Chapter 14

Development of the Unified Field in SU(2) and SU(3) Representation Spaces

by

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Abstract

Unified field theory is developed from the tetrad postulate of Cartan geometry by first deducing the Einstein Cartan Evans (ECE) Lemma and then factorizing the d'Alembertian in SU(2) and SU(3) representation spaces using respectively the Pauli and a development of the Gell-Mann matrices. It is shown that both dynamics and electrodynamics may be developed as first order differential equations in both SU(2) and SU(3) representation spaces. This means that the fundamental origin of bosons and fermions is geometrical. The key to deducing the first order differential equations is to factorize the d'Alembertian operator of the ECE Lemma. The parity operator produces another differential equation with reversed momentum. In the SU(2) representation space this procedure produces a fermion equation that is simpler than the Dirac equation, but equivalent to it. In the SU(3) representation space a novel equation is produced, both for dynamics and electrodynamics.

Keywords: ECE Lemma, SU(2) and SU(3) equations of the unified field.

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14.1 Introduction

Recently in this series of papers on the Einstein Cartan Evans (ECE) unified field theory [1]- [10] the equation of the fermion and anti-fermion have been deduced using the 2 x 2 Pauli matrices rather than the 4 x 4 Dirac matrices. The fundamental ECE postulate was used [11] to show that the equations of electrodynamics can be expressed in an SU(2) representation space, suggesting that there is a more fundamental meaning to the boson and fermion than hitherto suspected. In the twentieth century physics the term "fermion" was restricted to particles described by equations such as that of Dirac, the most well known examples being the electron and positron. The photon was described as a boson, and never developed in an SU(2) representation space. The use of the SU(n) representation space, n = 3, 4, 5, 6, was restricted to quark theory. The simplest three-quark theory was described with an SU(3) representation space [12]. In ECE theory there is only one field of force, the unified field, so it follows that the four fundamental fields that were thought to exist in the now obsolete standard model can be unified geometrically, and described in ECE theory in any representation space.

It is shown in Section 2 that the key to this procedure is again geometrical, the factorization of the d'Alembertian operator that appears in the ECE Lemma, itself deduced [1]- [10] from the very fundamental tetrad postulate [13], [14] of differential geometry. Using this method, the ECE Lemma can be factorized into two first order differential equations which are parity reversed images. In the SU(2) representation space these are the equations of the ECE fermion and anti-fermion. However, these equations hold not only for the fermion field but also for quantized electrodynamics, so the photon may be described by them. These equations also hold for the gravitational field, so they also describe the graviton.

In Section 3 this development is extended to the SU(3) representation space, the d'Alembertian is factorized using well defined combinations of the Gell-Mann matrices. It is shown that the ECE Lemma may be factorized in three different ways, giving six novel, first order differential equations in three parity reversed pairs. Not only can these be applied to the strong nuclear field (quark theory) but also to dynamics and electrodynamics, and also to the electron and positron. Therefore all elementary particles are generated by the unified field, which is generated from the most fundamental concept of geometry, the tetrad postulate. This is an original approach to elementary particle theory which is self consistent and simpler than the standard model, with many novel possibilities.

14.2 Factorization of the d'Alembertian Operator

In ECE theory there is only one field of force, the four fundamental fields thought to exist in the standard physics are limits of the unified field. The latter can be developed in any representation space. The method adopted in this section can be introduced through the well known Einstein energy equation

$$p^\mu p_\mu = m^2c^2 \tag{14.1}$$

of classical special relativity. Using the operator equivalence:

$$p^\mu = i\hbar\partial^\mu \tag{14.2}$$

Eq. (14.1) becomes:

$$(\Box + \kappa^2)\psi = 0 \tag{14.3}$$

where ψ is an eigenfunction and where:

$$\kappa = mc/\hbar \tag{14.4}$$

is the Compton wavenumber. Here m is mass, h is the reduced Planck constant, and c is the vacuum speed of light, considered to be a universal constant. Eq. (14.3) is a limit of the ECE Lemma and wave equation of unified field theory [1] - [10]. Therefore the wave equations of physics can be obtained from geometry in the manner of general relativity. The most fundamental theorem of differential geometry is the well known tetrad postulate [1]- [10]:

$$D_\mu q^a_\sigma = 0 \tag{14.5}$$

where q^a_σ is the Cartan tetrad. The latter can be re-expressed [1]- [10] as the fundamental geometrical identity:

$$\Box q^a_\mu := R\, q^a_\mu \tag{14.6}$$

known as the ECE Lemma. Here, $\Box$ is the well known d'Alembertian operator:

$$\Box = \partial^\mu\partial_\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \tag{14.7}$$

and R is the well defined scalar curvature:

$$R = q^\lambda_a \partial^\mu (\Gamma^a_{\mu\lambda} - \omega^a_{\mu\lambda}). \tag{14.8}$$

Here, $\Gamma^a_{\mu\lambda}$ is the connection of Riemann geometry and $\omega^a_{\mu\lambda}$ is the spin connection of Cartan geometry. In order to transform this pure geometry into physics, the postulate is made that:

$$R = -kT \tag{14.9}$$

where the Einstein constant is retained and where T is proportional to energy momentum density. The use of the Einstein constant is all that remains from the now obsolete Einsteinian era of cosmology and gravitational physics. Therefore the well known ECE wave equation is expressed as:

$$(\Box + kT)q^a_\mu = 0 \tag{14.10}$$

and by the property of the d'Alembertian, is a second order wave equation.

