

# The effect of gravitation on the Thomas precession and Sagnac effect: The gyro gravimeter and Doppler gravimeter.

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## Abstract

The Orbital Theorem of ECE theory is used to derive the Minkowski metric and the metric that describes the effect of gravitation. The Sagnac effect in ECE theory is a rotating frame effect, as shown in previous papers of the ECE theory. This rotating frame is shown to be equivalent to rotating the above metrics using cylindrical polar coordinates. Rotating the Minkowski metric produces the Thomas precession, and the Sagnac effect is the Thomas precession for a null geodesic. Rotating the gravitational metric produces the effect of gravitation on the Sagnac effect. This effect is a gravitational red shift of the photon frequency in the Sagnac inteferometer. Using these results two instruments are suggested: the gyro gravimeter and the Doppler gyro gravimeter.

*Keywords:* ECE theory, Thomas precession, Sagnac effect, effect of gravitation on the Sagnac effect.

## 1. Introduction

The Orbital Theorem was introduced in paper 111 of this series [1-10] and is a special case of the Frobenius Theorem for spherically symmetric spacetimes. In paper 111 it was used to describe the relativistic Kepler problem without the Einstein field equation, which has been shown to be geometrically incorrect during the course of development of ECE theory. In this paper the Orbital Theorem is used in Section 2 to derive the Minkowski metric and the metric that describes the effect of gravitation in ECE theory. The Minkowski metric is rotated to derive the Thomas precession [11], and a simple interpretation given of the Thomas precession. The Sagnac effect is derived as the Thomas precession for the photon (null geodesic). In Section 3 the gravitational metric is rotated to give the effect of gravitation on the Sagnac shift, and this effect is shown to be the gravitational red shift of the frequency of the photon used in the Sagnac effect. Rotation of the metric is equivalent to the rotating frame explanation of the Sagnac effect developed in earlier papers of this series. The Sagnac effect is therefore an effect of general relativity. In Section 4, two instruments are suggested based on the effect of gravitation on the ring laser gyro - the gyro gravimeter and Doppler gyro gravimeter.

## 2. Derivation of metrics from the orbital theorem

The method of calculation is to start with the Frobenius Theorem defining the most general line element [1-10]. The Theorem of Orbits is a special case of the Frobenius Theorem for the spherically symmetric spacetime and gives the line element:

$$ds^2 = n(r) c^2 dt^2 - m(r) dr^2 - r^2 d\varphi^2 - dz^2 \quad (1)$$

where

$$n(r) = 1 + \frac{\mu}{R} \quad , \quad m(r) = \left(1 + \frac{\mu}{R}\right)^{-1} \quad . \quad (2)$$

The Minkowski metric is described by the special case:

$$\mu = 0 \quad (3)$$

and the gravitational metric by:

$$\mu = -\frac{2MG}{c^2} \quad (4)$$

where  $M$  is the mass of the gravitating object and where  $G$  is Newton's constant:

$$G = 6.6726 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad (5)$$

Here  $c$  is the vacuum speed of light and  $R$  is the distance to the gravitating mass  $M$ . The Thomas precession is defined by rotating the Minkowski metric as follows:

$$d\varphi' = d\varphi + \omega dt \quad (6)$$

$$v = \omega r$$

giving the metric:

$$\frac{ds'^2}{c^2} = \left(1 - \frac{v^2}{c^2}\right) dt^2 - 2 \left(\frac{r}{c}\right)^2 \omega d\varphi dt - \left(\frac{r}{c}\right)^2 d\varphi^2 - (dr^2 + dz^2) / c^2 . \quad (7)$$

On the simplest level the Thomas precession is the dilation of angle due to the Lorentz boost as follows:

$$\theta' = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \theta \quad (8)$$

so

$$\Delta \theta = \theta' - \theta = \theta \left( \left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1 \right) \quad (9)$$

The relativistic angular velocity is:

$$\Omega = \omega \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad (10)$$

because:

$$\Omega = \frac{d\theta'}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \frac{d\theta'}{dt} \quad (11)$$

and

$$\omega = \frac{d\theta}{dt} \quad (12)$$

In ECE theory the rotating metric becomes a rotating spacetime [1 -10].

The effect of gravitation on the Sagnac effect is found by rotating the gravitational metric:

$$ds^2 = \left(1 - \frac{2MG}{c^2 R}\right) c^2 dt^2 - \left(1 - \frac{2MG}{c^2 R}\right)^{-1} dr^2 - r^2 d\varphi^2 - dz^2 \quad (13)$$

to give the metric:

$$ds'^2 = \left( \left( 1 - \frac{2MG}{c^2 R} \right) c^2 - v^2 \right) dt^2 - 2 r^2 \omega d\varphi dt - r^2 d\varphi^2 - \left( 1 - \frac{2MG}{c^2 R} \right)^{-1} dr^2 - dz^2 \quad (14)$$

