frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{D} - \frac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{E} - \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{B} - \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{H} - \mathbf{J} \cdot \mathbf{E} - \mu_0 c^2 \mathbf{J} \cdot \mathbf{D} + \mathbf{V} \cdot \mathbf{H} + \epsilon_0 c^2 \mathbf{V} \cdot \mathbf{B}$$ [19.8]

(9) $$\nabla \cdot \mathbf{P} = -\left(\frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{D} + \frac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{E} + \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{B} + \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{H}\right) - \mathbf{J} \cdot (\mathbf{E} + \mu_0 c^2 \mathbf{D}) + \mathbf{V} \cdot (\mathbf{H} + \epsilon_0 c^2 \mathbf{B})$$ [19.9]

(10) $$\nabla \cdot \mathbf{P} = -\left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H}\right) - \mathbf{J} \cdot (\mathbf{E} + \mu_0 c^2 \mathbf{D}) + \mathbf{V} \cdot (\mathbf{H} + \epsilon_0 c^2 \mathbf{B})$$ [19.10]

(11) $$\nabla \cdot \mathbf{P} = -\left(2\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + 2\mu_0 \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H}\right) - \mathbf{J} \cdot (\mathbf{E} + \mathbf{E}) + \mathbf{V} \cdot (\mathbf{H} + \mathbf{H})$$ [19.11]

(12) $$\nabla \cdot \mathbf{P} = -2\left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H}\right) - 2\mathbf{J} \cdot \mathbf{E} + 2\mathbf{V} \cdot \mathbf{H}$$ [19.12]

The energy volume density [18.37] is

$$\langle \mathbf{W} \rangle_0 = \mathbf{ED} + \mathbf{HB}$$ [18.37]

(13) $$\mathbf{W} = \mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B}$$ [19.13]

(14) $$\frac{\partial \mathbf{W}}{\partial t} = \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{D} + \frac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{E} + \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{B} + \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{H}$$ [19.14]

(15) $$\frac{\partial \mathbf{W}}{\partial t} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H}$$ [19.15]

(16) $$\frac{\partial \mathbf{W}}{\partial t} = 2\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + 2\mu_0 \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H}$$ [19.16]

(17) $$\frac{\partial \mathbf{W}}{\partial t} = 2 \left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + \mu_0 \frac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H} \right)$$ [19.17]

On substituting equation [19.17} into equation [19.12] gives

(18) $$\nabla \cdot \mathbf{P} = -\frac{\partial \mathbf{W}}{\partial t} \; - \; 2\mathbf{J} \cdot \mathbf{E} + 2\mathbf{V} \cdot \mathbf{H}$$ [19.18]

(19) $$\nabla \cdot \mathbf{P} \; \left(\frac{AV}{m^3} \right) = -\frac{\partial \mathbf{W}}{\partial t} \left(\frac{AV}{m^3} \right) - \; 2\mathbf{J} \cdot \mathbf{E} \left(\frac{AV}{m^3} \right) + 2\mathbf{V} \cdot \mathbf{H} \left(\frac{AV}{m^3} \right)$$ [19.19]

where

$\nabla \cdot \mathbf{P}$ is the power volume density flow out of the volume

$\frac{\partial \mathbf{W}}{\partial t}$ is the rate of change of the energy volume density

$2\mathbf{J} \cdot \mathbf{E}$ is the power volume density consumed by the outward spiralling magnetic current

$2\mathbf{V} \cdot \mathbf{H}$ is the power volume density released by the inward spiralling electric potential

The equation [19.19] represents the **COMPLETE POYNTING THEOREM.** The unaccounted power volume density term $2\mathbf{V} \cdot \mathbf{H}$ can been seen as the **HEAVISIDE ENERGY FLOW** or the **ENERGY FROM SPACETIME**.

On substituting Ohms law [1.53] and the dual Ohm law [1.58] in equation [19.19] gives

$$\mathbf{J} = \sigma_0 \mathbf{E}$$ [1.53]

$$\mathbf{V} = \eta_0 \mathbf{H}$$ [1.58]

(20) $\nabla \cdot \mathbf{P} \ = -\dfrac{\partial \mathbf{W}}{\partial t} - 2\sigma_0 \mathbf{E} \cdot \mathbf{E} + 2\eta_0 \mathbf{H} \cdot \mathbf{H}$ [19.20]

(21) $\nabla \cdot \mathbf{P} = -\dfrac{\partial \mathbf{W}}{\partial t} - 2\sigma_0 \mathrm{E}^2 + 2\eta_0 \mathrm{H}^2$ [19.21]

