Development of fundamental dynamics from differential geometry

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Abstract

The fundamental quantities of classical dynamics, such as position, velocity and acceleration, are expressed in terms of the Cartan tetrad, and the first Cartan structure equation is used to develop classical dynamics. The velocity is defined as the covariant exterior derivative of the position, and the acceleration as the exterior covariant derivative of the velocity. The resulting terms are analysed and classified.

Keywords: ECE theory, classical dynamics, accelerations.

1. Introduction

In paper 55 of the ECE series of paper [1-10], novel concepts in classical dynamics were introduced by developing the fundamental concepts of dynamics in terms of the Cartan tetrads as the basis elements of the vectors. The first Cartan structure equation was used to produce a relativistic version of the Euler equation, and the Cartan Bianchi identity was used to produce relativistic Coriolis and centripetal accelerations. These new equations of dynamics and new accelerations are likely to be observable in cosmology, and in situations where Newtonian and Einsteinian dynamics are not applicable. An example is the whirlpool galaxy, where Einsteinian general relativity fails completely, but where ECE dynamics has recently succeeded in giving a basic description without the use of adjustable parameters such as "dark matter". Nothing more than that is claimed at this stage of development but the initial work [11,12] appears promising.

In Section 2, the fundamental argument is given for the use of the Cartan tetrad as a basis element for the fundamental quantities of classical dynamics, and for the use of the covariant exterior derivative of Cartan as the fundamental derivative operator of general relativity. In Section 3, the fundamental velocities and accelerations of dynamics are defined using the first Cartan structure equation. There are forty eight fundamental accelerations, and these are classified in terms of known Newtonian and Eulerian accelerations in given limits.

2. Development of the general vector field

The position vector in three dimensions may be developed [1-10] as:

$$\boldsymbol{r} = \boldsymbol{r}^{(1)} + \boldsymbol{r}^{(2)} + \boldsymbol{r}^{(3)} \tag{1}$$

where the indices are those of the complex circular basis:

$$a = (0), (1), (2), (3)$$

Eq. (1) may be written as:

$$r_\mu^a = r q_\mu^a \tag{2}$$

where q_μ^a is a mixed index tensor which can be identified as the Cartan tetrad. The latter is defined by:

$$V^a = q_\mu^a V^\mu \tag{3}$$

where V is the vector field [1-12]. In three dimensions:

$$a = (1), (2), (3) \tag{4}$$

$$\mu = 1, 2, 3 \tag{5}$$

The index a is that of the complex circular basis, and μ is that of any other set of basis vectors in three dimensions, such as the Cartesian basis, spherical polar basis, cylindrical polar basis or general curvilinear basis. It has been shown [13-16] that the general vector field in three dimensions may always be developed as:

$$V = \boldsymbol{V}^{(1)} + \boldsymbol{V}^{(2)} + \boldsymbol{V}^{(3)} \tag{6}$$

and this is an extension of the Helmholtz Theorem [17]. In three dimensions the unit vectors of the complex circular basis [1-10] may be expressed as follows in terms of the unit vectors of the Cartesian basis:

$$\boldsymbol{e}^{(1)} = \frac{1}{\sqrt{2}} (\mathbf{i} - i\, \mathbf{j}) \tag{7}$$
$$\boldsymbol{e}^{(2)} = \frac{1}{\sqrt{2}} (\mathbf{i} + i\, \mathbf{j}) \tag{8}$$
$$\boldsymbol{e}^{(3)} = \mathbf{k} \tag{9}$$

Thus, $q_X^{(1)}$,..., $q_Z^{(3)}$are scalar valued elements of q_μ^a . The existence of the basis elements comes from the fact that a vector such as the position vector $\boldsymbol{r}$may be expressed in two ways:

$$\begin{aligned}\boldsymbol{r} &= r^{(1)}\boldsymbol{e}^{(1)} + r^{(2)}\boldsymbol{e}^{(2)} + r^{(3)}\boldsymbol{e}^{(3)} \\ &= r_X\, \mathbf{i} + r_Y\, \mathbf{j} + r_Z\, \mathbf{k}\end{aligned} \tag{10}$$