It is shown as follows that the Sagnac effect is the Thomas precession for the photon (null geodesic). Consider Eq. (7) in the case of the null geodesic and for the plane defined by:

$$ds' = dr = dz = 0 \quad (15)$$

Then:

$$\left( 1 - \frac{v^2}{c^2} \right) dt^2 = \left( \frac{r}{c} \right)^2 ( 2 \omega d\varphi dt + d\varphi^2 ) \quad (16)$$

i.e.:

$$A dt^2 - 2 \omega d\varphi dt - d\varphi^2 = 0 \quad (17)$$

where:

$$A = \left( \frac{c}{r} \right)^2 \left( 1 - \frac{v^2}{c^2} \right) \quad (18)$$

Eq. (17) is a quadratic of the type:

$$a x^2 + b x + c = 0 \quad (19)$$

so:

$$x = \frac{1}{2a} \left( -b \pm (b^2 - 4ac)^{1/2} \right) \quad (20)$$

Thus:

$$dt = \frac{1}{A} \left( \omega \pm (\omega^2 + A)^{1/2} \right) d\varphi \quad (21)$$

with:

$$v = \omega r \quad (22)$$

Here  $v$  is the tangential linear velocity,  $r$  the radius of the platform, and  $\omega$  the angular velocity of the platform.

Thus:

$$\omega^2 + A^2 = \frac{v^2}{c^2} + \left( \frac{c}{r} \right)^2 \left( 1 - \frac{v^2}{c^2} \right) = \left( \frac{c}{r} \right)^2 \quad (23)$$

and:

$$dt = \frac{(1 \pm v/c) r}{(1 - v^2/c^2) c} d\varphi \quad (24)$$

Now use:

$$(1 - v^2/c^2) = (1 - v/c) (1 + v/c) \quad (25)$$

so

$$dt = \frac{r/c}{(1 \pm v/c)} d\varphi \quad (26)$$

From Eq. (22):

$$dt = \frac{r/c}{(1 \pm \frac{r\omega}{c})} d\varphi = \frac{1}{\frac{c}{r} \pm \omega} d\varphi \quad (27)$$

Finally use:

$$\omega_0 = \frac{c}{r} \quad (28)$$

to obtain:

$$\frac{dt}{d\varphi} = \frac{1}{\omega_0 \pm \omega} \quad (29)$$

If

$$\int_0^{2\pi} d\varphi = 2\pi \quad (30)$$

then the time taken to cover a rotation of  $2\pi$  is:

$$t = \frac{2\pi}{\omega_0 \pm \omega} \quad (31)$$

which is the Sagnac effect. It is the null geodesic in a plane of the metric of the Thomas precession and so is the Thomas precession for the photon.

### 3. Effect of gravitation on the Sagnac shift

This is described by the null geodesic:

$$dt = \left(\frac{r}{c}\right)^2 \left(1 - \frac{v^2}{c^2} - \frac{2MG}{c^2 R}\right)^{-1} \left(\frac{v}{r} \pm \frac{c}{r} \left(1 - \frac{2MG}{c^2 R}\right)^{1/2}\right) d\varphi \quad (32)$$

which may be developed as follows:

$$dt = \frac{1}{\omega_0^2} (\omega \pm \omega_0 x) d\varphi / (x^2 - v^2/c^2) = \frac{1}{\omega_0^2} \left(\frac{\omega \pm \omega_0 x}{x^2 - (\frac{\omega}{\omega_0})^2}\right) d\varphi \quad (33)$$

where:

$$x = \left(1 - \frac{2MG}{c^2 R}\right)^{1/2} \quad (34)$$

Eq. (33) is:

$$\frac{dt}{d\varphi} = \frac{1}{x\omega_0 \pm \omega} \quad (35)$$

so:

$$t = \frac{2\pi}{x\omega_0 \pm \omega} \quad (36)$$

for a  $2\pi$  path. Therefore the effect of gravitation on the Sagnac shift is to produce the gravitational red shift:

$$\omega_0 \longrightarrow \left(1 - \frac{2MG}{c^2 R}\right)^{1/2} \omega_0 \quad (37)$$

a simple and self-consistent result, proving the correctness of the Orbital Theorem and method used. When a Sagnac interferometer is placed on the surface of the Earth, the photon of the interferometer is subjected to a tiny gravitational red shift which can be calculated as follows:

$$\left. \begin{aligned} M &= 5.98 \times 10^{24} \text{ kg} \\ G &= 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \\ c &= 3 \times 10^8 \text{ m s}^{-1} \\ R &= 6.37 \times 10^6 \text{ m} \end{aligned} \right\} \quad (38)$$

so

$$x = 1.39 \times 10^{-9} \quad (39)$$

Therefore the frequency of light in the vacuum of deep space is different from the frequency

of light on the Earth's surface by this tiny amount.