On substituting equation [19.17] in equation [19.21] gives

(22) $\nabla \cdot \mathbf{P} = -2\left(\epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + \mu_0 \dfrac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H}\right) - 2\sigma_0 \mathrm{E}^2 + 2\eta_0 \mathrm{H}^2$ [19.22]

For materials, ,equation [19.22] becomes

23) $\nabla \cdot \mathbf{P} = -2\left(\epsilon \dfrac{\partial \mathbf{E}}{\partial t} \cdot \mathbf{E} + \mu \dfrac{\partial \mathbf{H}}{\partial t} \cdot \mathbf{H}\right) - 2\sigma \mathrm{E}^2 + 2\eta \mathrm{H}^2$ [19.23]

24) $\displaystyle\int_0^{\mathbf{E}} \mathbf{E} \ \mathrm{d}\mathbf{E} = \frac{1}{2}\mathbf{E}^2 \longrightarrow \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{E}^2\right)$ [19.24]

25) $\displaystyle\int_0^{\mathbf{H}} \mathbf{H} \ \mathrm{d}\mathbf{H} = \frac{1}{2}\mathbf{H}^2 \longrightarrow \mathbf{H} \cdot \frac{\partial \mathbf{H}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{H}^2\right)$ [19.25]

On substituting equation [19.24] and [19.25] in equation [19.23] gives

26) $\nabla \cdot \mathbf{P} = -2\left(\epsilon \dfrac{\partial}{\partial t}\left(\dfrac{1}{2}\mathbf{E}^2\right) + \mu \dfrac{\partial}{\partial t}\left(\dfrac{1}{2}\mathbf{H}^2\right)\right) - 2\sigma \mathrm{E}^2 + 2\eta \mathrm{H}^2$ [19.26]

27) $\nabla \cdot \mathbf{P} = -\epsilon \dfrac{\partial \mathbf{E}^2}{\partial t} - \mu \dfrac{\partial \mathbf{H}^2}{\partial t} - 2\sigma \mathrm{E}^2 + 2\eta \mathrm{H}^2$ [19.27]

The electric energy stored in the capacitor is

28) $W = \displaystyle\iiint_V \left(\epsilon \mathbf{E}^2\right) \mathrm{d}V$ [19.28]

29) $$W = \left(\epsilon \frac{U^2}{d^2} \right) d \cdot A$$ [19.29]

30) $$W = \left(U^2 \right) \frac{\epsilon A}{d}$$ [19.30]

31) $$W = CU^2$$ [19.31]

where $C = \dfrac{\epsilon A}{d}$, is the capacitance of the capacitor

The magnetic energy stored in the inductor is

32) $$W = \iiint_V \left(\mu \mathbf{H}^2 \right) \mathrm{d}V$$ [19.32]

33) $$W = \left(\mu \frac{I^2}{s^2} \right) s \cdot A$$ [19.33]

34) $$W = \left(I^2 \right) \frac{\mu A}{s}$$ [19.34]

35) $$W = LI^2$$ [19.35]

where $L = \dfrac{\mu A}{s}$, is the inductance of the inductor loop

On substituting equation [19.31] and [19.35] in equation [19.27] gives

36) $$\nabla \cdot \mathbf{P} = - CU^2 - LI^2 - 2\sigma \mathsf{E}^2 + 2\eta \mathsf{H}^2$$ [19.36]

37) $$\underbrace{\nabla \cdot \mathbf{P}}_{\text{Power Volume Density}} = -\underbrace{CU^2}_{\text{Stored Electric Energy}} - \underbrace{LI^2}_{\text{Stored Magnetic Energy}} - \underbrace{2\sigma \mathrm{E}^2}_{\text{Energy Used by the Load}} + \underbrace{2\eta \mathrm{H}^2}_{\text{Total Energy Induced}}$$ [19.37]

The dual power area density [18.16] is

$$\mathbf{S}_{\perp(dual)} = c^2\mathbf{D} \times \mathbf{B}$$ [18.16]

38) $$\mathbf{S}_{\perp(dual)} = c^2\epsilon_0\mathbf{E} \times \mu_0\mathbf{H}$$ [19.38]

39) $$\mathbf{S}_{\perp(dual)} = c^2\epsilon_0\mu_0\mathbf{E} \times \mathbf{H}$$ [19.39]

40) $$\mathbf{S}_{\perp(dual)} = \mathbf{E} \times \mathbf{H}$$ [19.40]