This is a three dimensional example of the tetrad postulate of Cartan's differential geometry in n dimensions [1-10], the complete vector field $\boldsymbol{r}$ in Eq. (10) is the same. The elements $q_X^{(1)}$,..., $q_Z^{(3)}$are therefore elements of a basis set. For example, the scalar valued elements from the definition of the complex circular basis, Eq. (7), are:

$$e_X^{(1)} = \frac{1}{\sqrt{2}} , \quad e_Y^{(1)} = -\frac{1}{\sqrt{2}} \tag{11}$$

Having identified these basis elements are Cartan tetrads, the principles of Cartan geometry may be used to define the torsion form from the tetrad form:

$$T^a = d \wedge q^a + \omega_b^a \wedge q^b \tag{12}$$

where ω_b^a is the spin connection of Cartan. The right hand side of Eq. (12) is the exterior covariant derivative:

$$T^a = D \wedge q^a \tag{13}$$

which is the most general type of derivative in Cartan's differential geometry. The Cartan torsion defined in Eq. (12) is a fundamental property in n dimensions, and is therefore a fundamental property in three dimensions, and in the four dimensional spacetime of relativity. If the D ^ derivative is applied to the torsion, it produces a cyclic sum of Cartan curvature forms through the Cartan Bianchi identity as follows:

$$D \wedge \mathrm{T}^a := R_b^a \wedge q^b \tag{14}$$

where the Cartan curvature form is defined [1-12] as:

$$R^a_b = D \wedge \omega^a_b \tag{15}$$

The fundamental quantities in differential geometry are the tetrad, spin connection and exterior covariant derivative. These produce other fundamental quantities, the torsion and curvature.

3. Development of classical dynamics

Classical dynamics is developed from Cartan's differential geometry. As in Paper 55 of this series (www.aias.us), the fundamental quantities of dynamics are expressed as:

$$Q^a_\mu = Q\, q^a_\mu \tag{16}$$

each quantity is a scalar valued factor Q multiplied by a tetrad. The fundamental derivative operator of dynamics is the fundamental derivative operator of differential geometry, which is D ^ . Therefore the position, velocity and acceleration are tetrads, i.e. are vector valued one-forms:

$$v^a_\mu = v\, q^a_\mu \quad , \tag{17}$$

$$a^a_\mu = a\, q^a_\mu \quad , \tag{18}$$

$$r^a_\mu = r\, q^a_\mu \tag{19}$$

The velocity can also be expressed as a vector valued two form proportional to the Cartan torsion:

$$v^a_{\mu\nu} = c\,(D \wedge \mathrm{r}^a)_{\mu\nu} \tag{20}$$

where c is the speed of light in a vacuum. The reason for Eq. (20) is that the velocity is the derivative of the position tetrad, and the derivative operator must be D ∧ . Therefore velocity in tensor notation is:

$$v^a_{\mu\nu} = c\,(\partial_\mu\, r^a_\nu - \partial_\nu\, r^a_\mu + \omega^a_{\mu b}\, r^b_\nu - \omega^a_{\nu b}\, r^b_\mu\,) \tag{21}$$

where by definition:

$$v^a_{\mu\nu} = -\, v^a_{\nu\mu} \tag{22}$$

In the four dimensional spacetime of general relativity the indices are:

$$a = (0), (1), (2), (3) \tag{23}$$

$$\mu = 0, 1, 2, 3 \tag{24}$$

Since $v^a_{\mu\nu}$ is antisymmetric it may be written as:

$$v^a_{\mu\nu} = \begin{pmatrix} 0 & -v^a_X & -v^a_Y & -v^a_Z \\ v^a_X & 0 & -w^a_Z & w^a_Y \\ v^a_Y & w^a_Z & 0 & -w^a_X \\ v^a_Z & -w^a_Y & w^a_X & 0 \end{pmatrix} \tag{25}$$

in analogy to the antisymmetric field tensor in ECE electrodynamics. Since ECE is a unified field theory the structures of dynamics and electrodynamics are both based on the same geometry, so tensorial quantities such as (25) have the same structure. The position tetrad is:

$$r^a_\mu = (r^a_0, -\mathbf{r}^a) \tag{26}$$

and the partial derivative in four dimensions is:

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, \nabla \right) \tag{27}$$

The spin connection in four dimensions is:

$$\omega^a_{\mu b} = (\omega^a_{0b}, -\boldsymbol{\omega}^a_b) \tag{28}$$

With these definitions, Eq.(21) becomes two vector equations. Eq. (25) defines the spacelike components of $v^a_{\mu\nu}$ as follows:

$$\begin{aligned} v^a_X &= -v^a_{01} \quad , \quad v^a_{12} = -w^a_Z \quad , \\ v^a_Y &= -v^a_{02} \quad , \quad v^a_{13} = w^a_Y \quad , \\ v^a_Z &= -v^a_{03} \quad , \quad v^a_{23} = -w^a_X \quad . \end{aligned} \tag{29}$$

Since these are spacelike, then the indices a in Eq. (29) are also restricted to the spacelike:

$$a = (1), (2), (3) \tag{30}$$

The orbital part of this development is:

$$v_{01}^a = c(\partial_0 r_i^a - \partial_i r_0^a + \omega_{0b}^a r_i^b - \omega_{ib}^a r_0^b) \tag{31}$$

$i = 1, 2, 3$

and in vector notation the orbital velocity is:

$$\boldsymbol{v}^a = v_{01}^a \boldsymbol{i} + v_{02}^a \boldsymbol{j} + v_{03}^a \boldsymbol{k} \tag{32}$$

In vector component notation Eq. (31) is:

$$v_X^a = \frac{\partial r_X^a}{\partial t} + c\frac{\partial r_0^a}{\partial t} + c\omega_{0b}^a r_X^b - c\omega_{Xb}^a r_0^b \tag{33}$$

and so on. Therefore:

$$\boldsymbol{v}^a = \frac{\partial \boldsymbol{r}^a}{\partial t} + c\nabla r_0^a + c\omega_{0b}^a \boldsymbol{r}^b - c r_0^b \boldsymbol{\omega}_b^a \tag{34}$$

which in this theory is the most general expression for orbital velocity in classical dynamics.

The spin part of velocity is:

$$v_{ij}^a = c(\partial_i r_j^a - \partial_j r_i^a + \omega_{ib}^a r_j^b - \omega_{jb}^a r_i^b) \tag{35}$$

and the spin velocity vector is denoted as:

$$\boldsymbol{w}^a = w_{23}^a \boldsymbol{i} + w_{31}^a \boldsymbol{j} + w_{12}^a \boldsymbol{k} \tag{36}$$

In vector component notation Eq. (35) is:

$$w_z^a = c\left(\frac{\partial r_Y^a}{\partial X} - \frac{\partial r_X^a}{\partial Y} - \omega_{Yb}^a r_X^b + w_{Xb}^a r_Y^b\right) \tag{37}$$

and so on. Therefore:

$$\boldsymbol{w}^a = c(\nabla \times \boldsymbol{r}^a - \boldsymbol{\omega}_b^a \times \boldsymbol{r}^b) \tag{38}$$

is the most general expression for the spin velocity of dynamics in this theory. As shown in Paper 55, this expression for spin velocity is the relativistic generalization of the Euler equation. The latter does not enter into Einsteinian general relativity because the latter is restricted to linear acceleration.