The purpose of this section is to reduce the wave equation to a first order differential equation in SU(2) and SU(3) representation spaces. This procedure gives entirely novel equations of the unified field of force in physics.

To introduce these novel concepts consider the well known Dirac equation, which in the obsolete twentieth century physics was applied only to the fermion field. In condensed notation, the Dirac equation is:

$$(i\gamma^\mu \partial_\mu - \kappa)\psi = 0 \tag{14.11}$$

where γ^μ are the well known Dirac matrices [12]. These are 4 x 4 matrices made up of arrangements of 2 x 2 Pauli matrices. The Dirac equation can be thought of as a factorization of the d'Alembertian, because the latter can be expressed [12] in terms of Dirac matrices. However the Dirac equation contains more information than the wave equation of a fermion, Eq. (14.3). The reason is that the Dirac equation relates two senses of spin or handedness of the fermion. So if we extend this method to the unified ECE field, a lot of new physics will emerge.

Recently [1]- [10] it has been shown for the first time that the Dirac equation (14.11) can be written as an equation in the Pauli matrices without use of the Dirac matrices. This equation in 2 x 2 matrices may be expressed in terms of individual tetrad elements as:

$$\sigma^\mu p_\mu q_1^R = mc\sigma^0 q_1^L, \tag{14.12}$$

$$\sigma^\mu p_\mu q_2^R = mc\sigma^0 q_2^L. \tag{14.13}$$

Here σ^μ is a four vector made up of the four well known Pauli matrices [12]:

$$\sigma^\mu = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) \tag{14.14}$$

where:

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \ \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{14.15}$$

and p_μ is the covariant four-momentum:

$$p_\mu = (p_0, -\mathbf{p}). \tag{14.16}$$

Therefore:

$$\sigma^\mu p_\mu = \sigma^0 p_0 - \boldsymbol{\sigma} \cdot \mathbf{p} \tag{14.17}$$

where:

$$\boldsymbol{\sigma} = \sigma^1 \mathbf{i} + \sigma^2 \mathbf{j} + \sigma^3 \mathbf{k}, \tag{14.18}$$

$$\mathbf{p} = p_X \mathbf{i} + p_Y \mathbf{j} + p_Z \mathbf{k}. \tag{14.19}$$

Therefore equations (14.12) and (14.13) are:

$$(\sigma^0 p_0 - \boldsymbol{\sigma}\cdot\mathbf{p})q_1^R = mc\sigma^0 q_1^L, \tag{14.20}$$

$$(\sigma^0 p_0 - \boldsymbol{\sigma}\cdot\mathbf{p})q_2^R = mc\sigma^0 q_2^L. \tag{14.21}$$

Applying the parity inversion operator to these equation reverses the momentum **p** and reverses the handedness, so the right handed tetrad elements become left handed and vice versa. This procedure gives two more equations:

$$(\sigma^0 p_0 + \boldsymbol{\sigma}\cdot\mathbf{p})q_1^L = mc\sigma^0 q_1^R, \tag{14.22}$$

$$(\sigma^0 p_0 + \boldsymbol{\sigma}\cdot\mathbf{p})q_2^L = mc\sigma^0 q_2^R. \tag{14.23}$$

The overall effect is the following factorization:

$$\sigma^{02} p^\mu p_\mu = (\sigma^0 p_0 + \boldsymbol{\sigma}\cdot\mathbf{p})(\sigma^0 p_0 - \boldsymbol{\sigma}\cdot\mathbf{p}). \tag{14.24}$$

Using the fundamental operator equivalence of quantum mechanics, Eq. (14.2), then Eq. (14.24) becomes a factorization of the d'Alembertian operator:

$$\sigma^{02}\Box = (\frac{\sigma^0}{c}\frac{\partial}{\partial t} + \boldsymbol{\sigma}\cdot\boldsymbol{\nabla})(\frac{\sigma^0}{c}\frac{\partial}{\partial t} - \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}). \tag{14.25}$$

This is far from being a mathematical exercise, because the well known techniques of ESR, NMR and MRI arise from it. In ECE theory this technique can be applied not only to the fermion field but to the unified field, through the factorization of the ECE wave equation. This means that electrodynamics and dynamics can be expressed as subjects developed in a SU(2) representation space.