Therefore

41) $$\mathbf{S}_{\perp} = \mathbf{S}_{\perp(dual)} = \mathbf{E} \times \mathbf{H}$$ [19.41]

The effects of $\mathbf{S}_{\perp(dual)}$ are much more pronounced and easily observable when we work with media that are polarisable by electric fields and magnetisable by magnetic fields. These material properties evoke additional fields in the media, polarisation $\mathbf{P}$ and magnetisation $\mathbf{M}$. The resulting total electric displacement field is:

42) $$\mathbf{D}_{\perp(dual)} = \epsilon_0\mathbf{E} + \mathbf{P}$$ [19.42]

For magnetic materials, the induction is the sum of the magnetic field H and magnetisation M:$\mathbf{B}_{\perp(dual)} = \mu_0(\mathbf{H} + \mathbf{M})$ [19.43]

43) $$\mathbf{S}_{\perp(dual)} = c^2(\epsilon_0\mathbf{E} + \mathbf{P}) \times \mu_0(\mathbf{H} + \mathbf{M})$$ [19.44]

44) $\mathbf{S}_{\perp(dual)} = c^2\left(\mu_0\epsilon_0\mathbf{E}\times\mathbf{H} + \mu_0\epsilon_0\mathbf{E}\times\mathbf{M} + \mu_0\mathbf{P}\times\mathbf{H} + \mu_0\mathbf{P}\times\mathbf{M})\right)$ [19.45]

45) $\mathbf{S}_{\perp(dual)} = \mathbf{E}\times\mathbf{H} + \mathbf{E}\times\mathbf{M} + \frac{1}{\epsilon_0}(\mathbf{P}\times\mathbf{H} + \mathbf{P}\times\mathbf{M})$ [19.46]

For Magnetic Materials

46) $\mathbf{S}_{\perp(dual)M} = \mathbf{E}\times\mathbf{H} + \mathbf{E}\times\mathbf{M}$ [19.47]

The second term $\mathbf{E}\times\mathbf{M}$ in equation [19.47] gives the additional unaccounted power density when we work with magnetic material, for example, in an electric motor.

For Polarised Media

47) $\mathbf{S}_{\perp(dual)P} = \mathbf{E}\times\mathbf{H} + \frac{1}{\epsilon_0}\mathbf{P}\times\mathbf{H}$ [19.48]

The second term $\frac{1}{\epsilon_0}\mathbf{P}\times\mathbf{H}$ in equation [19.48] gives the additional unaccounted power density when we work with polarised material for example a polarised capacitor.

The real power clif is

$$\mathbf{P} = \langle\mathbf{EH}\rangle_2 + c^2\langle\mathbf{DB}\rangle_2 + c^2\langle\mathbf{DB}\rangle_4 + c^2\langle\mathbf{DB}\rangle_6 \quad [18.42]$$

The quadvector and sextvector power terms are always present in reality irrespective of our view. This is one of the key reasons that the prevalent theory of electromagnetism cannot be corroborated with free energy devices and many experimental results and continues to plague human collective consciousness.

19.1. Additional Analysis

The equations of transformation are

$$\mathbf{E} = \mathbf{v}\times\mathbf{B} \quad [1.13]$$

$$\mathbf{H} = -\,\mathbf{v}\times\mathbf{D} \quad [1.14]$$

Since $\mathbf{v} \perp \mathbf{B}$

48) $\mathbf{E} = \mathbf{vB}$ [19.49]

Since $\mathbf{v} \perp \mathbf{D}$

49) $\mathbf{H} = -\,\mathbf{vD}$ [19.50]

On substituting equation [19.49] and {1950} in equation [18.35] gives

$$\langle \mathbf{P} \rangle_1 = \mathbf{EH} + c^2\mathbf{DB} \quad [18.35]$$

50) $\mathbf{P} = -\,v^2\mathbf{BD} + c^2\mathbf{DB}$ [19.51]

51) $\mathbf{P} = \left(1 - \dfrac{v^2}{c^2}\right)c^2\mathbf{DB}$ [19.52]

52) $\mathbf{P} = \left(1 - \dfrac{v^2}{c^2}\right)\mathbf{EH}$ [19.53]

For $v < c \longrightarrow \dfrac{v^2}{c^2} < 1$ in case of special relativity, the magnetic field [14.35] and electric field [14.36] are

$$\mathbf{H} = \left(\frac{1}{1 - \frac{v^2}{c^2}}\right)\mathbf{H}_0 \quad [14.35]$$

$$\mathbf{E} = \left(\frac{1}{1 - \frac{v^2}{c^2}}\right)\mathbf{E}_0 \quad [14.36]$$