#### 4. Gyro gravimeter and Doppler gyro gravimeter

Under the conditions:

$$\omega_0 \gg \omega \quad (40)$$

and

$$\frac{2MG}{c^2 R} \ll 1 \quad (41)$$

the expression (36) is well approximated by:

$$\Delta t = 2\pi \left( \frac{1}{x\omega_0 - \omega} - \frac{1}{x\omega_0 + \omega} \right) \approx \frac{4\pi\omega}{\omega_0^2} \left( 1 + \frac{2MG}{c^2 R} \right) \quad (42)$$

and the relative shift due to a mass  $M$  placed a distance  $R$  away from the gravimeter is:

$$1 : \frac{2MG}{c^2 R} = 1.48 \times 10^{-27} \frac{M}{R} \quad (43)$$

If the Sagnac interferometer is placed one metre away from a mass of one kilogram, the frequency shift is one part in  $1.48 \times 10^{-27}$ . An instrument with a frequency resolution of this order would be able to detect the expected gravitational red shift. Such high accuracy instruments are available [12] and could therefore be used as gyro gravimeters. For example, such an instrument placed 100 metres away from a terrestrial mass of a million metric tonnes ( $10^9$  kilograms) would result in a relative frequency shift of  $1.48 \times 10^{-20}$ . This comes within range of contemporary high accuracy ring laser gyro's. The gyro gravimeter could be used to map mass distributions from an aircraft or satellite, using the effect of mass on guidance system designs.

A variation on this instrument uses the relativistic Doppler effect and is based on the Pound Rebka experiment [13]. Consider two high precision ring laser gyro's initially  $h$  apart. One moves away from the other at a velocity  $v$ . For gyro one:

$$\Delta t_1 \approx \left( \frac{4\pi\omega}{\omega_0^2} \right) \frac{1}{x_1^2} \quad (44)$$

where

$$x_1 = \left( 1 - \frac{2MG}{c^2(R+h)} \right)^{1/2} \quad (45)$$

and for gyro two:

$$\Delta t_2 \approx \left( \frac{4\pi\omega}{\omega_0^2} \right) \frac{1}{x_2^2} \quad (46)$$

where

$$x_2 = \left(1 - \frac{2MG}{c^2 R}\right)^{1/2} \quad (47)$$

Therefore:

$$\frac{\Delta t_1}{\Delta t_2} = \left(\frac{x_2}{x_1}\right)^2 \quad (48)$$

From the theory [1 -10] of the Pound Rebka experiment, there is a relativistic Doppler shift combined with gravitational shifts such that:

$$\left. \begin{aligned} \frac{\Delta t_2}{\Delta t_1} &= \frac{1+v/c}{1-v/c} = \frac{1-r_0/(R+h)}{1-r_0/R} \\ r_0 &= \frac{2MG}{c^2} \end{aligned} \right\} \quad (49)$$

This equation can be solved to find  $M/R^2$  in terms of  $v$  and  $h$ . From Eq. (49):

$$2 \frac{v}{c} = r_0 \left( \frac{1}{R} - \frac{1}{R+h} \right) \left( 1 - \frac{v}{c} \right) \quad (50)$$

If

$$v \ll c \quad (51)$$

then

$$2 \frac{v}{c} \approx \frac{hr_0}{R(R+h)} \quad (52)$$

If

$$h \ll R \quad (53)$$

then:

$$\frac{M}{R^2} \approx \frac{c}{G} \frac{v}{h} \quad (54)$$

$$\frac{M}{R^2} = 4.5 \times 10^{18} \frac{v}{h} \quad (55)$$

Knowing  $v$  and  $h$ , a map of  $M/R^2$  can be built up. Here  $R$  is the distance to the Earth's centre of mass from the lower gyro, corresponding to the receiver in the Pound Rebka experiment, and  $R+h$  is the distance from the Earth's centre of mass to the upper gyro, corresponding to the emitter in the Pound Rebka experiment. The upper gyro moves away from the lower gyro



at a velocity  $v$ . Eq. (55) can be regarded as a test of the Orbital Theorem.

A high accuracy speedometer can be designed using one part of Eq. (49) as follows:

$$\frac{\Delta t_2}{\Delta t_1} = \frac{1+v/c}{1-v/c} \quad (56)$$

If

$$v \ll c \quad (57)$$

then:

$$\frac{\Delta t_2}{\Delta t_1} \approx 1 + 2 \frac{v}{c} \quad (58)$$

There is a difference in the Sagnac shifts of one gyro moving with respect to another one, or simply in a moving gyro. Only one gyro is needed because the baseline Sagnac shift on the Earth's surface is known to very high accuracy [12].

A high accuracy altimeter can be developed from the equation:

$$\frac{\Delta t_2}{\Delta t_1} = \left(1 - \frac{r_0}{R+h}\right) \left(1 - \frac{r_0}{R}\right)^{-1} \quad (59)$$

Since

$$r_0 \ll R \quad (60)$$

this equation is

$$\frac{\Delta t_2}{\Delta t_1} \approx 1 + \frac{hr_0}{R(R+h)} \quad (61)$$

to a very good approximation. If

$$h \ll R \quad (62)$$

then

$$\frac{\Delta t_2}{\Delta t_1} = 1 + \left(\frac{r_0}{R^2}\right) h = 1 + 2.18 \times 10^{-16} h \quad (63)$$

and  $h$  can be measured from the difference between a gyro placed at  $h$  above the Earth's surface and the reference gyro placed on the Earth's surface.

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