This concept may be extended to any representation space. For example the well known [12] SU(3) representation space is described by 3 x 3 matrices rather than the 2 x 2 Pauli matrices.

14.3 Development of the Unified Field in SU(3)

This representation space was used in the obsolete twentieth century physics in 3-quark theory [12], so was confined to the nuclear strong field only. If ECE theory is accepted there is only one field of force in nature, and this may be described by any mathematical representation space. This inference may lead to new, directly observable, effects in physics. In SU(3) the Pauli matrices are replaced by eight 3 x 3 matrices λ^a , known in nuclear physics as the Gell-Mann matrices [12]. They are related by:

$$[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}] = i\mathrm{f}_{abc}\frac{\lambda^c}{2} \tag{14.26}$$

where f_{abc} is the structure factor of the SU(3) group. The eight matrices are:

$$\lambda^1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; \lambda^2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; \lambda^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\lambda^4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \; \lambda^5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}, \; \lambda^6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \tag{14.27}$$
$$\lambda^7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \; \lambda^8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

and the structure factors [12] are:

$$f_{123}{=}1; \; f_{147}{=}-f_{156} = f_{246}{=}f_{257}{=}f_{345}{=}-f_{367}{=}\frac{1}{2}; \quad f_{458}{=}f_{628} = \sqrt{3}/2. \tag{14.28}$$

The d'Alembertian may be factorized using these matrices with the following procedure. Define the unit 3 x 3 matrix as:

$$\lambda^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14.29}$$

and expand it as:

$$2\lambda^0 = \lambda^{01} + \lambda^{02} + \lambda^{03} \tag{14.30}$$

where

$$\lambda^{01} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; \lambda^{02} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; \lambda^{03} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{14.31}$$

Analyze the λ^8 matrix as follows:

$$\lambda^8 = \frac{1}{\sqrt{3}}(\lambda^9 + \lambda^{10}) = \frac{1}{\sqrt{3}} \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \right). \tag{14.32}$$

Now define the vectors:

$$\boldsymbol{\alpha}^1 = \lambda^1 \mathbf{i} + \lambda^2 \mathbf{j} + \lambda^3 \mathbf{k}, \tag{14.33}$$

$$\boldsymbol{\alpha}^2 = \lambda^4 \mathbf{i} + \lambda^5 \mathbf{j} + \lambda^9 \mathbf{k}, \tag{14.34}$$

$$\boldsymbol{\alpha}^3 = \lambda^6 \mathbf{i} + \lambda^7 \mathbf{j} + \lambda^{10} \mathbf{k}. \tag{14.35}$$

Thus:

$$\boldsymbol{\alpha}^1 \cdot \mathbf{p} = \begin{bmatrix} p_Z & p_X - ip_Y & 0 \\ p_X + ip_Y & -p_Z & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{14.36}$$

$$\boldsymbol{\alpha}^2 \cdot \mathbf{p} = \begin{bmatrix} p_Z & 0 & p_X - ip_Y \\ 0 & 0 & 0 \\ p_X + ip_Y & 0 & -p_Z \end{bmatrix}, \tag{14.37}$$

$$\boldsymbol{\alpha}^3 \cdot \mathbf{p} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & p_Z & p_X - ip_Y \\ 0 & p_X + ip_Y & -p_Z \end{bmatrix}. \tag{14.38}$$

and

$$\begin{aligned} (\boldsymbol{\alpha}^1 \cdot \mathbf{p})(\boldsymbol{\alpha}^1 \cdot \mathbf{p}) &= p^2 \lambda^{01}, \\ (\boldsymbol{\alpha}^2 \cdot \mathbf{p})(\boldsymbol{\alpha}^2 \cdot \mathbf{p}) &= p^2 \lambda^{02}, \\ (\boldsymbol{\alpha}^3 \cdot \mathbf{p})(\boldsymbol{\alpha}^3 \cdot \mathbf{p}) &= p^2 \lambda^{03}, \end{aligned} \tag{14.39}$$

$$p^2 = {p_X}^2 + {p_Y}^2 + {p_Z}^2. \tag{14.40}$$

Therefore:

$$(\boldsymbol{\alpha}^1 \cdot \mathbf{p})(\boldsymbol{\alpha}^1 \cdot \mathbf{p}) + (\boldsymbol{\alpha}^2 \cdot \mathbf{p})(\boldsymbol{\alpha}^2 \cdot \mathbf{p}) + (\boldsymbol{\alpha}^3 \cdot \mathbf{p})(\boldsymbol{\alpha}^3 \cdot \mathbf{p}) = 2p^2 \lambda^0 \tag{14.41}$$

where

$$p^2 \lambda^0 = ({p_X}^2 + {p_Y}^2 + {p_Z}^2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{14.42}$$