Therefore

53) $$\mathbf{EH} = \left(\frac{1}{1-\frac{v^2}{c^2}}\right)^2 \mathbf{E}_0\mathbf{H}_0$$ [19.54]

On substituting equation [19.54] in equation [19.53] gives

54) $$\mathbf{P} = \left(\frac{1}{1-\frac{v^2}{c^2}}\right) \mathbf{E}_0\mathbf{H}_0$$ [19.55]

55) $$\mathbf{P} = \left(\frac{1}{1-\frac{v^2}{c^2}}\right) \mathbf{P}_0$$ [19.56]

For $v > c \longrightarrow \frac{v^2}{c^2} > 1$ in case of general relativity there will be an ever increasing electric and magnetic field, which generates ever increasing power, for example, in the case of a decaying electric field in the conductor [6.40] and a magnetic field in the non conductor [6.106]. This is precisely what happens in the RA device during the trailing edge of the pulse in the input coil.

$$\mathbf{v}_{Ec(decay)} = 1.618c$$ [6.40]

$$\mathbf{v}_{Hnc(decay)} = 1.618c$$ [6.106]

20. Scalar Potential and Magnetic Vector Potential

The scalar potential ϕ and magnetic vector potential $\mathbf{A}$ is missing from the comprehensive table of duality.

This is due the fact that when William Thomson introduced the concept of vector $\mathbf{A}$ potential, it was purely based on mathematics by appropriately satisfying the Gauss law with no so called magnetic monopoles.

(1) $\nabla \cdot \mathbf{B} = 0$ [20.1]

(2) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ [20.2]

This is analogous to imaging a circle as a straight line and trying to figure out properties of the circle and build it with a ruler and a square set. Once it is realised that the original entity is a circle, all its properties can be determined without fragmentation and it can be built perfectly with simple compass (right tools).

Similarly the scalar potential ϕ was introduced purely based on mathematics by appropriately satisfying the Coulomb law.

(3) $\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}$ [20.3]

(4) $\nabla \cdot (-\nabla\phi) = \frac{\rho_e}{\epsilon_0}$ [20.4]

(5) $\nabla^2\phi = -\frac{\rho_e}{\epsilon_0}$ [20.5]

Both the scalar potential ϕ and magnetic vector potential $\mathbf{A}$, are poor substitute for electric potential ϕ_h and electric voltage area density $\mathbf{V}$ respectively. They distort reality and obscure the true nature of things. However, they do indirectly measure them in convoluted way. They did evolve our understanding and served their purpose well until now in the absence of so called magnetic monopoles i.e electric potential $\phi_h$and electric voltage area density $\mathbf{V}$.

The scalar potential ϕ represented electric potential ϕ_p, which is a perfect analogue to magnetic charge q. Similarly, magnetic vector potential $\mathbf{A}$ represented the electric voltage area density $\mathbf{V}$, which is a perfect analogue of then magnetic current area density $\mathbf{J}$.

Table 20.1

Duality					
Electricity			**Magnetism**		
Description	**Symbol**	**Units**	**Description**	**Symbol**	**Units**
~~Scalar potential~~	~~ϕ~~	~~V~~			
Electric potential	ϕ_p	$Wb = Vs$	Magnetic charge	q	$C = As$
~~Magnetic A potential~~	~~$\mathbf{A}$~~	$\frac{Vs}{m}$			
Electric voltage area density	$\mathbf{V}$	$\frac{V}{m^2}$	Magnetic current area density	$\mathbf{J}$	$\frac{A}{m^2}$

With the discovery of law of duality and new unknown quantities i.e. the electric potential ϕ_p, electric voltage area density $\mathbf{V}$ and parallel interaction between the electric field and the magnetic field in conjunction with electromagnetism in curved spacetime using the appropriate mathematical tools such as the Clifford algebra, we would be able to a make quantum leap in our understanding of nature of reality.

21. Horst Eckardt

Horst Eckardt, PhD, was born in Lower Saxony, Germany. After graduating from high school in 1973, he studied physics at the Technical University of Clausthal-Zellerfeld. There, he received his master's degree and his doctorate in theoretical solid state physics. His doctoral thesis dealt with the calculation of the optical properties of metals using numerical calculation methods. Thus, he also acquired basic knowledge in programming and working with computers. In 1985 he then switched to industry, to Siemens AG in Munich. There he worked in research and development, initially in the field of computer architectures, later in hardware and software application areas.