It is seen that both $\boldsymbol{v}^a$ and $\boldsymbol{w}^a$ are vectors in three dimensions and are the spacelike components of the four vectors:

$$v_\mu^a = (v_0^a\ , -\boldsymbol{v}^a\ ) \tag{39}$$

$$w_\mu^a = (w_0^a\ , -\boldsymbol{w}^a\ ) \tag{40}$$

However, we know that both these vectors are also tetrads:

$$v_\mu^a = v\ q_\mu^a \quad , \tag{41}$$

$$w_\mu^a = w\ q_\mu^a \tag{42}$$

So there exist the timelike quantities:

$$v_\mu^{(0)} = (\ v_0^{(0)}\ , \boldsymbol{0}\ ) \tag{43}$$

$$w_\mu^{(0)} = (\ w_0^{(0)}\ , \boldsymbol{0}\ ) \tag{44}$$

which in this theory have a meaning in classical dynamics. The scalar valued $v_0^{(0)}$ and $\omega_0^{(0)}$ are related to energy. They generalize the well known concept of four momentum:

$$p_\mu = (\frac{E_n}{c}\ , -\boldsymbol{p}\ ) \tag{45}$$

Similarly, acceleration is defined from the velocity tetrads (39) and (40) using the D ^ derivative. There are therefore two types of acceleration two-forms:

$$a^a = c\ D \wedge v^a \tag{46}$$

and

$$\alpha^a = c\ D \wedge w^a \tag{47}$$

Therefore there are four types of acceleration derivable from the two equations (46) and (47).

Both a^a and α^a have orbital and spin parts as follows:

$$a_{orbital}^a = \frac{\partial \boldsymbol{v}^a}{\partial t} + c\ \nabla v_0^a + c\ \omega_{0b}^a \boldsymbol{v}^b - c\ v_0^b\ \boldsymbol{\omega}_b^a\ ,$$

$$a_{spin}^a = c(\nabla \times \boldsymbol{v}^a - \boldsymbol{\omega}_b^a \times \boldsymbol{v}^b\ )$$

$$\alpha_{orbital}^a = \frac{\partial \boldsymbol{w}^a}{\partial t} + c\ \nabla w_0^a + c\ \omega_{0b}^a \boldsymbol{w}^b - c\ w_0^b\ \boldsymbol{\omega}_b^a\ ,$$

$$\alpha_{spin}^a = c(\nabla \times \boldsymbol{w}^a - \boldsymbol{\omega}_b^a \times \boldsymbol{w}^b\ ) \tag{48}$$

where:

$$\boldsymbol{v}^a = \frac{\partial \boldsymbol{r}^a}{\partial t} + c\ \nabla r_0^a + c\ \omega_{0b}^a \boldsymbol{r}^b - c\ r_0^b\ \boldsymbol{\omega}_b^a \quad , \tag{49}$$

and

$$\boldsymbol{w}^a = c(\nabla \times \boldsymbol{r}^a - \boldsymbol{\omega}_b^a \times \boldsymbol{r}^b) \tag{50}$$

So in general there are forty eight types of acceleration in this theory, sixteen from Eq. (48a), sixteen from Eq. (48c), eight from Eq. (48b) and eight from Eq. (48d).

There are new fundamental accelerations that do not appear in Newtonian or Einsteinian dynamics, and there are accelerations that develop Eulerian dynamics into a theory of general relativity. Some of these accelerations have been analysed in previous papers of this series, and from an inspection of eqs. (48). From the structure of these equations it is seen that they contain in certain limits (see paper 55) the well known accelerations of classical dynamics such as the Coriolis and centripetal accelerations, and as in recent papers of this series [1-10] they contain the accelerations used in fluid dynamics. These terms are produced from the fundamental structure of differential geometry, and may be analysed in given limits and classified, thus developing a new approach to the subject of relativistic dynamics in general. Notably the new accelerations apply in areas where it is known that Newtonian and Einsteinian dynamics do not apply, for example whirlpool galaxies in cosmology. They can also be applied to electrodynamics directly. Through the minimal prescription, the four-momentum is proportional to the potential, and dynamical force is proportional to the electromagnetic field of force. Therefore in electrodynamics there are forty eight fundamental types of field of force in this theory. They are constrained by the antisymmetry laws of ECE theory [1-10] and the new property of Cartan geometry discovered in Paper 142 of this series. This theory will be developed systematically in forthcoming papers.

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