It follows that $p^\mu \ p_\mu$ may be factorized in three ways:

$$(\alpha^{0i})^2 p^\mu p_\mu = (\alpha^{0i} p - \boldsymbol{\alpha}^i \cdot \mathbf{p})(\alpha^{0i} p + \boldsymbol{\alpha}^i \cdot \mathbf{p}) \tag{14.43}$$

$$i = 1, 2, 3 \ ,$$

$$\alpha^{0i} = \lambda^{0i}. \tag{14.44}$$

To check this result add the three equations to give:

$$\begin{aligned} ((\alpha^{01})^2 + (\alpha^{02})^2 + (\alpha^{03})^2) p^\mu p_\mu = ((\alpha^{01})^2 + (\alpha^{02})^2 \\ +(\alpha^{03})^2)(p^2 - ((\boldsymbol{\alpha}^1 \cdot \mathbf{p})(\boldsymbol{\alpha}^1 \cdot \mathbf{p}) + (\boldsymbol{\alpha}^2 \cdot \mathbf{p})(\boldsymbol{\alpha}^2 \cdot \mathbf{p}) + (\boldsymbol{\alpha}^3 \cdot \mathbf{p})(\boldsymbol{\alpha}^3 \cdot \mathbf{p})) \end{aligned} \tag{14.45}$$

i.e.

$$2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} p^\mu p_\mu = 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} (p^2 - \mathbf{p} \cdot \mathbf{p}), \tag{14.46}$$

Q.E.D.

The factorization of the d'Alembertian is accomplished by using the operator equivalence of quantum mechanics:

$$p^\mu = (p_0, \mathbf{p}) = i\hbar\partial^\mu = (\frac{1}{c}\frac{\partial}{\partial t}, -\boldsymbol{\nabla}), \tag{14.47}$$

$$p_0 = \frac{i\hbar}{c}\frac{\partial}{\partial t},\ \mathbf{p} = -i\hbar\boldsymbol{\nabla}. \tag{14.48}$$

Therefore the contravariant covariant product of four momenta becomes:

$$p^\mu p_\mu = -\hbar^2\Box. \tag{14.49}$$

Therefore the d'Alembertian can be factorized in SU(3) as follows:

$$(\alpha^{0i})^2\Box = (\frac{\alpha^{0i}}{c}\frac{\partial}{\partial t} + \boldsymbol{\alpha}^i\cdot\boldsymbol{\nabla})(\frac{\alpha^{0i}}{c}\frac{\partial}{\partial t} - \boldsymbol{\alpha}^i\cdot\boldsymbol{\nabla}) \tag{14.50}$$

where i = 1, 2, 3, and where:

$$\alpha^{01} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \alpha^{02} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \alpha^{03} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{14.51}$$

Therefore Eq. (14.11) of the SU(2) representation of the unified field becomes six equations in three parity inverted pairs:

$$(\frac{\alpha^{0i}}{c}\frac{\partial}{\partial t} - \boldsymbol{\alpha}^i\cdot\boldsymbol{\nabla})\phi^R = m\,c\,\alpha^{0i}\phi^L \tag{14.52}$$

$$(\frac{\alpha^{0i}}{c}\frac{\partial}{\partial t} + \boldsymbol{\alpha}^i\cdot\boldsymbol{\nabla})\phi^L = m\,c\,\alpha^{0i}\phi^R \tag{14.53}$$

where i = 1, 2, 3. Here ϕ^L and ϕ^R must be three spinors:

$$\phi^R = [q_1^R, q_2^R, q_3^R], \tag{14.54}$$

$$\phi^L = [q_1^L, q_2^L, q_3^L]. \tag{14.55}$$

The electromagnetic field in SU(3) representation space is therefore:

$$A^R = A^{(0)}\phi^R, \tag{14.56}$$

$$A^L = A^{(0)}\phi^L, \tag{14.57}$$

and the gravitational field in SU(3) representation space is:

$$\Phi^R = \Phi^{(0)}\phi^R, \tag{14.58}$$

$$\Phi^L = \Phi^{(0)}\phi^L. \tag{14.59}$$

Similar expressions may be obtained for the electromagnetic and gravitational fields in SU(n) representation spaces, where n is an integer.

In summary therefore, there is only one field of force in nature, and that field may be developed from Cartan geometry by using the tetrad postulate to give a fundamental wave equation of geometry, the ECE Lemma. This Lemma may be further developed in SU(n) representation space by factorizing the d'Alembertian. This factorization produces fundamental first order differential equations of geometry. The philosophy of general relativity means that each of these geometrical equations has meaning in physics. A well known example is the SU(2) factorization, which produces the idea of a fermion, and in physics produces ESR, NMR, MRI and other effects. The reader is referred to the accompanying background notes to paper 136 for more details.

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