On a private basis, Horst Eckardt dealt with the basics of alternative energy production, for which he was able to make good use of his knowledge in physics. As early as 2004 he recognized the great importance of the Einstein-Cartan-Evans (ECE) theory, which the Welsh physicist and chemist Myron Evans had been developing since 2003. From around 2005, Horst Eckardt actively contributed to the development of the ECE theory by introducing the use of computer algebra. This caused a significant acceleration in development and was a guarantee for the mathematical correctness of the results. Horst Eckardt then contributed to the further development of the content and became a co-author of most of the 450 scientific papers that Myron Evans produced until his sudden death in 2019. Horst Eckardt has recently expanded the theory so that it can also provide an answer to very fundamental natural-philosophical questions.

While working with Myron Evans, Horst Eckardt was appointed Director of the AIAS Institute (Alpha Institute for Advanced Studies), which Myron Evans founded in the 1990s. Horst Eckardt is also President of UPITEC (Unified Physics Institute of Technology), a non-profit association founded by Sean MacLachlan in the USA. Dr Eckardt is a leading scholar in the field of unified physics based on ECE theory, an area that has been pursued by practitioners for nearly 20 years. In addition to the development of the ECE theory, Horst Eckardt deals with the application of its results to technology, in particular to alternative methods of energy production. The AIAS Institute has developed various propositions for application in its articles. The aim of the institute is to promote technical progress in order to improve people's quality of life.

22. Anil Goel

Anil is the Founder and Creator of Ra Device - A Next Generation Renewable Energy Device.

Previously he was the Founder and Creator of Wayz - A Next Generation Mobility Platform and Ways - A Next Generation Tolling Platform. Ways Tolling Platform was commercially deployed at Rajiv Gandhi Sea Link Toll Plaza in Mumbai (attached MEP letter of appreciation and RGSL case study).

Previously he was the Founder and Creative Director of Meem Memory Cable. He was responsible for complete Product Development, Software and Hardware Development, ID and Interaction Design, Supply Chain, Manufacturing i.e. every aspect that is required to put the product into the box. He successfully converted his original concept into a fully certified product and has been granted patents for the same in EU, US and other countries.

With MEEM head office in London, Anil single handedly managed development office in Bangalore, design office in Milan and manufacturing in China. He also fostered strong relationships with top management in Toshiba, and other key partners.

Two of his innovations were Finalist in Bell Award 11th Edition (held in February 2021) in the Transport Tech Category namely Ways - Next generation tolling Platform (winner) and Wayz - Next Generation Mobility Platform (patent granted in UK).

He studied Post Graduate Diploma in Film & Television from University of Bristol, UK. He also holds an MBA from JBIMS, University of Bombay, and B.E. in Civil Engineering from Delhi College of Engineering, India.

Publications & Patents

1. **A method of collecting travel fares in a transport system**

   A Next Generation Mobility App to replace RFID card.

   Priority Date: 13 April 2018

   Status: **Granted in UK and US (Notice of Allowance).** Abandoned due to lack of funds in EU and Application Pending in India

2. **System and method of operating an email service for mobile telephone**

   A Next Generation Messenger App with mobile number as email ID.

   Priority Date: 19 December 2017

   Status: **Granted in UK, EU (Intention to Grant) and US (Notice of allowance)**. Application Pending in India

3. **Systems, methods and computer program product for operating electronic toll collection** Page 24

   A mobile App to prepay the toll fees for a journey and when the user vehicle arrives at the toll plaza, it is automatically recognised by its licence plate and opens the barrier gate if the toll fees has been prepaid.

   Priority Date: 19 April 2017

   Status: Applied only in India due to lack of funds. Just received the First Examination Report after 3 years with minor objections.

4. **Zero no-load usb power supply and a method for controlling the power consumption of a usb power supply**

   A USB charger that ejects the USB cable and switch itself off as soon as the mobile device is fully charged to save the vampire power.

   Priority Date: 1 October 2010

   Status: **Granted in EU and US**

5. **QWERTY keypad for handheld mobile computer**

   A ergonomic two thumb QWERTY keypad to significantly increase typing speed on mobile devices.

   Priority Date: 21 November 2007

   Status: Abandoned due to lack of funds

6. **Cable with memory**

   A USB cable that back-up all the data on the mobile device in the cable itself while charging.

   Priority Date: 30 October 2007

   Status: **Granted in EU, US and other countries**

23. Appendix

1) Mep - Letter of Appreciation
2) Graham Bell - Ways Tolling Award Certificate
3) Graham Bell - Wayz Mobility Award Certificate
4) Patent Grant Certificate - Wayz Mobility
5) US Patent Notice of Allowance - Wayz Mobility
6) Patent Grant Certificate - Xcess
7) US Patent Notice of Allowance - Xcess
8) Red Dot Awards-MEEM

23.1. Mep - Letter of Appreciation Mep

T +91 22 6741 2222 • B1 - 406, Boomerang, Chandivali Farm Road,
F +91 22 6741 2244 Near Chandivali Studio, Andheri (E), Mumbai - 400 072
E: investorrelations@mepinfra.com W: www.mepinfra.com CIN: L45200MH2002PLC136779

To Whosoever It May Concern

MEP Infrastructure has more than 18 years of experience in Toll Operations across India. Considering the complexity of the behaviour of Indian Road Users and challenges we faced every day to tackle those challenges, we worked very closely with Mr Anil Goel, who was developing the ANPR (Automatic Number Plate Reading) based toll collection system.

Based on our extensive experience in Toll Operations and working the Ways Tolling Platform, we can say that it is the best, quickest, convenient and most reliable toll collection system.

In India where there is no uniformity in vehicle number plates fonts, we achieved more than 95% accuracy in reading vehicle number plates. The system is tried and tested for a year at our RGSL toll plaza.

As per directives of MSRDC our principle employer for RGSL Toll plaza we had to stop using the Ways Tolling platform which required to implement and use Fastag RFID program only as per provision in our concessionaire agreement with MSRDC.

This event has no reflection on Ways Tolling Platform performance and Mr Anil Goel abilities to deliver on his commitments & contractual obligations to the finest degree.

We wish all the very best to Mr Anil Goel for his future ventures and look forward to work with him very soon.

Thanks

For MEP Infrastructure Developers Ltd

CEO (Toll Operations)
5th May 2021

mep INFRASTRUCTURE DEVELOPERS LTD.

23.2. Graham Bell - Ways Tolling Award Certificate

23.3. Graham Bell - Ways Tolling Award Certificate

Supported by

Ministry of Electronics and Information Technology Government of India

नीति आयोग

AEGIS GRAHAM BELL AWARDS

Award of Excellence

Supported by

Maharashtra State Innovation Society

Skill India

NICSI

FICCI

This is to certify that ANIL GOEL'S WAYZ NEXT GENERATION MOBILITY PLATFORM

is the finalist at the 11th Aegis Graham Bell Award 2020 in the category of

INNOVATION IN TRANSPORT TECH

Abhijit Gangopadhyay

Dean

Dr Abhijit Gangopadhyay

Initiative of

Aegis

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23.4. Patent Grant Certificate - Wayz Mobility

Intellectual Property Office

Certificate of Grant of Patent

Patent Number: GB2572816

Proprietor(s): Anil Goel

Inventor(s): Anil Goel

This is to Certify that, in accordance with the Patents Act 1977,

a Patent has been granted to the proprietor(s) for an invention entitled "**A method of collecting travel fares in a transport system**" disclosed in an application filed **13 April 2018.**

Dated 23 September 2020

Tim Moss
Comptroller-General of Patents, Designs and Trade Marks
Intellectual Property Office

The attention of the Proprietor(s) is drawn to the important notes overleaf.

Intellectual Property Office is an operating name of the Patent Office

23.5. US Patent Notice Of Allowance - Wayz Mobility

UNITED STATES PATENT AND TRADEMARK OFFICE

UNITED STATES DEPARTMENT OF COMMERCE
United States Patent and Trademark Office
Address: COMMISSIONER FOR PATENTS
P.O. Box 1450
Alexandria, Virginia 22313-1450
www.uspto.gov

NOTICE OF ALLOWANCE AND FEE(S) DUE

45812 7590 11/07/2022
Law Office of Michael D. Eisenberg
Intellectual Property Law
1991 Village Park Way
Suite 202C
Encinitas, CA 92024

EXAMINER	
VO, TUYEN KIM	
ART UNIT	PAPER NUMBER
2876	

DATE MAILED: 11/07/2022

APPLICATION NO.	FILING DATE	FIRST NAMED INVENTOR	ATTORNEY DOCKET NO.	CONFIRMATION NO.
17/047,288	10/13/2020	Anil Goel	GOEL-P003	7451

TITLE OF INVENTION: A method of collecting travel fares in a transport system

APPLN. TYPE	ENTITY STATUS	ISSUE FEE DUE	PUBLICATION FEE DUE	PREV. PAID ISSUE FEE	TOTAL FEE(S) DUE	DATE DUE
nonprovisional	MICRO	$300	$0.00	$0.00	$300	02/07/2023

THE APPLICATION IDENTIFIED ABOVE HAS BEEN EXAMINED AND IS ALLOWED FOR ISSUANCE AS A PATENT. PROSECUTION ON THE MERITS IS CLOSED. THIS NOTICE OF ALLOWANCE IS NOT A GRANT OF PATENT RIGHTS. THIS APPLICATION IS SUBJECT TO WITHDRAWAL FROM ISSUE AT THE INITIATIVE OF THE OFFICE OR UPON PETITION BY THE APPLICANT. SEE 37 CFR 1.313 AND MPEP 1308.

THE ISSUE FEE AND PUBLICATION FEE (IF REQUIRED) MUST BE PAID WITHIN THREE MONTHS FROM THE MAILING DATE OF THIS NOTICE OR THIS APPLICATION SHALL BE REGARDED AS ABANDONED. THIS STATUTORY PERIOD CANNOT BE EXTENDED. SEE 35 U.S.C. 151. THE ISSUE FEE DUE INDICATED ABOVE DOES NOT REFLECT A CREDIT FOR ANY PREVIOUSLY PAID ISSUE FEE IN THIS APPLICATION. IF AN ISSUE FEE HAS PREVIOUSLY BEEN PAID IN THIS APPLICATION (AS SHOWN ABOVE), THE RETURN OF PART B OF THIS FORM WILL BE CONSIDERED A REQUEST TO REAPPLY THE PREVIOUSLY PAID ISSUE FEE TOWARD THE ISSUE FEE NOW DUE.

HOW TO REPLY TO THIS NOTICE:

I. Review the ENTITY STATUS shown above. If the ENTITY STATUS is shown as SMALL or MICRO, verify whether entitlement to that entity status still applies.

If the ENTITY STATUS is the same as shown above, pay the TOTAL FEE(S) DUE shown above.

If the ENTITY STATUS is changed from that shown above, on PART B - FEE(S) TRANSMITTAL, complete section number 5 titled "Change in Entity Status (from status indicated above)".

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II. PART B - FEE(S) TRANSMITTAL, or its equivalent, must be completed and returned to the United States Patent and Trademark Office (USPTO) with your ISSUE FEE and PUBLICATION FEE (if required). If you are charging the fee(s) to your deposit account, section "4b" of Part B - Fee(s) Transmittal should be completed. If an equivalent of Part B is filed, a request to reapply a previously paid issue fee must be clearly made, and delays in processing may occur due to the difficulty in recognizing the paper as an equivalent of Part B.

III. All communications regarding this application must give the application number. Please direct all communications prior to issuance to Mail Stop ISSUE FEE unless advised to the contrary.

IMPORTANT REMINDER: Maintenance fees are due in utility patents issuing on applications filed on or after Dec. 12, 1980. It is patentee's responsibility to ensure timely payment of maintenance fees when due. More information is available at www.uspto.gov/PatentMaintenanceFees.

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PTOL-85 (Rev. 02/11)

23.6. Patent Grant Certificate - Xcess

Intellectual
Property
Office

Certificate of Grant of Patent

Patent Number:	GB2569784
Proprietor(s):	Anil Goel
Inventor(s):	Anil Goel

This is to Certify that, in accordance with the Patents Act 1977,

a Patent has been granted to the proprietor(s) for an invention entitled **"System and method of operating an email service for mobile telephones"** disclosed in an application filed **19 December 2017**.

Dated 1 January 2020

Tim Moss
Comptroller-General of Patents, Designs and Trade Marks
Intellectual Property Office

The attention of the Proprietor(s) is drawn to the important notes overleaf.

Intellectual Property Office is an operating name of the Patent Office

23.7. US Patent Notice Of Allowance - Wayz Mobility

UNITED STATES PATENT AND TRADEMARK OFFICE

UNITED STATES DEPARTMENT OF COMMERCE
United States Patent and Trademark Office
Address: COMMISSIONER FOR PATENTS
P.O. Box 1450
Alexandria, Virginia 22313-1450
www.uspto.gov

NOTICE OF ALLOWANCE AND FEE(S) DUE

15812 7590 08/18/2022
Law Office of Michael D. Eisenberg
Intellectual Property Law
1991 Village Park Way
Suite 202C
Encinitas, CA 92024

EXAMINER	
DU, ZONGHUA A	
ART UNIT	PAPER NUMBER
2448	

DATE MAILED: 08/18/2022

APPLICATION NO	FILING DATE	FIRST NAMED INVENTOR	ATTORNEY DOCKET NO	CONFIRMATION NO
16/955,245	06/18/2020	Anil Goel	GOEL-P002	1329

TITLE OF INVENTION: System And Method Of Operating An Email Service For Mobile Telephones

APPLN. TYPE	ENTITY STATUS	ISSUE FEE DUE	PUBLICATION FEE DUE	PREV. PAID ISSUE FEE	TOTAL FEE(S) DUE	DATE DUE
nonprovisional	MICRO	$300	$0.00	$0.00	$300	11/18/2022

THE APPLICATION IDENTIFIED ABOVE HAS BEEN EXAMINED AND IS ALLOWED FOR ISSUANCE AS A PATENT. PROSECUTION ON THE MERITS IS CLOSED. THIS NOTICE OF ALLOWANCE IS NOT A GRANT OF PATENT RIGHTS. THIS APPLICATION IS SUBJECT TO WITHDRAWAL FROM ISSUE AT THE INITIATIVE OF THE OFFICE OR UPON PETITION BY THE APPLICANT. SEE 37 CFR 1.313 AND MPEP 1308.

THE ISSUE FEE AND PUBLICATION FEE (IF REQUIRED) MUST BE PAID WITHIN THREE MONTHS FROM THE MAILING DATE OF THIS NOTICE OR THIS APPLICATION SHALL BE REGARDED AS ABANDONED. THIS STATUTORY PERIOD CANNOT BE EXTENDED. SEE 35 U.S.C. 151. THE ISSUE FEE DUE INDICATED ABOVE DOES NOT REFLECT A CREDIT FOR ANY PREVIOUSLY PAID ISSUE FEE IN THIS APPLICATION. IF AN ISSUE FEE HAS PREVIOUSLY BEEN PAID IN THIS APPLICATION (AS SHOWN ABOVE), THE RETURN OF PART B OF THIS FORM WILL BE CONSIDERED A REQUEST TO REAPPLY THE PREVIOUSLY PAID ISSUE FEE TOWARD THE ISSUE FEE NOW DUE.

HOW TO REPLY TO THIS NOTICE:

I. Review the ENTITY STATUS shown above. If the ENTITY STATUS is shown as SMALL or MICRO, verify whether entitlement to that entity status still applies.

If the ENTITY STATUS is the same as shown above, pay the TOTAL FEE(S) DUE shown above.

If the ENTITY STATUS is changed from that shown above, on PART B - FEE(S) TRANSMITTAL, complete section number 5 titled "Change in Entity Status (from status indicated above)".

For purposes of this notice, small entity fees are 1/2 the amount of undiscounted fees, and micro entity fees are 1/2 the amount of small entity fees.

II. PART B - FEE(S) TRANSMITTAL, or its equivalent, must be completed and returned to the United States Patent and Trademark Office (USPTO) with your ISSUE FEE and PUBLICATION FEE (if required). If you are charging the fee(s) to your deposit account, section "4b" of Part B - Fee(s) Transmittal should be completed. If an equivalent of Part B is filed, a request to reapply a previously paid issue fee must be clearly made, and delays in processing may occur due to the difficulty in recognizing the paper as an equivalent of Part B.

III. All communications regarding this application must give the application number. Please direct all communications prior to issuance to Mail Stop ISSUE FEE unless advised to the contrary.

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23.8. Red Dot Awards - MEEM

reddot network
members only

User Interface Design

MEEM

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MEEM, a charging cable and app, automatically backs up the data stored on a smartphone to the cable itself every time the phone's battery is charged. It can be used like any other charging cable, and neither a PC nor a cloud is needed. Backups according to individual settings are possible, comprising all content such as contacts, calendar, messages, photos, videos and music. Accidently deleted items or the entire phone's state of the last backup are easily restored. In addition, the app allows data to be selectively synced between different phones.

reddot award 2016
winner

Credits

Idea:
Anil Goel, AG, Delhi

Design:
jozeph forakis ... design, Milan

Client:
MEEM Memory Ltd, London

UI_/UX Design:
Stefanija Najdovska, MEEM Memory Ltd, Milan Takumi Yoshida, Simon Drexler, Jan-Cristoph Zoels, Experientia, Turin

Design Direction:
Jozeph Forakis